CM-P00061559

ARCHIVES

Ref.TH.2044-CERN

MASSIVE μ PAIR PRODUCTION
IN A VECTOR FIELD THEORY MODEL

I.G. Halliday ^{*)}
CERN - Geneva

A B S T R A C T

We treat massive electrodynamics as a model for the production of massive μ pairs in high energy hadronic collisions. The dominant diagrams in perturbation theory are identified and analyzed. These graphs have an eikonal structure which leads to enormous cancellations in the two-particle inclusive cross-section but not in the n particle production cross-sections. A Drell-Yan structure appears in the inclusive cross-section but the particles accompanying the μ pairs have a very different structure compared to the parton model. The pionization region is no longer empty of particles as in simple parton models.

*) On leave of absence from Imperial College, London.

1. INTRODUCTION

In the following the production of massive μ pairs in high-energy collisions is discussed. We shall assume that these are produced only through their coupling to a virtual photon. As a model for the strong interactions, we consider a single heavy vector meson coupling to a spin- $\frac{1}{2}$ particle through a conserved current. This model has previously given interesting results for the proton electromagnetic form factor ¹⁾, fixed angle elastic scattering ²⁾, $\bar{p}p$ annihilation at fixed angle ³⁾, and the inclusive production of particles at large transverse momenta ⁴⁾. In view of the possible emergence of scaling breakdown in e^+e^- and ep collisions ⁵⁾, the non-scaling results obtained by Gribov and Lipatov ⁶⁾ should be kept in mind.

The processes mentioned above are, by no accident, precisely those which the parton model, in its various guises, attempts to explain. The great virtue of the parton model is that many of the above processes are linked together. Thus the electroproduction and neutrino scattering distributions of quarks and antiquarks are fed into the Drell-Yan ⁷⁾ formula to give the massive μ pair cross-section.

To understand the results of our vector model, we must clearly separate the inclusive cross-section and the production cross-sections, for example, $\bar{p}p \rightarrow \mu^+\mu^- + NV$. We choose $\bar{p}p$ as the initial channel purely for simplicity, and V refers to the vector meson.

The cross-section for producing NV 's is given in the leading log approximation by the dominant diagrams shown in Fig. 1. Thus we have two vector mesons U, L which do not cross over the μ pair vertex. All other vector mesons, shown by dotted lines, only connect sectors A or B to C or D. These vector mesons are doubly logarithmic. In other words, each new vector meson added increases the contribution to σ_N by a factor of $\log^2 s$. Diagrams where we add a vector meson between A and B or C and D only give a singly logarithmic factor of $\log s$ provided, as we are assuming, $Q^2(\mu^+\mu^-) \simeq \tau s$, with τ held fixed. The vector mesons giving doubly logarithmic terms fall into an eikonal form and the leading contributions to σ_N may be summed. They give a form (schematically)

$$\sigma_N \approx e^{-g^2 \log^2 s} \frac{(g^2 \log^2 s)^N}{N!} \sigma_B$$

where σ_B corresponds to the lowest order term in g^2 . Notice that this is a very small cross-section coming from the equivalent of the fast fall-off in the nucleon form factor at large t ¹⁾.

On summing over N , we find

$$\frac{d\sigma^{ind}}{dQ^2} \simeq \sigma_B \cdot 1$$

where the unassuming factor of 1 contains the result of all the above eikonal dynamics.

Another way of stating the above result is that if we calculate the inclusive μ pair cross-section, then order by order in perturbation theory the leading doubly logarithmic contributions all cancel. Although we will demonstrate that this cancellation proceeds far beyond the leading logarithmic contribution, we have not been able to prove it to all orders in g^2 and $\log^2 s$. However, the cancellations we have proved are sufficiently powerful that we are led to the hypothesis that they are exact.

We may compare this calculation with fixed angle pp scattering as in Fig. 2a. Here an eikonal vector meson starting on the line p_1 may end on p_2 , p_3 or p_4 . The vector meson, drawn as a wiggly line, carries the large momentum transfer t . In $\bar{p}p \rightarrow 2V$ at fixed angle and high energy, an eikonal meson starting on p_1 in Fig. 2b must end on p_2 or it loses its doubly logarithmic eikonal nature. In the elastic case two of the sets of contributions (p_2 , p_4) cancel, leaving the third. In each case we are left with one uncanceled set of terms which sum to $\exp(-g^2 \log^2 s)$ as $s \rightarrow \infty$ ⁸⁾.

Thus in the μ pair cross-section, the corrections to σ_B , we believe, come from among others the diagrams of the form shown in Fig. 3. Here our vector mesons do not cross the line joining the two genuine photon vertices. These have the form of the Drell-Yan graphs shown in Fig. 4. Again there are eikonal ($\log^2 s$) vector mesons running between A or B and C or D. Again these all cancel.

The diagrams for which we have been unable to prove the cancellations between the different leading eikonal contributions, first appear in eighth order. They are shown in Fig. 5. The difference now is that the diagrams are not left-right symmetric. However, these diagrams are two powers of $\log s$ smaller than the diagrams of Fig. 1 with two eikonal vector mesons. Thus we must be able to calculate the terms in Fig. 1 up to terms two powers of $\log s$ down on the dominant term. We have been unable to organize this calculation into a form where our "hoped for" cancellations manifestly occur.

