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THE STRUCTURE OF FINAL STATES IN A SPACE - TIME
DESCRIPTION OF QUARKS AND EXTENDED HADRONS

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ABSTRACT

In a previous article we developed a space - time description of extended hadrons in terms of permanently confined triality zero quark systems or bags. This description was found to provide a more concrete formulation of the massive quark model. In this article we examine the structure of the final states emerging from super excited $q\bar{q}$ bag systems and discuss the implications for particle production in hadron-hadron collisions.

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1. INTRODUCTION

In a previous article ¹⁾ (hereafter referred to as I), we presented a description of hadrons in terms of excited states or orbits of confined triality zero quark bag systems. The $q\bar{q}$ bag system is the simplest to describe and we shall see it is responsible for the bulk of particle production in hadronic collisions. In I we concluded that the spectrum of highly excited $q\bar{q}$ meson bag states was characterized by

$$M_N^2 = \frac{2\pi N}{R^2} \quad ; \quad N = n + J/2 \quad (1.1)$$

and

$$J \leq MR_\perp / 2$$

where J is the spin of the meson state. These states correspond to a spherical bag of radius $R(M) = MR^2$ as $M^2 \rightarrow \infty$. However, if we sum up coherently all degenerate states with mass M , along some chosen z axis, then the quark distribution corresponds to a cylindrical region of length $R_L \simeq MR^2$ and radius $R_\perp$. The purpose of the present article is to determine the nature of particle production from the decay of these super excited $q\bar{q}$ states.

In I we argued that extended hadrons interact through a three-bag-vertex and we showed that the high lying states decay preferentially in steps emitting the lowest mass bag states ($\pi, \rho, \omega, \dots$) as "quanta". This leads to a process, in which the super excited $q\bar{q}$ system cascades down in energy by the chain emission mechanism depicted in Fig. 1.

Fig. 1

The linear cascade decay for the super excited $q\bar{q}$ system

This kind of process has been considered in a vector meson model for $e^+e^- \rightarrow \text{hadrons}$ ²⁾, where it was shown that such a mechanism can be described

in terms of an integral equation in much the same way as the multi-peripheral mechanism ³⁾.

In I we showed that, if we saturate the discontinuity of the $q\bar{q}$ Green's function with the above super excited $q\bar{q}$ bag states, we obtained at high energies

$$\text{disc}_s G(s, t, \sigma_1, \dots, \sigma_4) = \prod_{i=1}^4 \delta_{R^2}[\sigma_i - m_q^2] s \sigma_{q\bar{q}}(s) f_{R_1}[t] \quad (1.2)$$

where

$$\sigma_{q\bar{q}}(s) = \frac{(2\pi)^3}{\mu^2 Z} \frac{\pi R_1^2}{R^2} \text{Im} \psi\left(1 - \frac{R^2}{\pi}(s - 2i\gamma)\right) \quad (1.3)$$

$$f_{R_1}[t] = \frac{2 J_1(R_1 \sqrt{-t})}{R_1 \sqrt{-t}} \quad (1.4)$$

$\delta_{R^2}(\sigma)$ is an entire function peaked around zero, and has the properties

$$\int_{-\infty}^{\infty} d\sigma \delta_{R^2}[\sigma] = 1 \quad ; \quad \delta_{R^2}[0] = R^2/\pi \quad (1.5)$$

$\{\sigma_i\}$ are the four-momentum squared variables of the quark legs, and the constants μ^2 and Z are defined in I. $f_{R_1}[t]$ is a strongly peaked function at $t=0$, characterized by the slope $b = (df_{R_1}/dt)|_{t=0} = R_1^2/8$. The spacing of the super excited $q\bar{q}$ states is given by

$$\Delta M^2 = \frac{2\pi}{R^2} \quad (1.6)$$

and in the Appendix we shall show that the asymptotic width parameter $\gamma = \lim_{N \rightarrow \infty} \frac{M_N \Gamma_N}{N}$ for their decay into the continuum of ground state hadrons is given by

$$\gamma = 2\kappa \frac{2\pi}{R^2} \quad \text{with} \quad \kappa \gg 1$$

Hence, these states overlap only locally. Nevertheless, $\sigma_{q\bar{q}}(s)$ tends smoothly to a constant. In Fig. 2 we give $\sigma_{q\bar{q}}(s)$ for the different values of $\kappa = 1, 2.5, 5$ and 10 , respectively. For $\kappa < 1$, $\sigma_{q\bar{q}}(s)$ will oscillate asymptotically.

Fig. 2

The behaviour of $\sigma_{q\bar{q}}(s)$ for different values of $\kappa = \sqrt{2\Delta M^2}$

Equation (1.2) corresponds to a fixed pole at $J=1$ and leads to scaling in deep inelastic phenomena ⁴⁾. It also gives rise to the Pomernanchuk mechanism in hadron-hadron scattering ⁵⁾. However, if we want to examine, for example, the inclusive spectra corresponding to this scaling fixed pole behaviour, we have to recast $\text{disc}_s G$ in a more transparent form reflecting the mechanism, by which the intermediate super excited bag states decay. This involves writing the cascade process in Fig. 1 in the form of an integral equation; this will be discussed in the next section. The basic picture of particle production that emerges from the description of hadronic interaction we have in mind, lies somewhere between the statistical cluster models of Hagedorn and Montvay ⁶⁾ and the multiperipheral model ³⁾. In the former, the particles emerge from clusters or fireballs which decay statistically, populating most of the available phase space.

Whereas in the case of the multiperipheral picture, only a limited region of phase space is populated with the particles spread out in longitudinal momentum, giving rise to a plateau in rapidity.

