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PROBLEMS WITH SPONTANEOUS BREAKDOWN IN
REGGEON FIELD THEORIES

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A B S T R A C T

We prove a theorem which states that if a Reggeon field theory undergoes spontaneous breakdown as a result of the Pomeron intercept being above one, then the shifted Lagrangian must in general develop terms which are non-canonical from the point of view of Reggeon Lagrangians. We discuss the interpretation of these terms and point out some of the problems associated with them.

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1. INTRODUCTION

There has been considerable interest recently in Gribov's Reggeon calculus¹⁾ as a theory of high-energy scattering consistent with cross-channel unitarity. Much work has been done studying the possibility that t-channel effects renormalize the Pomeron intercept to $\alpha_p(0) = 1$. In this case, the theory looks like a field theory with zero mass gap¹⁾, or a statistical system near its critical point, and renormalization group and critical phenomenon²⁾ ideas can be used to sum all t-channel cut iterations. This makes possible calculations³⁻⁶⁾ of the critical exponents of the theory which determine the rise in the total cross-section and other aspects of the asymptotic behaviour of diffractive and non-diffractive processes. The applicability of universality to this theory makes it a particularly attractive possibility. Other papers⁷⁾ have discussed the Reggeon calculus with $\alpha_p(0) > 1$, arguing that s-channel iterations generate "grey disc" behaviour with $\sigma_{\text{tot}} \sim (\ln s)^2$. It has been shown that such behaviour is characteristic of a first-order phase transition while the standard Reggeon calculus with $\alpha_p(0) = 1$ is expected to exhibit the behaviour of a second-order phase transition⁸⁾.

Quite apart from phenomenological considerations, it is important to find theoretical reasons for preferring one of these options over the other, particularly in the absence of group symmetrical or field theoretical arguments for the *a priori* exceptional cases $\alpha_p(0) = 1$. No compelling symmetry or gauge invariance has yet emerged, although it has been suggested⁹⁻¹¹⁾ that since $\alpha_p(0) > 1$ corresponds, in the field theory formulation, to a negative (mass)², a spontaneous breakdown to a stable vacuum may enforce the condition $\alpha_p(0) \leq 1$, and automatically lead to cross-sections satisfying the Froissart bound.

In this paper we examine¹²⁾ this interesting possibility by first formulating a set of conditions which a Reggeon Lagrangian, like those derived from hybrid Feynman diagrams or topological expansions¹³⁾, might be expected to obey. We then show that a non-trivial vacuum shift of the field theory from one such Lagrangian to another is, in general, impossible. It thus constitutes a sort of no-go theorem, and like other famous such theorems¹⁴⁾, cries out to be evaded by a relaxation of the conditions under which it was derived.

When discussing the possibility of spontaneous breakdown, one encounters problems of interpretation. Should external particles be coupled to the Reggeon fields before the spontaneous breakdown shift, or to the shifted fields? In either case, is it possible to relax the conditions of the no-go theorem so as to allow a spontaneous breakdown and still have a sensible theory? As for the first question, we give arguments that the *unshifted* fields are coupled to external particles, based on the way particle scattering amplitudes are obtained from

Reggeon field theory Green's functions, and based on the interpretation of field-particle couplings in terms of hybrid Feynman graph ladders or topological (dual model) tubes. Unfortunately, whether it is the shifted or unshifted Reggeon fields which couple to external particles, certain problems remain if there is a spontaneous breakdown violating the conditions of the theorem. We will discuss these problems later, but it is worth mentioning that a way out of the dilemma may conceivably be related to the recent work of Alessandrini, Amati, Caneschi and Jengo¹⁵⁾.

Section 2 contains a derivation of the theorem about spontaneous breakdown, and Section 3 discusses its interpretation and implications.