Thus the results of our calculation may be expressed as follows. For $g^2 \log^2 s$ small, but finite, the inclusive cross-section is dominated by the graph of Fig. 1 with no eikonal vector mesons. The leading terms in $(g^2 \log^2 s)^n$ cancel and it is plausible, but unproven, that we actually only have a series in $g^2 \log s$ for the inclusive cross-section. Such contributions will certainly come from the graphs of Figs. 3 and 5. The open question is whether there are terms of the form $(g^2 \log s)^m$ coming from the non-cancelling parts of Fig. 1, etc., with eikonal vector mesons giving AD, etc.

The above does not apply to the fixed multiplicity cross-sections. The clean part of our calculation is therefore the calculation of these partial cross-sections. Here the $(g^2 \log^2 s)$ terms do not cancel.

Assuming complete cancellation of the eikonal vector meson contributions, we shall calculate the leading diagrams in a $g^2 \ln s$ expansion. This then takes an almost Drell-Yan form. The sum is also almost identical to that performed by Gribov and Lipatov for electroproduction.

In Section 2 we discuss the simple set of graphs of Fig. 3 which are of the Drell-Yan form, and examine the momentum space distribution of the particles accompanying the μ pair. In Section 3 we discuss the dominant graphs in the N body cross-sections and demonstrate the applicability of an eikonal technique. The massive cancellations occurring in the inclusive cross-sections are demonstrated and elaborated in Section 4. Finally, in Section 5 we sum the contributions of graphs with no eikonal mesons. Section 6 contains a final discussion of our results.

2. THE DRELL-YAN GRAPHS

The general idea of the Drell-Yan ⁷⁾ model is that the production of a massive photon in an inclusive cross-section can be visualized as in Fig. 4. In other words, the photon is produced only when a quark in $\underline{a}$ annihilates an antiquark in $\underline{b}$, or vice versa. The two bubbles are proton-quark (anti-quark) amplitudes which also appear in the scaling predictions for electroproduction.

The simplest such graph in our theory is shown in Fig. 6. There are other equally large graphs of the Drell-Yan form which we shall treat later. To minimize the trace complexity, we have considered only $\bar{p}p \rightarrow \gamma + (N \text{ vector mesons})$. This is merely for pedagogical ease.

The calculation of this graph, in the limit where $Q^2 = \tau s$ with τ a fixed constant < 1 , is essentially identical to the Gribov-Lipatov calculation of electroproduction. We repeat it here for completeness and contrast

with our later calculations. The momenta k_i, k'_j are parametrized by

$$\begin{aligned} k_i &= \alpha_i p_1 + \beta_i p_2 + \underline{k}_i \\ k'_j &= \gamma_j p_1 + \delta_j p_2 + \underline{\Lambda}_j \end{aligned} \quad (2.1)$$

where $\underline{k}_i, \underline{\Lambda}_i$ are the usual transverse momenta orthogonal to p_1, p_2 . If we insist on all the vector mesons being on mass shell, we find

$$(\alpha_i - \alpha_{i-1})(\beta_{i-1} - \beta_i)S - (\underline{k}_{i-1} - \underline{k}_i)^2 = m^2 \quad (2.2)$$

The dominant region of integration has $\underline{k}_i^2 \gg \underline{k}_{i-1}^2$ and $\alpha_i \sim \alpha_{i-1} \sim \text{constant}$. Then by (2.2) $|\beta_i| \gg |\beta_{i-1}|$ and

$$\beta_i = - \frac{m^2 + \underline{k}_i^2}{(\alpha_{i-1} - \alpha_i)S} = - \frac{\mu_i^2}{(\alpha_{i-1} - \alpha_i)S} \quad (2.3)$$

The β_i are clearly negative and $\alpha_{i-1} \geq \alpha_i$. The denominators of the nucleon propagators become

$$k_i^2 - m^2 \cong - \frac{\alpha_{i-1}}{(\alpha_{i-1} - \alpha_i)} \mu_i^2 \quad (2.4)$$

The trace obtained by summing over the nucleon spins gives us a contribution

$$\begin{aligned} 4.(2) & \prod_1^{m+n} \underline{k}_i^2 \prod_1^n \underline{\Lambda}_j^2 \left[p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu} p_1 \cdot p_2 \right] \\ &= \prod \underline{k}_i^2 \prod \underline{\Lambda}_j^2 \cdot F^{\mu\nu} \end{aligned} \quad (2.5)$$

Here μ, ν are the labels on the γ 's coupling the genuine photons.

From (2.4) and (2.5) we clearly have a logarithmic divergence in each of the $\underline{k}_i, \underline{\Lambda}_i$ integrals. Each of these gives rise to a $(\log s)$ factor.

Because of the strong ordering, if we write $Q = \alpha p_1 + \beta p_2 + \underline{\kappa}$, $\beta \sim \delta_n$, $\alpha \sim \alpha_m$. Thus if we fix α , β and $\underline{\kappa}$, we obtain

$$I \approx \left[\int_0^1 d\alpha_1 \frac{(1-\alpha_1)}{\alpha_1^2} \int_0^{\alpha_1} d\alpha_2 \frac{(\alpha_1-\alpha_2)}{\alpha_2^2} \dots \int d\alpha_m (\alpha_{m-1}-\alpha_m) \right. \\ \left. \delta(\alpha_m-\alpha) \int_0^{Q^2} \frac{d^2 K_m}{K_m^2} \int \frac{d^2 K_{m-1}}{K_{m-1}^2} \dots \int \frac{d^2 K_1}{K_1^2} \right] \times \\ \left[\text{Ditto } \alpha \leftrightarrow \delta, K \leftrightarrow \Lambda, m \leftrightarrow n \right] \frac{\bar{F}_{\mu\nu}}{s} \delta^2(\underline{K}_m + \underline{\Lambda}_m' - \underline{K})$$