In Section 2 we examine the basic equations governing the one-particle distribution in the super excited $q\bar{q}$ bag system, restricting ourselves to the case when only the ground state hadrons (pions) are emitted. In this section we also comment on the large transverse momentum behaviour of the one-particle distribution. The analysis is extended in Section 3 to the two-particle distribution, from which the nature of the multiplicity distribution and rapidity correlations among the pions are deduced. In Section 4 we briefly discuss the production in addition of low-lying excited bag states (i.e., resonances), and the influence this has on, for example, the approach to the asymptotic regime. The inclusion of an $SU(n)$ symmetry is considered in Section 5; in particular we deduced the charge multiplicity distribution in the case of $SU(2)$. Finally, we devote Section 6 to a discussion of the structure of the final states that emerge in hadron-hadron collisions in such a description of hadrons and quarks. Some technical details are left to the Appendix.

2. INTEGRAL EQUATION FOR disc G AND THE ONE-PARTICLE DISTRIBUTION

We shall take into account here only pion emission in the cascade decay of the super excited $q\bar{q}$ bag system. Later in Section 4, we will consider the effect of the production in addition to low-lying excited states. The basic pion emission vertex is shown in Fig. 3 and

Fig. 3

The vertex for $N \rightarrow N' + \pi$

corresponds to the expression

$$M(N \rightarrow N' + \pi) = (\mu^2)^3 \int \frac{d^4 l}{(2\pi)^4} Z_N(p, \frac{1}{2}p + l) Z_{N'}^*(p', \frac{1}{2}p' + l - k) Z_{\pi}^*(k, \frac{1}{2}k + l)$$

$$\begin{aligned} \ell &= \alpha P + \beta P' + \ell_{\perp} \\ \ell' &= \alpha' P + \beta' P' + \ell'_{\perp} \\ k &= x P + y P' + k_{\perp} \end{aligned} \quad (2.4)$$

A straightforward analysis, in which we use the δ_{R^2} distributions to integrate out the α, β, α' and β' dependence and $\delta^+(k^2 - m_\pi^2)$ the y dependence, yields the following eigenvalue condition [defining $q = (0, q_1, 0, 0)$]

$$S \sigma_{q\bar{q}} \left[f_{R_\perp}(q_\perp) - \frac{\lambda}{4\pi} \int_0^1 \frac{dx}{x} (1-x) \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 l_\perp}{(2\pi)^2} \frac{d^2 l'_\perp}{(2\pi)^2} K[x, k_\perp, l_\perp, l'_\perp, q_\perp] f_{R_\perp}(q_\perp - l_\perp + l'_\perp) \right] = 0 \quad (2.5)$$

The explicit form of the kernel $\underline{K}$ is given in the Appendix. We note here two of its important properties

$$i) \quad \lim_{x \rightarrow 0} \frac{K}{x} [x, k_1, l_1, l'_1, q_1] \rightarrow 0$$

and

$$\text{ii)} \quad \lim_{k_{\perp}^2 \rightarrow \infty} \frac{1}{k_{\perp}^2} [x, k_{\perp}, \ell_{\perp}, \ell'_{\perp}, a_{\perp}] \rightarrow O(x/k_{\perp}^2)^4$$

We denote the non-forward missing mass discontinuity for $q\bar{q} + \pi \rightarrow q\bar{q} + \pi$ by

$$A_6(s, k, q) = \begin{array}{c} q \text{ --- } \bigcirc \text{ --- } q \\ \pi \text{ --- } \bigcirc \text{ --- } \pi \\ \bar{q} \text{ --- } \bigcirc \text{ --- } \bar{q} \end{array}$$

The corresponding equation for A_6 is given by

(2.6)

We shall consider separately the regions $x>0$ and $x<0$, by concentrating on $x>0$. Noting (see Appendix)

$$A_6(s, k, q) = A_6[s, x, k_\perp] f_{R_\perp}(q_\perp) \quad (2.7)$$

where $\vec{k} = (\underline{k}_\perp, x\sqrt{s}/2)$, we can write (2.7) in the form (setting $q_\perp = 0$)

$$A_6 [s, x, k_\perp] = \frac{\lambda}{2(2\pi)^3} \sigma_{q\bar{q}} K[x, k_\perp] + \frac{\lambda}{4\pi} \int_0^{1-x} \frac{dx'}{x'} \int \frac{d^2 k'_\perp}{(2\pi)^2} K[x', k'_\perp] A_6[(1-x)s, \frac{x}{1-x}, k_\perp - k'_\perp]$$

where

$$K[x, k_\perp] = \int \frac{d^2 l_\perp}{(2\pi)^2} \frac{d^2 l'_\perp}{(2\pi)^2} K[x, k_\perp, l_\perp, l'_\perp, 0] f_{R_\perp}(l_\perp - l'_\perp) \quad (2.8)$$

From (2.8) we see that there is the obvious scaling solution

$$F[x, k_\perp] = x \frac{d^3 \sigma_{q\bar{q}}}{dx d^2 k_\perp} = \frac{1}{s} A_6[s, x, k_\perp] \quad (2.9)$$

Integrating over $d^2 k_\perp$ and defining $F(x) = \int d^2 k_\perp / (2\pi)^2 F[x, k_\perp]$, we see that the x dependence is determined by the integral equation

$$F(x) = F_0(x) + \frac{\lambda}{4\pi} \int_0^{1-x} \frac{dx'}{x'} (1-x') K(x') F\left(\frac{x}{1-x'}\right)$$

where

$$F_0(x) = \frac{\lambda}{4\pi} \sigma_{q\bar{q}} (1-x) K(x)$$

and

$$K(x) = \int \frac{d^2 k_\perp}{(2\pi)^2} K[x, k_\perp] \quad (2.10)$$

2.1 The central region $x \rightarrow 0$

In order to extract the leading behaviour as $x \rightarrow 0$, we take the Mellin transform of (2.10) by defining

$$\tilde{F}(\omega) = \int_0^1 \frac{dx}{x} x^{\omega} F(x)$$

with its inverse

$$F(x) = \frac{1}{2\pi i} \int_{-i\infty+0^+}^{i\infty+0^+} d\omega \left(\frac{1}{x}\right)^{\omega} \tilde{F}(\omega) \quad (2.11)$$