2. SYMMETRIES OF THE THEORY AND THE FORM OF SPONTANEOUS BREAKDOWN

As originally proposed by Gribov¹⁾ and collaborators, we formulate the Reggeon calculus as an analogue field theory with a free Lagrangian:

$$\mathcal{L}_{\text{free}} = \frac{i}{2} \psi^+ \overleftrightarrow{\frac{\partial}{\partial \tau}} \psi - \alpha'_p \nabla \psi^+ \cdot \nabla \psi - \Delta_p \psi^+ \psi \quad (2.1)$$

where $\tau \equiv iY = i \ln s$ is a "time" variable and $\nabla \equiv \partial/\partial \underline{x}$ where $\underline{x}$ is a D-dimensional transverse space variable. The parameter α'_p is the Pomeron trajectory slope and $\Delta = 1 - \alpha_p(0)$ is the difference of the Pomeron intercept from unity. Corresponding to the free Lagrangian (2.1) is the free propagator

$$G_{\psi\psi^+}(E, t) \equiv i \int d^D \underline{x} d\tau e^{-iE\tau} e^{-i\underline{k} \cdot \underline{x}} \langle 0 | T(\psi^+(\underline{x}, \tau) \psi(0, 0)) | 0 \rangle$$

$$= \frac{i}{E - \alpha'_p \underline{k}^2 - \Delta_p + i\epsilon} \quad (2.2)$$

where $t = -\underline{k}^2$. Propagators $G_{\psi\psi} \equiv G_{2\psi}$ and $G_{\psi^+\psi^+} \equiv G_{2\psi^+}$ can be defined, involving the T-product of 2 ψ or ψ^+ fields. For the theory of Eq. (2.1) these propagators are zero, and the Pomeron propagates in a unique direction in time τ (rapidity Y).

The effects of t-channel unitarity are taken into account by adding to the Lagrangian an interaction term $\mathcal{L}_{\text{int}}(\psi^+, \psi)$ and then computing its consequences using field theory techniques such as perturbation theory¹⁾, Schwinger-Dyson equations, or the renormalization group^{3, 4)}.

To proceed further, we need to ask if there are any general conditions which should be satisfied by $\mathcal{L}_{\text{int}}$ in order for the field theory to make sense. In particular, what is the meaning of a bare Pomeron with $\alpha_p(0) > 1$? Our point of view is as follows: Suppose we understood the underlying field theory of

the strong interactions. We could then proceed *à la* Gribov¹⁾ and sum certain infinite subsets of Feynman diagrams to generate the bare Pomeron of the Reggeon calculus^{*)}. Proceeding further, we tie these sums of graphs together in all possible ways allowed by the underlying field theory to produce Gribov's hybrid Feynman diagrams, which are in one to one correspondence with the diagrams generated by the Reggeon calculus Lagrangian. Hence $\mathcal{L}_{\text{int}}$ should contain only those terms which will generate the topologies expected from the underlying field theory. If the couplings of the underlying field theory were such that the bare Pomeron had $\alpha_p(0) > 1$, the Reggeon calculus perturbation theory might well be unstable, and we might have to shift the theory to a new vacuum. In this case we must distinguish between $\mathcal{L}_{\text{int}}^{(B)}$ (before the shift) and $\mathcal{L}_{\text{int}}^{(A)}$ (after the shift). In any event, it is clear that $\mathcal{L}_{\text{int}}^{(B)}$ should obey the constraints which follow from the Gribov analysis. These are:

- $\alpha)$ Terms in $\mathcal{L}_{\text{int}}$ which are even powers of ψ and ψ^+ must be purely real.

Examples are

$$(\psi^+\psi)^2, \quad \psi^+\psi^3, \quad \dots$$

- $\beta)$ Terms in $\mathcal{L}_{\text{int}}$ which are odd powers of ψ and ψ^+ must be purely imaginary.

An example is the familiar triple Reggeon coupling

$$ig \psi^+\psi^2. \quad (2.3)$$

This and the previous condition are imposed by the signature properties of multi-Regge amplitudes¹⁾ and can also be derived from cross-channel unitarity arguments¹⁶⁾.

- $\gamma)$ $\mathcal{L}_{\text{int}}$ must be symmetric under the interchange of ψ and ψ^+ . Thus the interaction (2.3) can only depend on the combinations

$$\psi^+\psi, \quad (\psi^+ + \psi).$$

This condition reflects the Lorentz symmetry of the underlying field theory, and preserves the interchangeability of target and projectile.

- $\delta)$ Every term in $\mathcal{L}_{\text{int}}$ which is not a c-number constant must contain a factor $\psi^+\psi$. This means, for instance, that the only allowed three-point coupling has the form

$$ig(\psi^+\psi)(\psi + \psi^+) \quad (2.4)$$

illustrated in Fig. 1.