(2.6)

The $\underline{\kappa}$ of the external photon is now integrated out using the δ function. Each $\underline{\kappa}_i, \underline{\Lambda}_j$ integral gives rise to an extra power of $\log s$. Thus finally we obtain $(m+n)$ powers of $\log s$. Notice that the α 's in the top half of the diagram are forced to be finite since we must carry a finite fraction α of p_1 down to the genuine photon. Notice that if $s \gg Q^2 \gg m^2$ so that $\tau = \alpha\beta \rightarrow 0$, this restriction would no longer hold and we would obtain logarithms from the α_i integrals as well as the $\underline{\kappa}_i$ integrals ⁴⁾.

The formula (2.6) is now in the Drell-Yan form. We must add the genuine photon propagator and the $\gamma \rightarrow \mu^+ \mu^-$ vertex and spin sum. This is left to the reader.

However, there are other graphs which will contribute at the same level as the ladder graphs. Two simple examples are shown in Fig. 5. Before starting this calculation, we stress that in the ladder calculation above, the trace of Eq. (2.5) only involved the $\underline{\kappa}_i$ from the nucleon propagators. In other words, the $g^{\mu\nu}$ of the vector meson propagators always ended up performing scalar products between $\underline{\kappa}_i$'s. This will not be true for Fig. 5. In Fig. 5a we must first all decide which of (ab) and (cd) will carry $\underline{\kappa}_i$ and which will carry p_1 or p_2 . We, arbitrarily, pick a, c to carry p_1 and p_2 , respectively. Now label the momenta by a, $\underline{\kappa}_m$, c, $\underline{\kappa}_n'$. Consider first the numerator of the $(p_1 - a)$ propagator. It

contains a term $[1 - \alpha(a)] \gamma \cdot p_1$. If this is commuted past the γ^μ at the left-hand end of the a line, it gives $\sim m$ on meeting $u(p_1)$. Thus it must be carried across the diagram by the $g^{\mu\nu}$ of a . The numerator of the $(k_m + a)$ line contains a term $[\beta_m + \beta(a)](\gamma \cdot p_2)$. On commuting with the above term, we obtain $[1 - \alpha(a)][\beta_m + \beta(a)]S$. However,

$$(k_m + a)^2 \simeq (\alpha_m + \alpha(a))(\beta_m + \beta(a))S$$

Therefore, if as before, we take the α 's finite, we see that the numerators from $(p_1 - a)$, $(k_m + a)$ effectively cancel $1/[(k_m + a)^2 - m^2]$. The term $(p_1 - a)^2$ in the denominator is now given by $-[1 - \alpha(a)]\beta(a) \cdot s - \kappa_a^2 \propto \kappa_a^2$, since $\beta_1 \ll \beta(a)$ if $\kappa_a^2 \gg 1$ and $\alpha(a) \sim 1$.

The k_m, k_n^1 propagators now behave as before and we obtain a very similar form to (2.6) with the square bracket replaced by

$$U = \int_0^{1-\alpha} \frac{d\alpha(a)}{(\alpha_m + \alpha(a))} \int_0^{1-\alpha(a)} d\alpha_m \frac{(1 - \alpha(a) - \alpha_m)}{1 - \alpha(a)} \delta(\alpha_m - \alpha) \int_{\sqrt{s}}^{\frac{d^2 K_a}{K_a^2}} \int_0^{K(a)} \frac{d^2 K_m}{K_m^2} \quad (2.7)$$

The crucial property of this diagram compared to the ladder diagrams is that the momenta in $(p_1 - a)$ and $(k_m + a)$ are predominantly p_1 with coefficients $[1 - \alpha(a)]$ and $[\alpha_m + \alpha(a)]$, respectively. The four-momenta squared are, however, very different

$$\begin{aligned} (p_1 - a)^2 &\simeq -K_a^2 / \alpha(a) \\ (k_m + a)^2 &\simeq -K_m^2 / (1 - \alpha(a) - \alpha_m) \end{aligned} \quad (2.8)$$

In (2.7) the dominant region has $Q^2 \gg \kappa_m^2 \gg \kappa_a^2$. The graph of Fig. 5b may be calculated in a similar way.

3. THE DOMINANT GRAPHS

We now wish to calculate the dominant graphs in a given order. These have a very different structure compared to the above graphs. In the above, for each new vector meson added, we only obtained one extra $\log s$. This is due to the fact that the α_i could not be strong ordered. The graphs of Fig. 1 are, however, capable of giving an extra factor of $\log^2 s$ for each vector meson added.

The vector mesons which give $\log^2 s$ factors only join the p_1 sectors (A,B) to the p_2 sectors (C,D). The equivalent of these graphs is therefore missing in electroproduction.

Rather than performing a heroic calculation on the general graph of Fig. 1, we shall look at the simplest case in detail and then merely describe the strategy of calculation for higher terms. We have checked large classes of diagrams both in momentum and Feynman parameter space, and are confident about the general answer. This will be partially motivated by our eikonal technique. The summation and cancellation will also be proved by this method. The logic of the procedure is that graph by graph the dominant terms come from certain, very restricted, momentum flows. In these regions of phase space we may simplify the diagram. This simplification allows us to sum the eikonal subset of diagrams to a simple form whose final asymptotic behaviour is easily calculated.