We obtain the algebraic equation

$$\tilde{F}(\omega) = \tilde{F}_0(\omega) + \frac{\lambda}{4\pi} \tilde{K}(\omega) \tilde{F}(\omega)$$

and hence

$$F(x) = \frac{1}{2\pi i} \int_{-i\infty+0^+}^{i\infty+0^+} d\omega \left(\frac{1}{x}\right)^{\omega} \frac{\tilde{F}_0(\omega)}{1 - \frac{\lambda}{4\pi} \tilde{K}(\omega)} \quad (2.12)$$

where

$$\tilde{F}_0(\omega) = \frac{\lambda}{4\pi} \sigma_{q\bar{q}} \int_0^1 \frac{dx}{x} x^{\omega} (1-x) K(x)$$

and

$$\tilde{K}(\omega) = \int_0^1 \frac{dx}{x} (1-x)^{1+\omega} K(x)$$

The leading behaviour as $x \rightarrow 0$ is determined by the nearest pole in the ω plane, which comes from a zero in the denominator of (2.12), i.e.,

$$1 - \frac{\lambda}{4\pi} \tilde{K}(\omega) = 0 \quad (2.13)$$

Comparing (2.13) with the eigenvalue condition (2.5), we see that the leading pole is located at $\omega = 0$. Extracting the residue of this singularity, we obtain

$$F(x) \xrightarrow{(x \rightarrow 0)} c \tilde{F}_0(0)$$

with

$$c = \tilde{K}(0) / \left[- \frac{\partial}{\partial \omega} \tilde{K}(\omega) \right]_{\omega=0}$$

and

$$\tilde{F}_0(0) = \sigma_{q\bar{q}} \quad (2.14)$$

The last relation follows also from (2.5). If we express (2.14) in terms of the usual rapidity variable y , defined by

$$- \text{Log } x = \frac{1}{2} Y - y, \quad \text{with} \quad Y = \frac{1}{2} \text{Log } (S/S_0)$$

we obtain

$$\frac{1}{\sigma_{q\bar{q}}} \frac{d\sigma_{q\bar{q}}}{dy} = c \quad (2.15)$$

Hence we see we have a plateau in rapidity of height c , which implies a mean multiplicity $\bar{n}_q = c Y$.

In the quark fragmentation region ($x \rightarrow 1$), we see on inspecting (2.10) that

$$F(x) \underset{(x \rightarrow 1)}{\simeq} F_0(x) \propto (1-x) \quad (2.16)$$

This equation simply reflects the lack of a leading particle effect. The rapidity distribution is sketched in Fig. 4.

Fig. 4

One-pion distribution emerging from the super excited $q\bar{q}$ system
as a function of rapidity

2.2 The $k_{\perp}$ distribution

In order to extract the properties of the $k_{\perp}$ distribution, in particular $k_{\perp}^2$ in the central region, we return to (2.8) and define the moments

$$\begin{aligned} F_{k_{\perp}}^{(n)}(x) &= \int \frac{d^2 k_{\perp}}{(2\pi)^2} (k_{\perp}^2)^n F[x, k_{\perp}] \\ &= \overline{k_{\perp}^{2n}}(x) F(x) \end{aligned} \quad (2.17)$$

For $n=1$ we obtain from (2.8) the integral equation [making use of $F(x, k_{\perp}) = F(x, -k_{\perp})$]

$$\begin{aligned} F_{k_{\perp}}^{(1)}(x) &= F_{k_{\perp}0}^{(1)}(x) + \frac{\lambda}{4\pi} \int_0^{1-x} \frac{dx'}{x'} (1-x') K(x') F_{k_{\perp}}^{(1)}\left(\frac{x}{1-x'}\right) \\ &\quad + \frac{\lambda}{4\pi} \int_0^{1-x} \frac{dx'}{x'} (1-x') K_{n_{\perp}}^{(1)}(x') F\left(\frac{x}{1-x'}\right) \end{aligned} \quad (2.18)$$

where $F_{k_{\perp}0}^{(1)}(x)$ and $K_{n_{\perp}}^{(1)}(x)$ are defined the same way as $F_{k_{\perp}}^{(1)}(x)$ in Eq. (2.17).

Taking the Mellin transform of (2.18) leads to the algebraic equations

$$\tilde{F}_{k_{\perp}}^{(1)}(\omega) = \tilde{F}_{k_{\perp 0}}(\omega) + \frac{\lambda}{4\pi} \tilde{K}(\omega) \tilde{F}_{k_{\perp}}^{(1)}(\omega) + \frac{\lambda}{4\pi} \tilde{K}_{k_{\perp}}^{(1)}(\omega) \tilde{F}(\omega) \quad (2.19)$$

from which we readily deduce

$$F_{k_{\perp}}^{(1)}(x) = c \tilde{F}_{k_{\perp 0}}^{(1)}(0) + c^2 \frac{\partial}{\partial \omega} \left[\frac{\lambda}{4\pi} \tilde{K}_{k_{\perp}}^{(1)}(\omega) \tilde{F}_0(\omega) \right]_{\omega=0}$$

$$\text{i.e.,} \quad \overline{k_{\perp}^2} = \overline{k_{\perp 0}^2} + \frac{c}{\sigma_{q\bar{q}}} \frac{\partial}{\partial \omega} \left[\frac{\lambda}{4\pi} \tilde{K}_{k_{\perp}}^{(1)}(\omega) \tilde{F}_0(\omega) \right]_{\omega=0} \quad (2.20)$$

From (2.20) we see that the mean $\overline{k_{\perp}^2}$ is independent of rapidity in the central region.