*) Instead of a field theory formulated *à la* Gribov, one could imagine other theories underlying the Reggeon calculus. Candidates are dual models and topological classifications of Feynman graphs¹³⁾. These give the same form for $\mathcal{L}_{\text{int}}$ as does Gribov's analysis discussed below.

Clearly the free Lagrangian (2.1) also respects this condition. It disallows interactions of the form illustrated in Fig. 2. Such graphs never arise in the derivation of the Reggeon calculus from hybrid Feynman diagrams¹⁾ (or dual models¹³⁾). This reflects the understanding that Pomerons cannot simply pop out of the vacuum, but must have the external particles as their sources. What is the general form of $\mathcal{L}_{\text{int}}$ obeying the conditions (α) to (δ)? Clearly condition (γ) implies

$$\mathcal{L}_{\text{int}} = \mathcal{L}_{\text{int}}(\psi^+\psi, \psi + \psi^+) \quad (2.5)$$

whereas condition (δ) further implies

$$\mathcal{L}_{\text{int}} = (\psi^+\psi) \bar{\mathcal{L}}_{\text{int}}(\psi^+\psi, \psi + \psi^+) + \text{constant} \quad (2.6)$$

where $\bar{\mathcal{L}}_{\text{int}}$ is analytic in both its variables at the origin. In order to obey conditions (α) and (β) we must impose the further condition

$$\mathcal{L}_{\text{int}} = (\psi^+\psi) \bar{\mathcal{L}}_{\text{int}}(\psi^+\psi, i(\psi + \psi^+)) + \text{constant} \quad (2.7)$$

where $\mathcal{L}_{\text{int}}(y, z)$ is understood to be a real function analytic at the origin, i.e. a double power series with real coefficients. An alternative way of rewriting (2.7) is to use the variables ϕ, χ :

$$\psi = \phi + i\chi, \quad \psi^+ = \phi - i\chi \quad (2.8)$$

in terms of which (2.7) becomes

$$\mathcal{L}_{\text{int}} = (\phi^2 + \chi^2) \bar{\mathcal{L}}_{\text{int}}(\phi^2 + \chi^2, 2i\phi) + \text{constant} \quad (2.9)$$

We can now state our theorem as follows: If we require that conditions (α)-(δ) are obeyed by $\mathcal{L}^{(B)}$, (we call these properties $\alpha^{(B)}$ - $\delta^{(B)}$) then in general $\mathcal{L}^{(A)}$ must have the property that it violates at least one of these conditions $[\alpha^{(A)}$ - $\delta^{(A)}]$. There may be some exceptions to this theorem, but they will require very special forms of $\mathcal{L}^{(B)}$ as we shall see. We first prove the theorem, and then discuss its implications in Section 3.

We consider the most general possibility for a shift in the fields^{*)} written in the form

$$\left. \begin{aligned} \psi &\rightarrow \psi + i\lambda + i\mu \\ \psi^+ &\rightarrow \psi^+ + i\lambda - i\mu \end{aligned} \right\} \quad (2.10)$$

or

$$\left. \begin{aligned} \phi &\rightarrow \phi + i\lambda \\ \chi &\rightarrow \chi + \mu \end{aligned} \right\} \quad (2.11)$$

with no prejudice as to whether λ or μ are real or complex. It is convenient^{**)} to introduce the Fourier transform of the non-derivative Lagrangian terms $\mathcal{L}_{\text{int}} - \Delta_p \psi^+ \psi$:

$$\tilde{\mathcal{L}}(p, q) = \int d\phi d\chi e^{ip\phi} e^{iq\chi} (\mathcal{L}_{\text{int}}(\phi^2 + \chi^2, 2i\phi) - \Delta_p(\phi^2 + \chi^2)) \quad (2.12)$$