Thus, we now turn to the graph of Fig. 7. This is the simplest graph which demonstrates the eikonal method of generating two powers of $\log s$. Use the momenta of 1, 2, 3 to fix all the momenta. Then the squared four momenta of Fig. 7 may be written

$$p_a^2 = -(1-\alpha_1)\beta_1 s - \underline{k}_1^2$$

$$p_b^2 = -(1-\alpha_1-\alpha_2)(\beta_1+\beta_2)s - (\underline{k}_1+\underline{k}_2)^2$$

$$p_c^2 = -\alpha_3(1-\beta_3)s - \underline{k}_3^2$$

$$p_d^2 = -(\alpha_2+\alpha_3)(1-\beta_2-\beta_3)s - (\underline{k}_2+\underline{k}_3)^2 \quad (3.1)$$

The lines 1, 2, 3 are all on-shell, and so their α, β are all positive definite. Now look at the region

$$\begin{aligned} m &\ll K_1 \ll K_2 \sim K \\ m &\ll K_3 \ll K_2 \sim K \\ \alpha_3 &\ll \alpha_2 \ll \alpha_1 \sim \alpha \\ \beta_1 &\ll \beta_2 \ll \beta_3 \sim \beta \end{aligned} \quad (3.2)$$

The $\alpha(\beta)$ entirely above (below) the line Q have finite values, but the line 2 which straddles Q has small α and β .

The four-momentum conservation δ functions now force $\alpha_1 = 1 - \alpha$, $\beta_3 = 1 - \beta$. The on-shell δ functions for 1, 2 and 3 allow us to carry out the β_1, α_3 and β_2 integrals.

In the numerator trace we keep, as before, the transverse parts of $\gamma \cdot p_a, \gamma \cdot p_c$, but in b, d we keep, respectively $(1 - \alpha_1 - \alpha_2)p_1$ and $(1 - \beta_2 - \beta_3)$. If, for simplicity, we sum over the photon polarizations with $g^{\mu\nu}$ only, we finally obtain

$$I' \approx S \int \frac{d\alpha_2}{\alpha_2} \int \frac{\frac{3}{11} d^2 K_i \delta^2(K_2 + K) K_1^2 K_3^2 \alpha \beta}{(1 - \alpha)(1 - \beta) P_a^2 P_b P_d P_c^2} \quad (3.3)$$

where

$$\begin{aligned} P_a &\approx -K_1^2 / (1 - \alpha) \\ P_b &\approx -\alpha K_2^2 / \alpha_2 \\ P_c &\approx -K_3^2 / (1 - \beta) \\ P_d &\approx -\alpha_2 \beta S \end{aligned} \quad (3.4)$$

So that after integrating over $d^2 K$ we obtain

$$I(\alpha, \beta) = \int \frac{d\alpha_2}{\alpha_2} \int \frac{\frac{3}{11} d^2 K_i (1 - \alpha)(1 - \beta)}{\frac{3}{11} K_i^2} \quad (3.5)$$

This is very similar to (2.6) with $F_{\mu\nu} \sim s$, except that $\alpha_2 \rightarrow 0$ and we pick up an extra $\log s$. The limits on α_2 are set by our approximation for P_d , for example in (3.4).

One can check that (3.2) is the only important region for this graph. Notice that the vector meson 2 transmits p_1 across the graph in the trace by means of its $g^{\mu\nu}$. The lines 1 and 3 must still carry a finite $\alpha_1 = 1 - \alpha$ and $\beta_3 = 1 - \beta$ to conserve four momentum. However, 2 may have very small and comparable α and β . In other words, 2 lies in the pionization region of the rapidity plot. The validity of our eikonal technique will depend on the property that $p_b^2 \simeq [(p_1 - 1) - 2]^2 \gg (p_1 - 1)^2$. Thus, in p_b^2 we need only keep the cross term. This is one of the conditions for the validity of the eikonal approximation, as we shall see.

For any such meson between sectors A(B) with C or D, we similarly obtained $\log^2 s$ factors. Always in the vertical propagators we can ignore the squares relative to the cross terms. The momenta in the left and right propagators of Fig. 6 are clearly equal and opposite. However, if we dress the diagrams of Fig. 5 with eikonal mesons, this symmetry does not exist.

4. THE EIKONAL TECHNIQUE

In this section we assume that we can use the eikonal to sum over the vector mesons. Thus we assume that the numerators in the propagators of A above U may be approximated by $\gamma \cdot p_1$, while the denominators may be approximated by $(p_1 - \Sigma q_i)^2 \simeq -2p_1 \cdot (\Sigma q_i)$ where q_i are the momenta of the eikonal vector mesons attached to A. Below U, p_1 is replaced by $(p_1 - p_u)$ in both of these formulae. Similarly, we approximate B, C, D with respect to p_1, p_2 and p_3 , respectively.

We have explicitly checked that these assumptions are valid in the dominant phase space regions for all graphs up to 8th order and many graphs beyond.