It is difficult to obtain a simple analytic expression for $\overline{k_{\perp}^2}$, because of the complexity of the kernel in (2.5). However, a very crude calculation gives $\overline{k_{\perp}^2} \sim \pi/R_{\perp}^2$, which corresponds to pions being emitted from a transverse region of radius $\sim R_{\perp}$. A similar analysis of the higher moments $k_{\perp}^{2n}(x)$ show they too are independent of x in the central region. This means the rapidity and $k_{\perp}$ distributions are independent in the central region. Furthermore, this implies, unlike a pure multiperipheral mechanism, that, within each $q\bar{q}$ system, the transverse momentum is not compensated locally in rapidity.

2.3 The large $k_{\perp}$ limit

A complete discussion of large transverse momentum phenomena is outside the scope of the present article. However, we can readily deduce the basic power laws arising from particle production in the excited $q\bar{q}$ bag system. By examining the behaviour of the kernel $K[x, k_{\perp}]$ for large $k_{\perp}^2$, we see that the inclusive distribution falls like $k_{\perp}^{-3}$. In Ref. 4) it was argued, however, that this power law - which comes essentially from the driving term in Eq. (2.6) - should only hold in an intermediate region. When s and $k_{\perp}^2$ are large enough, the MQM diagram in Fig. 5a should eventually determine the large $k_{\perp}$ behaviour.

Fig. 5

- a) MQM diagram for leading large $k_{\perp}$ behaviour
 b) The $q\bar{q}$ bag mechanism for generating the $q\bar{q} \rightarrow q\bar{q} + q\bar{q}$ Green's function

This diagram involves the Green's function for $q\bar{q} \rightarrow q\bar{q} + q\bar{q}$ and, within the framework set up in I, the latter is generated by the three-bag-vertex through the mechanism shown in Fig. 5b, in which we sum over all possible intermediate bag states in much the same way as in the case of the $q\bar{q}$ four-point function. When all three $q\bar{q}$ invariant masses m^2 , m_1^2 and m_2^2 become large, the Green's function is highly suppressed due to the behaviour at the vertex. However, after an initial sharp fall-off, the three-bag-vertex develops a power behaviour as $m^2 \sim m_1^2 m_2^2 \rightarrow \infty$, due to the properties of the $\delta_{R^2}(\sigma)$ distributions which behave asymptotically like

$$\delta_{R^2}^2(\sigma) \xrightarrow{(\sigma \rightarrow \infty)} \frac{1}{2\pi^2} \frac{1}{\sigma^2} \quad (2.21)$$

The dependence on the invariant masses is essentially contained in the factor (see I)

$$f(m^2, m_1^2, m_2^2) = \int_{-1}^1 dx \delta_{R^2}^2(a - bx)$$

where

$$\begin{aligned} a &= \frac{1}{4} [m^2 - m_1^2 - m_2^2] \\ b &= \frac{1}{4} [m^4 + (m_1^2 - m_2^2)^2 - 2m^2(m_1^2 + m_2^2)]^{1/2} \end{aligned} \quad (2.22)$$

Using (2.21) we obtain asymptotically

$$f(m^2, m_1^2, m_2^2) \sim 1/(m_1^2 m_2^2) \text{ as } m^2 \sim m_1^2 m_2^2 \rightarrow \infty \quad (2.23)$$

Since in Fig. 4a the maximum transverse mass squared $m_{\perp}^2 = m_{\pi}^2 + k_{\perp}^2$ is proportional to m_2^2 and the full amplitude is proportional to f^2 , we obtain for this diagram the power law $k_{\perp}^{-4}$. The driving term in Eq. (2.6), which gives the $k_{\perp}^{-8}$ behaviour and can be thought of as an exclusive limit of Fig. 5a, will dominate up to quite large energy and $k_{\perp}^2$, because the latter involves a much higher threshold and the $k_{\perp}^{-4}$ behaviour has a very small numerical coefficient due to the initial sharp fall-off in the invariant masses at the three-bag vertex. A more complete discussion of the emergence of such power laws is contained in Ref. 4).

3. THE TWO-PARTICLE DISTRIBUTION IN THE $q\bar{q}$ SYSTEM

The basic equation governing the two-particle distribution can be obtained from that governing the one-particle distribution, and is given graphically by

$$(3.1)$$

Defining

$$F_2[x_1, k_{1\perp}; x_2, k_{2\perp}] = \frac{1}{6} \text{disc}_{M^2} G_8 \quad (3.2)$$

where $\vec{k}_i = (k_{i\perp}, x_i \sqrt{s}/2)$, $i = (1, 2)$, and following the same analysis as for the one-particle distribution, leads to the following equation for

$$F_2(x_1, x_2) = \int \frac{d^2 k_{1\perp}}{(2\pi)^2} \frac{d^2 k_{2\perp}}{(2\pi)^2} F_2[x_1, k_{1\perp}; x_2, k_{2\perp}]$$

namely (taking $x_1, x_2 > 0$)

$$F_2(x_1, x_2) = \frac{\lambda}{4\pi} (1-x_1) K(x_1) F\left(\frac{x_2}{1-x_1}\right) + \frac{\lambda}{4\pi} (1-x_2) K(x_2) F\left(\frac{x_1}{1-x_2}\right) + \frac{\lambda}{4\pi} \int_0^{1-x_1-x_2} \frac{dx'}{x'} (1-x') K(x') F_2\left(\frac{x_1}{1-x'}, \frac{x_2}{1-x'}\right) \quad (3.3)$$

The allowed phase space in x_1 and x_2 is shown in Fig. 6 and can be divided into the five regions I-IV, I(III) and II(IV) being reflections of each other.