We notice that because of the symmetry properties of $\mathcal{L}_{\text{int}}$,

$$\tilde{\mathcal{L}}(p, q) = \tilde{\mathcal{L}}_S(p, q) + i \tilde{\mathcal{L}}_A(p, q) \quad (2.13)$$

where

$$\left. \begin{aligned} \tilde{\mathcal{L}}_S(p, q) &= \int d\phi d\chi \cos p\phi \cos q\chi \\ &\quad \left[\frac{1}{2} \mathcal{L}_{\text{int}}(\phi^2 + \chi^2, 2i\phi) + \frac{1}{2} \mathcal{L}_{\text{int}}(\phi^2 + \chi^2, -2i\phi) - \Delta_p(\phi^2 + \chi^2) \right] \\ \text{and} \\ \tilde{\mathcal{L}}_A(p, q) &= \int d\phi d\chi \sin p\phi \cos q\chi \\ &\quad \left[\frac{1}{2} \mathcal{L}_{\text{int}}(\phi^2 + \chi^2, 2i\phi) - \frac{1}{2} \mathcal{L}_{\text{int}}(\phi^2 + \chi^2, -2i\phi) \right] \end{aligned} \right\} \quad (2.14)$$

*) In the Reggeon field theory, the interaction Lagrangian is non-Hermitian, and so one might wonder about the meaning of the vacuum. Although it is not crucial for the discussion here, we comment that the vacuum can be defined as the highest saddle point of the generating functional in the complex ϕ and χ planes which can be reached by a deformation of the integrals $\delta\phi \delta\chi$ from the real axis. The direction of stability is the direction of steepest descent from the saddle point. This will be discussed in more detail elsewhere¹⁷⁾.

**) We thank Y. Aharonov for suggesting to us this Fourier transform trick.

Further, because of the reality properties of $\tilde{\mathcal{L}}_{\text{int}}$, $\tilde{\mathcal{L}}_S$ must be purely real and $\tilde{\mathcal{L}}_A$ purely imaginary, so that $\tilde{\mathcal{L}}$ itself is purely real as well as being symmetric in the variable q . The spontaneous shift (2.10) or (2.11) thus corresponds to the transformation

$$\tilde{\mathcal{L}}(p, q) \rightarrow \tilde{\mathcal{L}}(p, q) e^{p\lambda} e^{-iq\mu} \quad (2.15)$$

Now we see that the reality property of $\tilde{\mathcal{L}}$ before and after the transformation (2.15) implies that λ is purely real and μ is purely imaginary. Further, the symmetry of $\tilde{\mathcal{L}}$ in the q variable means that $\mu = 0$. Hence the only shift of the vacuum of the theory which retains the properties (α) to (δ) of the Reggeon field theory is

$$\phi \rightarrow \phi + i\lambda \quad \text{or} \quad (\psi, \psi^+) \rightarrow (\psi, \psi^+) + i\lambda \quad (2.16)$$

That is, the "real" part of the field takes on an imaginary vacuum expectation value. In this case the non-derivative Lagrangian terms are

$$\begin{aligned} & (\psi^+ \psi) \left[\tilde{\mathcal{L}}_{\text{int}}(\psi^+ \psi, i(\psi + \psi^+)) - \Delta_p \right] \rightarrow \\ & (\psi^+ \psi + i\lambda(\psi + \psi^+) - \lambda^2) \times \\ & \times \left[\tilde{\mathcal{L}}_{\text{int}}(\psi^+ \psi + i\lambda(\psi + \psi^+) - \lambda^2, i(\psi + \psi^+) - 2\lambda) - \Delta_p \right] \end{aligned} \quad (2.17)$$

If the transformed Lagrangian is to obey the condition (δ), it is apparent that apart from a possible constant piece,

$$\tilde{\mathcal{L}}_{\text{int}}(\psi^+ \psi + i\lambda(\psi + \psi^+) - \lambda^2, i(\psi + \psi^+) - 2\lambda) - \Delta_p$$

should contain a factor $(\psi^+ \psi)$ and hence, apart from a possible piece generating this constant term in $\mathcal{L}$, $\tilde{\mathcal{L}}_{\text{int}}[\psi^+ \psi, i(\psi + \psi^+)] - \Delta_p$ must contain a factor

$$(\psi^+ - i\lambda)(\psi - i\lambda) = (\psi^+ \psi - i\lambda(\psi + \psi^+) - \lambda^2) \quad (2.18)$$

