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TESTS OF ABSORPTIVE MODELS IN HIGH ENERGY
REACTIONS ON NUCLEI

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A B S T R A C T

We discuss two relations connecting the inclusive cross-section on nuclei to the particle-nucleus elastic amplitude and to the particle-nucleon total cross-section, respectively. These relations are valid in the general framework of absorptive models. Possible tests of these models in the scattering of two heavy nuclei are briefly examined.

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An outstanding feature of particle-nucleus interactions is the absence of strong cascading effects which would lead to a violent atomic number (A) and energy (s) dependence of the average multiplicity $\bar{n}$.

To account for the weak A- and s-dependence of $\bar{n}$, Gottfried¹⁾ has developed an interesting space-time picture of the evolution of hadronic final states. Alternatively, it has been recognized²⁾ that such an effect is very natural in absorptive models (of the Gribov calculus type) thanks to very delicate cancellations among the different discontinuities of Reggeon interaction diagrams³⁾. In these kinds of models⁴⁾, even the qualitative features of the cross-section, and of the multiplicity, depend on the choice of the Pomeron interaction (reflecting the relative weight of elastic and inelastic absorptive rescatterings).

In the present note we want to stress two relations that hold in this class of models, independent of the details of the Pomeron interaction. We will limit ourselves in a first approximation to summing the tree diagrams studied in Ref. 4, neglecting Pomeron loop corrections. In fact, the Pomeron loop effects are the same as in p-p scattering where they are known to be very small at present energies, whereas the Pomeron-nucleus coupling being proportional to $A^{1/3}$ *) can be made large.

In the approximation of treating the nucleus as a set of independent nucleons, we obtain for the sum of the tree diagrams⁴⁾ (Fig. 1):

$$T_{aA} = 1 - S_{aA} = 1 - \exp(-g_a U_A(Y, \vec{B})) \quad (1)$$

Here, T_{aA} is the imaginary part of the scattering amplitude, $U_A(Y, \vec{B})$ (Fig. 2) is the sum of the connected tree diagrams, and g_a is the Pomeron coupling to the projectile a.

A typical diagram contributing to the U^2 term is shown in Fig. 3 as an old-fashioned perturbation diagram and as a Reggeon diagram. This diagram is of the Mandelstam type (completely nested) at every stage. It contains "projectile rescattering" and a "rescattering" of one of the produced particles. The interactions end on three nucleons which can be chosen as any triplet out of $A^{1/3}$, i.e. in $\binom{A^{1/3}}{3}$ ways. All the diagrams corresponding to the different triplets would be coherent provided the chains start before the interacting particle reaches the edge of the nucleus. One obtains, therefore, from the standard time evolution of multiperipheral diagrams, the coherence condition:

*) The appearance of the $gA^{1/3}$ effective coupling is a consequence of the simple fact that the Pomeron can couple to any of the $A^{1/3}$ nucleons present at a fixed impact parameter, and all the diagrams with the Pomerons coupled to different nucleons are coherent, provided the energy is large enough. The exact coherence condition will be discussed below.

$$\exp(y_1) \gg \exp(y_{\text{critical}}) = m_0 A^{1/3}$$

where y_1 is the rapidity of the ramification point in Fig. 3.

An additional complication⁶⁾ could be the coupling of the Pomerons to the nucleons not directly, but through AFS type cuts present at low energy (for y less than y_{critical}). The transition from $y > y_{\text{cr}}$ to $y < y_{\text{cr}}$, should be more carefully studied, but effectively we could incorporate the aforementioned coupling in our formalism by lowering the value of y_{cr} .

In the limit of small Pomeron-Pomeron interaction, Eq. (1) reduces to the usual eikonal. The form (1) automatically satisfies elastic unitarity, and in turn can be obtained as an iterative solution to the elastic unitarity equation⁵⁾. Since we expect the elastic cross-section to be relatively more important in the scattering on a nucleus than on a nucleon, we think that this constraint is essential.

The important property of the absorptive models that we want to stress here is that, thanks to the Gribov calculus discontinuity rules³⁾, the sum of the connected diagrams U , rather than its exponential S , is measured independently by the one particle inclusive distribution (Fig. 4). This property has two important phenomenological consequences that do not depend on the explicit form of U and therefore, test essentially (1) and the discontinuity rules³⁾. From Fig. 4, we see that

$$\left. \frac{d\sigma(\vec{B})}{dy} \right|_A = g_a U(Y-y) P(y) \Theta(R - |\vec{B}|)$$

where

$$\int d^2 \vec{B} \left. \frac{d\sigma}{dy} \right|_1 = g_a g_N P(Y-y) P(y)$$

Here, g_N is the Pomeron-nucleon coupling, $R = r_0 A^{1/3}$ is the nuclear radius, $P(y) = e^{[\alpha(0)-1]y}$ and $(d\sigma/dy)|_A$ and $(d\sigma/dy)|_1$ are the single particle inclusive cross-sections on the nucleus and on a single nucleon, respectively. Hence

$$g_a U(Y-y) = \frac{g_{aN}(Y-y)}{\pi R^2} \frac{\left. \frac{d\sigma}{dy} \right|_A}{\left. \frac{d\sigma}{dy} \right|_1}$$

or

$$\log S = - \frac{g_{aN}}{\pi R^2} \frac{\frac{1}{g_{aN}} \left. \frac{d\sigma}{dy} \right|_A}{\frac{1}{g_{aN}} \left. \frac{d\sigma}{dy} \right|_1} \quad (2)$$

The values of Y, y in Eq. (2) should be chosen safely larger than y_{cr} defined above. In the following, we will always use $(d\sigma/dy)$ in this region.