Fig. 6

The (x_1, x_2) phase space

The central region V is a circle of radius $\sim s^{-1/2}$ which contracts to a point as $s \rightarrow \infty$. The shaded areas are threshold regions, in which one or both the hadrons produced are close to the quarks in rapidity. By examining Eq. (3.3), we see that it simplifies in the latter regions. We shall concentrate on the extreme regions, in particular the central region, from which the nature of the multiplicity distribution can be deduced.

3.1 Central region $x_1, x_2 \rightarrow 0$

The leading behaviour in this region is obtained by taking the double Mellin transform of (3.4), which is defined by

$$\tilde{F}_2(\omega_1, \omega_2) = \int_0^1 \frac{dx_1}{x_1} x_1^{\omega_1} \int_0^1 \frac{dx_2}{x_2} x_2^{\omega_2} F_2(x_1, x_2) \quad (3.4)$$

We obtain

$$\begin{aligned}\tilde{F}_2(\omega_1, \omega_2) &= \frac{\lambda}{4\pi} \tilde{K}(\omega_1) \tilde{F}(\omega_2) + \frac{\lambda}{4\pi} \tilde{K}(\omega_2) \tilde{F}(\omega_1) \\ &+ \frac{\lambda}{4\pi} \tilde{K}(\omega_1 + \omega_2) \tilde{F}_2(\omega_1 + \omega_2)\end{aligned}\quad (3.5)$$

and hence

$$F_2(x_1, x_2) = \int \frac{d\omega_1}{2\pi i} \left(\frac{1}{x_1}\right)^{\omega_1} \int \frac{d\omega_2}{2\pi i} \left(\frac{1}{x_2}\right)^{\omega_2} \frac{\lambda}{4\pi} \left[\frac{\tilde{K}(\omega_1) \tilde{F}(\omega_2) + \tilde{K}(\omega_2) \tilde{F}(\omega_1)}{1 - \frac{\lambda}{4\pi} \tilde{K}(\omega_1 + \omega_2)} \right] \quad (3.6)$$

Again making use of the eigenvalue condition (2.5), we see that the leading behaviour as $x_1, x_2 \rightarrow 0$ is given by

$$F_2(x_1, x_2) \rightarrow 2c^2 \tilde{F}_0(0)$$

or alternatively,

$$\frac{1}{\sigma_{q\bar{q}}} \frac{d^2\sigma_{q\bar{q}}}{dy_1 dy_2} \rightarrow 2c^2 \quad (3.7)$$

From (3.7) we obtain

$$\begin{aligned}\langle n(n-1) \rangle &= 4 \int_0^{\frac{1}{2}Y} dy_1 \int_0^{y_1} dy_2 \frac{1}{\sigma_{q\bar{q}}} \frac{d^2\sigma_{q\bar{q}}}{dy_1 dy_2} \\ &= c^2 \text{Log}^2 s/s_0\end{aligned}\quad (3.8)$$

The dispersion of the multiplicity distribution is given by

$$D^2 = \langle n^2 \rangle - \langle n \rangle^2 = \langle n \rangle = c \text{Log} s/s_0 \quad (3.9)$$

Equation (3.9) indicates that we have a Poisson multiplicity distribution and an independent emission of particles in rapidity in the central region. However, as in the multiperipheral model, we expect short-range correlations. The correlations are defined by

$$C(y_1 - y_2) = 4 \int_0^{\frac{1}{2}\gamma} dy_1' \int_0^{y_1'} dy_2' \delta[y_1' - y_2' - (y_1 - y_2)] \left\{ \frac{1}{\sigma_{q\bar{q}} dy_1' dy_2'} \frac{d^2\sigma}{d^2\sigma} - \frac{1}{\sigma_{q\bar{q}} dy_1'} \frac{d\sigma}{d\sigma} \frac{1}{\sigma_{q\bar{q}} dy_2'} \frac{d\sigma}{d\sigma} \right\} \quad (3.10)$$

and using (3.6) we obtain

$$C(y_1 - y_2) = 2 \int \frac{d\omega_1}{2\pi i} \int \frac{d\omega_2}{2\pi i} \frac{e^{\frac{1}{2}\gamma(\omega_1 + \omega_2)} - 1}{\omega_1 + \omega_2} e^{\omega_2(y_1 - y_2)} \times \left\{ \frac{1}{\tilde{F}_0(0)} \frac{\lambda}{4\pi} \frac{\tilde{K}(\omega_1)\tilde{F}(\omega_2) + \tilde{K}(\omega_2)\tilde{F}(\omega_1)}{[1 - \frac{\lambda}{4\pi} \tilde{K}(\omega_1 + \omega_2)]} - 2 \frac{\tilde{F}(\omega_1)\tilde{F}(\omega_2)}{[\tilde{F}_0(0)]^2} \right\} \quad (3.11)$$

The leading singularity at $\omega_1 = \omega_2 = 0$ cancels, and the next-to-leading singularity leads to the form

$$C(y_1 - y_2) \propto e^{-p|y_1 - y_2|} \quad (3.12)$$

Similar short-range correlations are found in the multiperipheral, where $p = 2\alpha'$; α' being the Regge slope. It is difficult to determine the value of p because of the complexity of the kernel. However, we can expect it to be of the order of unity.