Thus, under conditions $\alpha^{(B)}_{-\delta^{(B)}}$ and $\alpha^{(A)}_{-\delta^{(A)}}$ the most general form of a non-derivative Reggeon field theory Lagrangian which can undergo spontaneous breakdown to another Reggeon field theory Lagrangian is

$$(\psi^+\psi) \left[(\psi^+\psi - i\lambda(\psi^+\psi^+) - \lambda^2) \bar{\bar{L}}(\psi^+\psi, i(\psi^+\psi^+)) + \frac{M}{\lambda^2 + i\lambda(\psi^+\psi^+)} \right] \quad (2.19)$$

where $\bar{\bar{L}}(x,y)$ is a real function of x and y . Clearly Reggeon Lagrangians do not in general have the form (2.19), and so seem incapable of breaking down spontaneously to another pukka Reggeon Lagrangian obeying conditions (α) to (δ) . In particular, the simplest interaction Lagrangian with the dominant infrared behaviour

$$\mathcal{L}_{int} = ig(\psi^+\psi)(\psi^+\psi^+) \quad (2.20)$$

is not of the form (2.19).

Similar conclusions on the difficulty of spontaneous symmetry breakdown in the Reggeon field theory have been reached by S.A. Jackson¹⁵⁾. Conflicting conclusions have been reached by Abarbanel, Bronzan, Schwimmer and Sugar¹⁶⁾, who propose an $\mathcal{L}_{int}^{(A)}$ which does not satisfy conditions α - δ .

3. INTERPRETATION AND DISCUSSION

There are two questions of interpretation which we would like to discuss in the light of the theorem of the previous section. They are:

- i) How is spontaneous breakdown to be interpreted? In particular, do external particles couple to shifted or unshifted Reggeon fields?
- ii) How can the theorem be evaded, and spontaneous breakdown achieved by relaxing some of the conditions we assumed?

As far as the first question is concerned, it seems to us that external particles should be regarded as coupling to the *unshifted* fields. Suppose we consider the (2 to 2) scattering amplitude in Reggeon field theory, which can be written as

$$A = \sum_{n,m=1}^{\infty} g_n g_m^+ G_{n\psi, m\psi^+} \quad (3.1)$$

where g_n and g_m^+ are defined diagrammatically in Fig. 3. In Eq. (3.1) we understand $G_{n\psi, m\psi^+}$ to include both connected and disconnected parts of the (n,m)

Green's function. In the underlying models from which Reggeon field theory has been abstracted, such as hybrid Feynman diagrams or topological expansions of dual models, the couplings g_n and g_m^+ of Reggeons to external particles have precise meanings, as indicated in Fig. 4. For example, in Fig. 4a we see a ϕ^3 ladder with *bona fide* real intermediate particles coupling to the external particles. What happens now if the Reggeon vacuum is unstable and there is a spontaneous breakdown $\psi = \psi' + v$, $\psi^+ = \psi^{+'} + v_+$ with $\langle 0 | \psi' | 0 \rangle = \langle 0 | \psi^{+'} | 0 \rangle = 0$? The fields ψ' and $\psi^{+'}$ no longer have simple interpretations as ladders (Fig. 4a) or tubes (Fig. 4b), but the Green's functions $G_{n\psi', m\psi^{+'}}$ can easily be calculated using, for example, the standard generating functional formalism. The fields ψ' and $\psi^{+'}$ are then the elementary quantized excitations about the vacuum state. There is one difference from regular field theory (e.g. a spontaneously broken gauge theory): in that case there is (as far as we know?) no external source with a coupling to the fields specified from exterior considerations. In Reggeon field theory the g_n and g_m^+ are so specified, and we believe the correct procedure^{*)} to be to rewrite (3.1) as

$$A = \sum_{n,m} g'_n g^{+'}_m G_{n\psi', m\psi^{+'}} \quad (3.2)$$

where

$$\left. \begin{aligned} g'_n &= \sum_{N=1}^{\infty} \binom{N}{n} g_N v^{N-n} \\ g^{+'}_m &= \sum_{M=1}^{\infty} \binom{M}{m} g^+_M v_+^{M-m} \end{aligned} \right\} \quad (3.3)$$

The shift in Reggeon fields is then regarded as an intermediate but essential step in the calculation of the original amplitude A in Eq. (3.1).