The standard technique ²⁾ is to consider each emitting segment separately. Thus in Fig. 8 we show the top segment of A. Now summing over the different ways of attaching the q_i to A by means of the formula

$$\sum_{P \in \text{mes}} \frac{1}{a_{\pi(1)}} \frac{1}{(a_{\pi(1)} + a_{\pi(2)})} \dots \frac{1}{(a_{\pi(1)} + \dots + a_{\pi(n)})} = \prod_{i=1}^n \frac{1}{a_i} \quad (4.1)$$

we find that the sum gives $\prod_{i=1}^n (p_1 - q_i)^2$ for the new denominator. The momenta q_i are now in factorized form. Thus insertion of an extra vector

meson and summation over where it is inserted, leads to the graphs shown in Fig. 9.

In Fig. 9a, which is obtained by joining sectors AD or BC, r_1 is given by p_1 or $p_1 - p_u$ depending on whether the vector meson is coupled above or below U. Similarly, for r_2 . In this term, notice that q is necessarily on-mass-shell. Remember that we must calculate the cross-section or, in Mueller language, the $(p_1 + p_1 - Q)^2$ discontinuity of the six-point function. In Fig. 10b the vector meson line is not on-shell. This trivial fact will make a crucial sign change for us. Notice that $p_1^2 = m^2$, but $(p_1 - p_u)^2 \simeq \alpha^2 u^2 / (1 - \alpha) \rightarrow \infty$ in the dominant part of the integral. Thus we shall need to evaluate the above graphs with r_1^2, r_2^2 also large, although always separately smaller than $(r_1 \pm r_2)^2 \propto s$. The diagrams of Fig. 9 contain no numerators as the traces are performed separately from the q integrations.

It turns out that there are four separate cases which we shall require:

- a) r_1^2, r_2^2 finite;
- b) $r_1^2 \rightarrow \infty, r_1^2/s \rightarrow 0, r_2^2$ finite;
- c) $r_1^2, r_2^2 \rightarrow \infty, r_1^2$ and $r_2^2 \ll s, r_1^2 r_2^2 \ll s$;
- d) $r_1^2, r_2^2 \rightarrow \infty, r_1^2$ and $r_2^2 \ll s$, but $r_1^2 r_2^2 \gg s$.

Since the graphs of Fig. 9a,b are scalar quantities, it is clear that they only depend on r_1^2, r_2^2 and $V = (r_1 \mp r_2)^2$, respectively. Then for the term with q on-shell, we find for the function I, which contains only $1/(q^2 - m^2)$ propagator factors and $d^4 q$

$$\begin{aligned}
 \text{a)} \quad I &\simeq \frac{\pi^2 i}{2V} \log^2 V \\
 \text{b)} \quad I &\simeq \frac{\pi^2 i}{2V} \left[\log^2 V - \log^2 r_1^2 \right] \\
 \text{c)} \quad I &\simeq \frac{\pi^2 i}{2V} \left[\log^2 V - \log^2 r_1^2 - \log^2 r_2^2 \right] \\
 \text{d)} \quad I &\simeq \frac{\pi^2 i}{2V} \left[\log \frac{V}{r_1^2} \log \frac{V}{r_2^2} \right]
 \end{aligned}$$

The term with $+\delta^+(q^2 - m^2)2\pi i$ in Fig. 2 has opposite signs for all these terms. To understand these effects, we show how the sign change in a) occurs and calculate b) for the on-shell term. The other terms are calculated in a similar way.

In a) $r_1 = p_1$, $r_2 = p_2$ and $V = (p_1 - p_2)^2 \simeq -s$. Using Sudakov variables

$$I = \frac{1}{2} \int \frac{s d\alpha d\beta d^2k}{[\alpha\beta s - k^2 - m^2][-(1-\alpha)\beta s - k^2 - m^2][-\alpha(1-\beta)s - k^2 - m^2]} \quad (4.3)$$

Here α, β are integrated over their whole range $(-\infty, \infty)$. Thus we may apply Cauchy's theorem to the α integral. For a non-zero answer there must be poles on each side of the real axis. Thus we must have $0 \leq \beta \leq 1$. Then the $(r_2 - q)^2 - m^2$ pole is above the axis and the other two are below. The easiest calculation is to move the contour to infinity in the upper half plane, thus picking up only the $(r_2 - q)^2 - m^2$ pole term. The contour is in the correct direction, and the residue is negative. Thus the answer may be written

$$I = -\pi^2 i s \int_0^1 d\beta \int_{-\infty}^0 d\alpha \frac{d^2k \delta(-\alpha(1-\beta)s - k^2 - m^2)}{[\alpha\beta s - k^2 - m^2][-(1-\alpha)\beta s - k^2 - m^2]} \quad (4.4)$$

remembering that $\delta(\alpha x) = \delta(x)/|\alpha|$. Now do the d^2k integral

$$I \simeq -\pi^2 i s \int_0^1 d\beta \int_{-\infty}^0 d\alpha \frac{1}{\alpha s [-\beta s + \alpha s]} \quad (4.5)$$

The over-all answer is negative imaginary. Some tedious integrals then give

$$I = \frac{-\pi^2 i}{s} \cdot \frac{1}{2} \cdot \log^2 s \quad (4.6)$$

This agrees with (4.2) since $V \simeq -s$.

If we had pulled the α contour the other way, we would have obtained pole contributions from $(q^2 - m^2)$ and $[(r_1 - q)^2 - m^2]$ pole terms. As one can easily check, the second term gives no contribution to the leading

logarithm term. The first term has a factor of $(-2\pi i)$ from the fact that the Cauchy contour is pulled down and so the circle round the pole is taken clockwise.