3.2 The threshold regions

From the original Eq. (3.3), it is simple to see that when either $x_1 \rightarrow 1$ or $x_2 \rightarrow 1$, the two-particle distribution vanishes like $(1 - x_1)$ or $(1 - x_2)$, respectively. As a particular example we consider the limit $x_1 \rightarrow 1$ and $x_2 \rightarrow 0$, in which we see that

$$F(x_1, x_2) \underset{x_1 \rightarrow 1, x_2 \rightarrow 0}{\simeq} \frac{\lambda}{4\pi} K(1) (1 - x_1) C \sigma_{q\bar{q}}$$

i.e.,

$$\frac{1}{\sigma_{q\bar{q}}} \frac{d^2\sigma}{dy_1 dy_2} \underset{y_1 \rightarrow \frac{1}{2}\gamma, y_2 \ll \frac{1}{2}\gamma}{\simeq} \frac{1}{\sigma_{q\bar{q}}} \frac{d\sigma}{dy_1} \frac{1}{\sigma_{q\bar{q}}} \frac{d\sigma}{dy_2} \quad (3.13)$$

Equation (3.13) simply reflects the lack of long-range correlations in rapidity and the independent emission of particles separated by long distances in rapidity. There is an interesting intermediate threshold region where one expects non-vanishing correlations, namely when $x_1 + x_2 \simeq 1$. In this case

$$\begin{aligned}
 F(x_1, x_2) &\simeq \frac{\lambda}{4\pi} \kappa(x_1)(1-x_1)F\left(\frac{x_1}{1-x_2}\right) + \frac{\lambda}{4\pi} \kappa(x_2)(1-x_2)F\left(\frac{x_2}{1-x_1}\right) \\
 &\quad x_1 + x_2 \rightarrow 1 \\
 &\simeq \left(\frac{\lambda}{4\pi}\right)^2 \mathcal{O}_{q\bar{q}} \kappa(1)(1-x_1-x_2) [\kappa(x_1) + \kappa(x_2)]
 \end{aligned}
 \tag{3.14}$$

It is interesting to compare (3.14) with the Mueller-Regge approach within the MQM ⁴⁾. In conclusion, we note that in situations where correlations occur, the particles are close to one another in rapidity and in the cascade chain. However, unlike the multiperipheral model, the particles with different rapidity, completely overlap in impact parameter. If we plot the b space rapidity density distribution ⁷⁾ for the cascade mechanism, we obtain a picture like that shown in Fig. 7a. This should be contrasted with a purely Regge multiperipheral model which leads to a picture like that in Fig. 7b.

Fig. 7

Weber b space rapidity plots for the one-particle distribution, corresponding, respectively, to
a) chain emission mechanism
b) multiperipheral mechanism

4. THE PRODUCTION OF LOW-LYING EXCITED BAG STATES

So far we have exclusively made use of the fact that the super excited $q\bar{q}$ states decay preferentially by emitting the lightest bag states (pions) at each vertex (Fig. 8).

Fig. 8

The three-bag vertex

The dependence on m^2 , the mass squared of the lightest bag state emitted, is approximately given by (see I)

$$f(s, (1-x)s, m^2) \underset{s \rightarrow \infty}{\simeq} f(s, (1-x)s, 0) e^{-\frac{2}{\pi} R^2 \frac{m^2}{x}} \quad (4.1)$$

where $R^2 \simeq \pi$. Equation (4.1) reflects the initial strong fall-off in m^2 . For sufficiently large m^2 (i.e., $R^2 m^2 \gg 2$), we have the power dependence given in Section 2. For m^2 in the region 0.5 to 1 GeV^2 , allowing for all possible states in this region, we expect the probability of resonance production to be about 10% that for pions. The inclusion of such contributions leads to additional terms in the integral equation for the inclusive distributions, which have a slightly different character than those discussed in Sections 2 and 3. Equations (2.3) and (2.7) become, respectively,

$$\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} + \sum_R \begin{array}{c} \text{Diagram 5} \\ \text{Diagram 6} \end{array} \quad (4.2)$$

$$\begin{array}{c} \text{Diagram 7} \\ \text{Diagram 8} \end{array} = \begin{array}{c} \text{Diagram 9} \\ \text{Diagram 10} \end{array} + \sum_R \begin{array}{c} \text{Diagram 11} \\ \text{Diagram 12} \end{array} \quad (4.3)$$

Equation (4.2) leads to the modified eigenvalue condition

$$1 - \frac{\lambda_\pi}{4\pi} \tilde{K}(\omega) - \sum_R \frac{\lambda_R}{4\pi} \tilde{K}_R(\omega) = 0 \quad (4.4)$$

For $\lambda_\pi \gg \lambda_R$, it is simple to see that the production of clusters gives a small component to the increasing part of the pion multiplicity. This can be written in the form

$$\bar{n} = c \log S/s_0 + n_0$$

where

$$c = c_\pi + \frac{1}{\tilde{K}(0)} \sum_R n_R \frac{\lambda_R}{4\pi} \tilde{K}_R(0) \quad (4.5)$$

n_R is the pion multiplicity associated with a given resonance R . From (4.1) we see that these contributions to the pion multiplicity are not likely to be much more than 20%. A precise statement would require a rather complex analysis and in which one should also allow for final state interactions among the primary pions produced close to one another in rapidity.

The main source of low mass clusters at intermediate energies will come from the end process in the cascade decay, which goes through the low-lying excited states ρ , ω , ρ' , A_2 We denoted these contributions in Eq. (2.2) by Γ_0 and they contain a whole host of sub-processes, which are depicted in Fig. 9.