It is apparent from the structure of Eq. (3.3) that, in general, g_n' and $g_m^{+'}$ may be non-zero. Also in a general spontaneous breakdown conditions (γ^A) and (δ^A) of Section 2 may be violated, so that some of the Green's functions $G_{n\psi', 0\psi^{+'}}$ and/or $G_{0\psi', m\psi^{+'}}$ may be non-zero. The unfamiliar menagerie of diagrams shown in Fig. 5 may then appear in the theory. Their appearance raises further problems of interpretation, but first we should discuss the general question of the evasion of the theorem of Section 2.

*) With a caveat mentioned later.

If relaxation of the conditions (α^A) to (δ^A) is considered, it seems that the most vulnerable conditions are (γ^A) and (δ^A) . Although the original derivations of (α) and (β) via hybrid Feynman diagrams are not applicable to the interactions of ψ' and $\psi^{+'}$, there are alternative derivations¹⁶⁾ based on the t-channel unitarity analysis of J-plane singularities. These are likely to be valid for interactions of ψ' and $\psi^{+'}$ at least as long as the shifted intercept is near 1.

A model has recently been proposed¹¹⁾ in which conditions (γ^A) and (δ^A) are violated by the presence of a ψ'^2 term in the shifted Lagrangian and the absence of a $\psi^{+'2}$ term. As mentioned above, we believe the external particles should be regarded as coupling to the unshifted fields, in which case the diagrams of Fig. 5 are non-zero. It is easy to check^{*)} that Fig. 5a makes a contribution $\propto s\delta(t)$ to the amplitude A, whereas Fig. 5b has no such pathological behaviour. It seems reasonable^{**) to subtract the graphs of Fig. 5a on the grounds that the top half of the diagram just represents a renormalization of the self-interactions of the external particle which has no physical relevance to scattering. It may also be possible to argue away all or part of Fig. 5b by similar reasoning.}

There is at least one further problem which arises if condition (δ^A) is violated and external particles couple to the shifted fields, which is typified by Fig. 6. In this diagram the vertex ψ'^2 acts outside the rapidity range $[0, Y]$ of the scattering process. As long as the s-channel decomposition of Fig. 6a is not attempted, this is just formal. However, if final-state particles are coupled to Reggeons via a $\psi^{+'}\psi'$ vertex¹⁸⁾, then the graph of Fig. 6b appears, where the particle may be emitted infinitely far from the range $[0, Y]$.

This problem does not arise if external particles are coupled to $\psi^+\psi$, but there is one further problem which does. If the shifted Lagrangian does not obey (γ^A) , then the amplitude A will not in general be symmetric under interchange of the target and projectile. Thus, for example, in the model with a ψ'^2 term the Green's functions $G_{2\psi', \psi^{+'}}$ and $G_{\psi', 2\psi^{+'}}$ are not symmetric^{***)}, and their couplings to the external particles are different if the unshifted fields are used as in Eqs. (3.1) and (3.2).

If the conditions (α^A) to (δ^A) are difficult to violate without raising serious problems of interpretation, how may the theorem be evaded so that t-channel unitarity enforces $\alpha_p(0) \leq 1$? One possibility is to discontinue the search for a homogeneous spontaneous breakdown, and look for soliton-like solutions. Alessandrini, Amati, Caneschi and Jengo¹⁵⁾ have formulated just such a model in a space with zero transverse dimensions.

*) With the aid of someone who wishes to remain anonymous.

**) Daniele Amati emphasized this point to us.

***) This may not be true in more general models -- Wojtek Zakrzewski, private communication.

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Figure captions

- Fig. 1 : The form of triple Reggeon interaction (2.4) allowed in the standard models¹⁾.
- Fig. 2 : Some multi-Pomeron interactions disallowed in the standard models.
- Fig. 3 : The (2-2) scattering amplitude and the couplings g_n, g_m^+ .
- Fig. 4 : The coupling g_1 in (a) Hybrid Feynman diagrams; (b) a topological expansion.
- Fig. 5 : Unfamiliar Reggeon diagrams (a) involving $g_0^{+'}$, and (b) involving a disconnected part of $G_{(n+1)\psi'\psi^{+'}}$.
- Fig. 6 : A contribution to the $g_1' g_2' G_{\psi', 2\psi^{+'}}$ portion of A.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6