Now consider the calculation of b). Here we take $r_1 = p_1$, but $r_2 = (p_2 - q_3) = [-\alpha_3, 1 - (1 - \beta), -\underline{k}_3]$ in Sudakov variables using the notation of (3.1)

$$I' = \pi i \int \frac{s d\alpha_2 d\beta_2 d^2 k_2 \delta(\alpha_2 \beta_2 s - \underline{k}_2^2 - m^2)}{[-(1 - \alpha_2)\beta_2 s - \underline{k}_2^2 + m^2][(\alpha_2 + \alpha_3)(\beta_2 - \beta)s - (\underline{k}_2 + \underline{k}_3)^2 - m^2]} \quad (4.7)$$

Now perform the $d^2 k_2$ integral. Then the first term in the denominator becomes $-\beta_2 s$. The interesting region of β_2 integration is near 0. Thus $\beta_2 \ll \beta \sim \text{constant}$. For a logarithmic integral we need a factor of $\sim 1/\alpha_2$. Thus we must have $\alpha_2 \gg \alpha_3$ [cf. (3.2)] and $\alpha_2 \beta s \gg \underline{k}_3^2$. Thus the dominant part of the integral gives

$$I' \simeq \frac{\pi^2 i}{\beta s} \int \frac{d\alpha_2}{\alpha_2} \int \frac{d\beta_2}{\beta_2} \quad (4.8)$$

where $\epsilon \gg \beta_2 \gg m^2/\alpha_2 s$, $\epsilon \gg \alpha_2 \gg \underline{k}_3^2/\beta s$

set the limits

$$\begin{aligned} I' &= \frac{\pi^2 i}{\beta s} \left[\log^2 s - \log^2 \underline{k}_3^2 \right] \\ &= -\frac{\pi^2 i}{\beta s} \left[\log^2 V - \log^2 \underline{k}_2^2 \right] \end{aligned} \quad (4.9)$$

Similar calculations give the other results of (4.2). We have also checked these results by multiple Mellin transforms of the Feynman parametric representation. Since the expression for I is known to be pure imaginary for negative V , we can immediately extract also the next-to-leading log terms in the expressions for I .

Now we add all the eikonal contributions. We start with the dressing of Fig. 1. Then in the usual way we find ⁸⁾

$$\Delta T = T_L \exp \left[J(A_U, D_L) + J(A_U, C_L) + J(A_U, D_U) + J(A_U, C_U) + J(A_L, D_L) + J(A_L, C_L) + J(A_L, D_U) + J(A_L, C_U) + (A \leftrightarrow B, C \leftrightarrow D)^* \right] \quad (4.10)$$

J now contains all the factors of π, i omitted in I plus the trace contributions. A_U, A_L refer to the portions of A above and below U , and similarly for the other terms. We leave it to the reader to prove that these terms have opposite signs in pairs in leading log approximation. This sign change comes from:

- a) the numerator trace does not change sign;
- b) in $J(A_U, C_L)$, $V \simeq +s$, but in $J(A_U, D_L)$, $V \simeq -s$;
- c) the vector meson propagator $-ig^{\mu\nu}/q^2$ becomes $-g^{\mu\nu} 2\pi \delta(q^2 - m^2)$ on-shell.

Thus the $J(A_U, D_L)$ terms are positive definite. This is clearly necessary since they are the dominant terms in a cross-section.

From the symmetry of Fig. 1, we see that all the leading logarithms cancel. Moreover, in this eikonal approximation, the non-leading terms also cancel. It takes no great leap of the imagination to see that all the eikonal corrections to Fig. 6 are also mutually cancelling. This is clear since the momentum flows down left and right sides of the diagram are clearly opposite. Thus the J function arguments are clearly equal or opposite. The analysis is, however, very different when we turn to non-symmetric graphs as in Fig. 5.

In this case to consider leading log terms, we must again choose which photon of (a, b) and (c, d) is transverse and which is longitudinal. If we choose (a, c) as transverse, then only the lines $(k_m + a), (k'_n + c)$ may be joined by eikonal vector mesons. The apparently cancelling terms from, for example, $(k_m + a)(p_2 - c)$ are in fact a power less as $s \rightarrow \infty$. This is because the eikonal trace structure is incompatible with the traces required in the analysis of Fig. 7 in Section 2.

This term will then contribute an exponentially growing cross-section, unless it is cancelled by the lower order terms in the eikonal for Fig. 1. This clearly must happen, but our technical ability to handle these lower terms is very weak.

However, if we do not sum over the final states, but instead pick a given set of final states, we do not obtain the above cancellations. Thus the exclusive channels containing a massive μ pair are all damped exponentially ($\exp[-A(\alpha, \beta) \log^2 s]$) for a fixed multiplicity. Notice that this multiplicity distribution is Poisson, but in $\log^2 s$ not $\log s$. Clearly the non-leading terms, as in the multiperipheral model, will alter this neat conclusion. Only the existence of the large number of open channels eventually cancels this rapid damping and lets a slow fall-off eventually occur for the inclusive cross-section.

The analysis of our kinematics in the previous section shows that these eikonal vector mesons are strongly ordered both in rapidity and transverse momentum. Thus the pionization region in rapidity is in no sense depleted. Indeed the mean multiplicity is growing faster with energy than in normal hadronic collisions. These facts are in startling contrast with the naive parton model which empties the pionization region. More sophisticated versions do, however, put particles in the pionization region by means of Pomeron exchange diagrams ⁹⁾. These do not contribute to the inclusive cross-section.