It is clear from Eq. (2) that if the nucleus becomes black for large A , this can happen only if

$$\frac{1}{\sigma_{aA}^{un}} \frac{dS}{dy} \Big|_A / \frac{1}{\sigma_{aA}^{un}} \frac{dS}{dy} \Big|_1$$

increases with increasing A . Hence the necessary and sufficient condition for $(\sigma_{el}/\sigma_{tot}) \xrightarrow{A \rightarrow \infty} \frac{1}{2}$ [or more exactly that $(\sigma_{el} + \sigma_{DD})/\sigma_{tot} \rightarrow \frac{1}{2}$, where σ_{DD} is the low mass diffraction dissociation⁷⁾] is that $(1/\sigma_{aA}^{in}) (d\sigma/dy)|_A$ increases without limit as a function of A .

Recently, Koplik and Mueller⁸⁾ derived a bound similar in spirit to our Eq. (2). The additional assumption that allows us to obtain an equality (rather than a bound) is the connection among the different terms in the expansion of Eq. (1) in powers of U^n . In reference 8, only inequalities among the different terms were required, in order to make the contribution to the inclusive cross-section coming from n chains separately positively definite. This happens automatically in our framework, but only if we consider Eq. (1) at all orders in U^n . Therefore, our Eq. (2) satisfies the bound of Ref. 8 only in the large n limit.

Since elastic unitarity plays an essential role for reactions on nuclei, and it leads to the assumed connection among the terms in the multiple scattering expansion of Eq. (1), we feel that our relation (2) is in fact on firmer ground than the bound of Ref. 8.

Let us stress that the normalization in Eq. (2) is provided by $1/\sigma_{aA}^{in}$. If σ_{aA}^{tot} is used as a normalization, there is in the black disk limit, a difference of a factor of 2. This is numerically very important: for instance, if

$$\frac{1}{\sigma_{aA}^{tot}} \frac{dS}{dy} \Big|_A / \frac{1}{\sigma_{aA}^{tot}} \frac{dS}{dy} \Big|_1$$

is of order 1, S can be still as small as e^{-2} , and therefore the nucleus is almost black. Hence, a test of Eq. (2) is, in fact, rather delicate, since it involves a very precise measurement of the particle-nucleus elastic and low mass diffraction dissociation cross-sections.

There is, however, a second test that involves only measuring one particle-spectra with different projectiles. From Fig. 4, we see that the one particle-spectra should be strictly proportional to g_a . This is independent of the validity of Eq. (1), and tests only the discontinuity rules.

Since g_a is in turn proportional to the σ -nucleon total cross-section, we obtain $(d\sigma/dy)|_A$ measured with different projectiles must be proportional to σ_{aN}^{tot} . In particular, if the cross-section of a on the nucleus is A independent (as certainly happens if the nucleus is black), for large Y , $\bar{n}_{aA}$ must be proportional to σ_{aN} . Some indications that such an effect may be present are reported in Ref. 9.

The above considerations can be extended to the case of nucleus-nucleus scattering¹⁰⁾, when the diagram of Fig. 5 determines the one-particle spectrum. No relation of the form (2) can be obtained here, but we can relate this spectrum to the one obtained in the particle a - A_1 and the particle b - A_2 collisions:

$$\frac{d\sigma^{A_1 A_2}}{dy} = \frac{d\sigma^{a A_1}}{dy} \frac{d\sigma^{b A_2}}{dy} \left(\frac{d\sigma^{ab}}{dy} \right)^{-1} \quad (3)$$

Coming back to particle-nucleus scattering, we have stressed that the validity of Eq. (2) is independent of the choice of the Pomeron interaction Lagrangian. However, depending on whether elastic or inelastic absorptive rescattering dominates, we obtain very different features. We refer to Ref. 4 for detailed calculations.

In general, whenever elastic rescattering dominates, the nucleus becomes black at large A , and $\bar{n}_A$ is proportional to $A^{1/3}$ (and obviously to σ^{aN}) also for $Y \rightarrow \infty$. If inelastic rescattering is important (as in the case of $L_I = \lambda \phi^+ \phi^2$) we obtain an asymptotic greyness that is A independent (but depends on the projectile). The multiplicity is A independent, too, but only in the limit of a very transparent nucleus ($\bar{n}_A/\bar{n}_1 \rightarrow 1$), and becomes projectile independent.

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Figure captions

- Fig. 1 : Pictorial representation of Eq. (1).
- Fig. 2 : Tree diagrams contributing to U .
- Fig. 3 : A diagram contributing to T and its time evolution.
- Fig. 4 : Contribution to the one-particle inclusive distribution in particle-nucleus scattering.
- Fig. 5 : Contribution to the one-particle inclusive distribution in nucleus-nucleus scattering.

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5