Fig. 9

The end processes of the cascade decay

The pions emerging from these kinds of processes populated the very small x region as the energy increases, and their presence will lead to a breaking

of scaling in the inclusive distribution at low energies. Through the energy sum rule,

$$\sigma_{q\bar{q}}(s) = 2 \int_0^1 dx \, x \frac{d\sigma}{dx} q\bar{q} \quad (4.6)$$

we see also that these terms will influence the approach of $\text{disc}_s G(s,t)$ to its asymptotic limit. In the $q\bar{q}$ system, the latter effect is enhanced because the small x region is emphasized, due to the absence of a leading particle effect (see Section 2). The mean multiplicity is expected to be modified at low energies, where we should write

$$\bar{n} = c \log s/s_0 + n_0(s)$$

where

$$n_0(s) = n_0(s_0) \left[1 - a (s_0/s) \right] \quad (4.7)$$

The s dependence of n_0 is due to the threshold behaviour arising from the opening of the low-lying channels, and the presence of these terms can considerably distort (e.g., broaden) the multiplicity distribution at low mean multiplicities. It is interesting to note that these processes are very important in $e^+e^- \rightarrow \text{hadrons}$ at lower energies ⁸⁾.

5. INCLUSION OF AN INTERNAL $SU(n)$ SYMMETRY

The inclusion of an internal $SU(n)$ symmetry is, in principle, straightforward, since the scaling behaviour in Green's function for $q_\alpha \bar{q}_\beta \rightarrow q_\alpha q_\beta$ is built up by a coherent sum of intermediate states, to which the super excited $q_\alpha \bar{q}_\beta$ system couples. This means that the basic kernel preserves the $SU(n)$ indices of the quarks (neglecting a possible mixing). The basic equation is given graphically by

$$\text{Diagram (5.1)} \quad (5.1)$$

Allowing for $SU(n)$ mass splitting, Eq. (5.1) leads to the eigenvalue condition

$$\alpha_{\alpha\beta} - \frac{\lambda}{4\pi} \sum_i \tilde{K}(0, m_i^2, m_\alpha^2, m_\beta^2, m_\gamma^2) [\lambda_{\alpha\gamma}^i \lambda_{\gamma\alpha}^i \sigma_{\gamma\beta} + \lambda_{\beta\gamma}^i \lambda_{\gamma\beta}^i \sigma_{\alpha\gamma}] = 0 \quad (5.2)$$

where $\lambda_{\alpha\beta}^i$ are the λ matrices of the $SU(n)$ group, i referring to the basic pseudoscalar multiplet, which we assume includes the lightest mesons. $\{m_\alpha\}$ refers to the effective mass of the quark (see I).

The corresponding equation for the one-particle distribution is given graphically by

$$\quad \quad \quad (5.3)$$

and can be written in the form

$$\tilde{F}_{\alpha\beta}^i(\omega) = \tilde{F}_{\alpha\beta}^i(\omega) + \frac{\lambda}{4\pi} \sum_j \tilde{K}(\omega, m_j^2, m_\alpha^2, m_\beta^2, m_\gamma^2) [\lambda_{\alpha\gamma}^j \lambda_{\gamma\alpha}^j \tilde{F}_{\alpha\gamma}^i(\omega) + \lambda_{\beta\gamma}^j \lambda_{\gamma\beta}^j \tilde{F}_{\gamma\beta}^i(\omega)] \quad (5.4)$$

Equation (5.2) again ensures that the leading singularity in $\tilde{F}_{\alpha\beta}^i(\omega)$ is at $\omega=0$, which gives rise to the central region plateau. However, by virtue of the small mass of the pion and the large mass splitting between it and other members of the $SU(n)$ multiplet, the main contribution to the above equations will come, in fact, from the pions. By writing the kernel

$$\tilde{K}(0, m_j^2, m_\alpha^2, m_\beta^2, m_\gamma^2) = \int_0^1 \frac{dx}{x} (1-x) K(x, m_j^2, m_\alpha^2, m_\beta^2, m_\gamma^2)$$

in the approximate form (see Section 4)

$$\tilde{K}(0, m_j^2, \dots) = e^{-\frac{4}{\pi} R^2 \frac{m_j^2 - m_\pi^2}{\langle x \rangle}} \tilde{K}(0, m_\pi^2, \dots) \quad , \quad (5.5)$$

we can parametrize the probability for the production of, for example, kaons relative to pions, by the coupling constant

$$\lambda_K = \lambda_\pi e^{-\frac{2}{\pi} R^2 \frac{m_K^2 - m_\pi^2}{\langle x \rangle}} \quad (5.6)$$

Since $x^{-1} K(x; m_\pi^2, \dots) \rightarrow 0$ as $x \rightarrow 0$, we expect $\langle x \rangle \simeq 0.5$. Using (5.6) and treating (5.4) perturbatively, we obtain the following estimate of the mean multiplicity of the kaons

$$\bar{n}_K = \frac{\lambda_K}{\lambda_\pi} \bar{n}_\pi \quad (5.7)$$

i.e., we expect the kaon yield to be approximately 15% of that of the pions.