5. NON-EIKONAL GRAPHS

These cancellations and subsequent non-eikonalization of the inclusive cross-section then make it interesting to sum the graphs of Figs. 5 and 6, etc., without the eikonal vector mesons. We should also include intermediate states with nucleons.

This calculation is very similar to the Gribov-Lipatov electroproduction calculation. We will therefore omit all details and merely stress the differences.

Thus we now set up two integral equations. The kernels adding terms above and below the two genuine photons in the centre.

For Fig. 6, the calculation is identical. For the diagrams of Fig. 5 we must alter the Gribov calculation slightly. By judicious use of Ward's identity, the "crossing" line contributions in electroproduction are cancelled against each other leaving only diagrammatic contributions as in Fig. 10. This involves the line A in a crucial way. However, the same

effect holds in $\mu\mu$ pair production by a cancellation identical to the eikonal calculation. Thus we add to the kernels, Gribov and Lipatov's expression for dotted lines. Similarly, we add the nucleon loop terms. Thus our kernels are identical to Gribov and Lipatov. In $\bar{p}p$ annihilation the Born term is also very similar. The lowest order term is shown in Fig. 11. Clearly, both α and β for the photon must be one. Thus the Born term contribution to the inclusive cross-section for a massive photon is given by

$$S \sigma(s) \propto \int dQ^2 \delta(\alpha\beta s - Q^2) \frac{s}{2} \delta(\alpha-1) \delta(\beta-1) \delta^2(k) \frac{1}{Q^4} F_{\mu\nu} d\alpha d\beta d^2k \quad (5.1)$$

where α, β are the longitudinal Sudakov components of the massive photon momentum. Thus

$$\frac{d\sigma}{dQ^2} \propto \frac{1}{s^2} \int d\alpha d\beta \delta(\alpha-1) \delta(\beta-1) \delta(\alpha\beta s - Q^2) \quad (5.2)$$

This is exactly the form found in (2.6) if we use the δ function to introduce Q^2 in terms of α, β .

Thus in the variables α, δ of Section 2, we have an integral equation exactly in the Gribov-Lipatov form which we solve in the same way.

The Born terms for $pp \rightarrow \mu^+ \mu^- + X$ are somewhat more complicated. We shall not treat them here.

We introduce

$$\begin{aligned} d_c(k^2) &= \left(1 - \frac{g^2}{12\pi^2} \log \frac{k^2}{m^2}\right)^{-1} = e^{\frac{4}{3}\xi} \\ g(k^2) &= e^{\xi} \\ \chi(k^2) &= e^{-\xi} \\ \xi(k^2) &= -\frac{3}{4} \log \left(1 - \frac{g^2}{12\pi^2} \log \frac{k^2}{m^2}\right) \end{aligned} \quad (5.3)$$

In the leading log calculation, these are the correction factors to the vector and spinor propagators and the vertex γ for large values of $1c$? We must clearly include the equivalent graphs, since our calculation has these propagators for off-shell.

The kernel of our integral equation is given by

$$\begin{aligned}
 & 2 \int_{\tilde{\xi}_k}^{\tilde{\xi}} d\tilde{\xi}_{k'} \int_0^{\omega_k} d\omega'_k \frac{(\omega_k - \omega'_k)}{\omega_k \omega'_k} f_{\mu\nu}(\omega'_k, \tilde{\xi}_{k'}) \\
 & + 2 \int_{\tilde{\xi}_k}^{\tilde{\xi}} d\tilde{\xi}_{k'} \int_0^{\omega_k} d\omega''_k \frac{(\omega_k - \omega''_k)}{\omega''_k} \left[\frac{2\omega_k(\omega_k - \omega''_k)}{\omega''_k{}^2} + 1 \right] \\
 & \times \int_0^{\omega''_k} \frac{d\omega'_k}{\omega'_k} \left[\frac{2\omega'_k(\omega'_k - \omega''_k)}{\omega'_k{}^2} + 1 \right] f_{\mu\nu}(\omega'_k, \tilde{\xi}_{k'}) \\
 & - 4 \int_{\tilde{\xi}_k}^{\tilde{\xi}} d\tilde{\xi}_{k'} \int_0^{\omega_k} d\omega'_k \frac{\omega'_k}{\omega_k - \omega'_k} \left[\frac{f_{\mu\nu}(\omega_k, \tilde{\xi}_k)}{\omega_k} - \frac{f_{\mu\nu}(\omega'_k, \tilde{\xi}_{k'})}{\omega'_k} \right]
 \end{aligned} \tag{5.4}$$

where

$$\tilde{\xi}_k = \frac{g^2}{16\pi^2} \int_{m^2}^{-k^2} \frac{d(-K_{\perp}^{'2})}{-K_{\perp}^{'2}} d_c(-K_{\perp}^{'2})$$

$$\tilde{\xi} = \tilde{\xi}(Q^2)$$

(5.5)

The first term corresponds to (2.6) for adding one rung to m with $\omega_k = \alpha_k/\alpha$.

The κ^2 integrals have also been changed by inserting complete propagators and vertices (5.3).

The second term corresponds to nucleon loop addition, and the third to (2.7) and its generalizations. These equations are now soluble by means of Laplace transforms. Again we refer the interested reader to Gribov and Lipatov ⁶⁾ for details. The final answer is that

$$\frac{d\sigma}{dQ^2} \simeq \left[\frac{d\sigma}{dQ^2} \right]_{D.Y.} f(\tau, \xi) \quad (5.6)$$

where $(d\sigma/dQ^2)_{DY}$ contains the usual Drell-Yan Q^2 dependence. All the complexity of our diagram sum is continued in

$$f(\tau, \xi) \simeq \int d\alpha d\beta \delta(\alpha\beta - \tau) \exp \left[\left(\frac{81\xi}{2\log\xi} \log^2\alpha \right)^{1/3} + \left(\frac{81\xi}{2\log\xi} \log^2\beta \right)^{1/3} \right] \quad (5.7)$$

and as $Q^2 \rightarrow \infty$, $\xi \rightarrow -\infty$ so that we obtain an asymptotic answer marginally greater than the Drell-Yan formula.

This should also be compared with the Gribov-Lipatov result that the electroproduction structure functions are asymptotically greater than the naive parton model predictions. Thus they predict, for example, at fixed ω .

$$W_1 \simeq \frac{1}{2m} d(\xi) \exp \left[81\xi \log^2\omega / 2 \log\xi \right]^{1/3} \quad (5.8)$$

6. CONCLUSIONS

We see that μ pair production in a massive vector theory is a cross-section of great interest. Although the exclusive channels are dominated by an eikonal vector meson structure when we calculate the inclusive cross-section, the exponential cut-offs implied by these vector mesons are, we believe, exactly cancelled by the sum over the exclusive channels. Thus the eikonal structure disappears leaving a series of graphs very like those found in electroproduction.

Clearly, to prove the above hypotheses requires a much more powerful calculational technique than we have applied. The final result might lead one to guess that in spite of the vector mesons, a renormalization group technique should be applicable.

It would clearly be interesting also to study this problem in the context of gauge vector theories. In fixed angle scattering they apparently retain the exponential form of massive electrodynamics ¹⁰⁾, while retaining scaling in electroproduction.

In a previous paper ⁴⁾, a subset of diagrams was studied for the production of large $p_{\perp}$ particles. The analysis of this paper can clearly be extended to that case with its very similar kinematics. The momentum structure claimed in that paper will clearly be altered by the above eikonal results. Thus all the rapidity range will now be filled. However, the large $p_{\perp}$ structure of the other mesons should be preserved. Moreover, the eikonal mesons will give rise to a much larger $(\log^2 s)$ multiplicity.

ACKNOWLEDGEMENTS

I would like to thank Chris Sachrajda and John Huskins for discussions and comments on the above work.

REFERENCES

- 1) T. Appelquist and J.R. Primack, Phys. Rev. D4, 2454 (1971);
E.M. Lifshitz and L.P. Pitaevskii, Relativistic quantum theory
(part II) (Pergamon, London, 1974).
- 2) I.G. Halliday, J. Huskins and C.T. Sachrajda, Phys. Letters B47,
509 (1973); Nuclear Phys. B83, 189 (1974).
- 3) I.G. Halliday and J. Huskins, Nuclear Phys. (to be published).
- 4) I.G. Halliday, J. Huskins and C.T. Sachrajda, Nuclear Phys. B87,
93 (1975).
- 5) D.J. Fox et al., Phys. Rev. Letters 33, 1504 (1974).
- 6) V.N. Gribov and L.N. Lipatov, Sov. J. Nuclear Phys. 15, 438 (1972).
- 7) S.D. Drell and T.M. Yan, Phys. Rev. Letters 25, 316 (1970).
- 8) It has been claimed by P.M. Fishbane and L.M. Simmons, Phys. Rev. D11,
150 (1975), that this exponentiation is invalid even at 4th order
in fixed angle scattering. We have checked that their "different"
 γ matrices may be reduced to the usual form ²).
- 9) P.V. Landshoff and J.C. Polkinghorne, Nuclear Phys. B33, 221 (1971);
F. Henyey and R. Savit, Phys. Letters 52B, 71 (1974);
J.L. Cardy and G. Winbow, Phys. Letters 52B, 95 (1974);
C.E. De Tar, S.D. Ellis and P.V. Landshoff, Nuclear Phys. B87, 176
(1975).
- 10) J. Carazzone, E.C. Poggio and H.R. Quinn, Harvard Preprint.
J.M. Cornwall and G. Tiktopoulos, UCLA Preprint UCLA/75/TEP/1.

FIGURE CAPTIONS

Fig. 1 : The dominant graphs.

Fig. 2 : The dominant graphs for $pp \rightarrow pp$ and $p\bar{p} \rightarrow 2V$. The dotted lines represent the eikonal vector mesons.

Fig. 3 : The simplest ladder graphs.

Fig. 4 : The Drell-Yan structure.

Fig. 5 : The non-cancelling graphs.

Fig. 6 : The momentum flows for the simple ladder graphs.

Fig. 7 : The simplest eikonal graph.

Fig. 8 : The graphical eikonal sum.

Fig. 9 : The triangle graphs required in the calculation.

Fig. 10 : Terms in the integral equations.

Fig. 11 : The $p\bar{p} \rightarrow \mu^+ \mu^- - X$ Born term.

FIG.1

FIG.2

FIG. 3

FIG. 4

FIG. 5

FIG. 6

FIG. 7

FIG.8

(a)

(b)

FIG.9

FIG.10

FIG.11