If we restrict our attention to pion production and $SU(2)$, we can readily determine the charge multiplicity distribution. Remembering that the quarks form an iso-doublet system and that the asymptotic quark four-point function conserves the isospin indices of the quarks - as far as the internal symmetry is concerned - the production mechanism reduces to pion emission from a single quark line. This implies a simple binomial charge multiplicity distribution, and the cross-section for the production of a n pion system (comprised of n_c charged pions) can be written in the form

$$\sigma(n, n_c) = P(n, n_c) \sigma(n)$$

where

$$P(n, n_c) = \binom{n}{n_c} P^{n_c} (1-P)^{n-n_c} \quad ; \quad P = 2/3 \quad (5.8)$$

Remembering that $\sigma(n)$ should follow asymptotically a Poisson distribution, we see that the inclusive prong cross-section is given by

$$\frac{1}{\sigma_{q\bar{q}}} \left(\frac{d\sigma}{dy} \right)_{n_c} = c \frac{(\bar{n}_c)^{n_c}}{n_c!} e^{-\bar{n}_c} \quad (5.9)$$

with

$$\bar{n}_c = \frac{2}{3} \bar{n} = \frac{2}{3} c \log s/s_0$$

6. IMPLICATION FOR HADRONIC PRODUCTION PROCESSES

The way by which hadronic interactions evolve from the basic $q\bar{q}$ dynamics was outlined in I and in Ref. 4). We are now in a position to say something about the structure of the final states emerging from such a description. We begin by considering the mechanism responsible for particle production in the central region. This is shown in Fig. 10, and is common to all hadron-hadron collisions. The basic kernel or Born term (Fig. 10a) involves the simultaneous emission of pions from two $q\bar{q}$ systems, and corresponds to a fixed pole at $J=1$. The iterations shown in Figs. 10a and 10b result from the t channel unitarization of this fixed pole, and lead to a moving Pomeron pole with intercept $\alpha_P(0) = 1 + \epsilon$ where $\epsilon \simeq 0.1$.

Fig. 10

The basic mechanism for the central region

At low energies ($\epsilon \log s/s_1 \ll 1$), the first term dominates, and the Pomeron mechanism involves simply the independent emission of particles in

rapidity from two totally overlapping central clusters, namely the two $q\bar{q}$ systems depicted in Fig. 9a. If we fold in the longitudinal momentum distributions of the quarks which we denote by $f(x)$, then recalling that within each cluster the multiplicity distribution is approximately Poisson, we obtain an over-all multiplicity distribution for the first term of the form

$$P_c(n, \bar{n}_c) = \int_0^1 dx f(x) \int_0^1 dy f(y) \sum_{m=1}^n P_{q\bar{q}}[n-m, \bar{n}_q(s_1)] P_{q\bar{q}}[m, \bar{n}_q(s_2)]$$

where

$$P_{q\bar{q}}[n, \bar{n}_q] \underset{\bar{n}_q \gg 1}{\approx} \frac{\bar{n}_q^n}{n!} e^{-\bar{n}_q} \quad (6.1)$$

with

$$\bar{n}_q = c \log s/s_0 + n_{q0}$$

$$\int_0^1 dx f(x) = 1$$

and

$$s_1 = xys, \quad s_2 = (1-x)(1-y)s$$

Writing (6.1) in the form

$$P_c(n, \bar{n}_c) = \int_0^1 dx dy f(x) f(y) [\bar{n}_q(s_1) + \bar{n}_q(s_2)]^n \frac{e^{-[\bar{n}_q(s_1) + \bar{n}_q(s_2)]}}{n!} \quad (6.2)$$

we can readily deduce for large $\bar{n}_c$ that

$$\bar{n}_c = 2c \log s/s_0 + n_0 \quad (6.3)$$

and

$$D^2 = \langle n^2 \rangle - \langle n \rangle^2 = \bar{n}_c + \text{constant} \quad (6.4)$$

At low mean multiplicities, we should expect a significant distortion of the multiplicity distribution due to threshold effects in the $q\bar{q}$ system and a non-Poisson low multiplicity tail. In particular, n_0 should be replaced by the non-asymptotic form

$$n_0(s) = n_0(\infty) [1 - k s_0/s] \quad (6.5)$$

Fig. 11

Diagram involving N central clusters

From the analysis of Ref. 5), we expect, at higher energies, that the particles are emitted from a string of such double clusters corresponding to the t channel iteration shown in Figs. 10b and 10c. In the weak coupling solution to this iteration [see Ref. 5)], the cluster multiplicity distribution is given by (see Fig. 11)

$$P_{cl}(N, \bar{N}_{cl}) = \frac{\bar{N}_{cl}^{(N-1)}}{(N-1)!} e^{-\bar{N}_{cl}} \quad (6.6)$$

with $\bar{N}_{cl} = \epsilon \log s/s_1$.

The scale s_1 depends essentially on the transvers "size" of the extended hadrons and will in fact be larger than s_0 , which depends on the effective mass of the quark legs and the threshold for particle production in the $q\bar{q}$ system. Typically, we expect $s_0 \sim 1 \text{ GeV}^2$, while $s_1 \sim 4 \text{ GeV}$.

The mean multiplicity, corresponding to particle emission from the above chain, is given by

$$\begin{aligned} \bar{n}(s) &= \sum_{N=1}^{\infty} \frac{\bar{N}_{cl}^{N-1}}{(N-1)!} e^{-\bar{N}_{cl}} N \left[2c \log(s/s_0)^{1/N} + n_0 ([s/s_0]^{1/N}) \right] \\ &= 2c \log s/s_0 + n_0 \epsilon \log s/s_1 + \text{constant} \\ \bar{N}_{cl} &\rightarrow \infty \end{aligned} \quad (6.7)$$

If we allow for the finite energy behaviour in the $q\bar{q}$ system and include the threshold behaviour by replacing n_0 by its non-asymptotic form (6.5), we will obtain a threshold behaviour in $\bar{n}(s)$ and a rise in the height of the plateau

$$h = \frac{1}{\sigma_{tot}} \left. \frac{d\sigma}{dy} \right|_{y=0}$$

This rise will occur over the same region as the rise in the total cross-section, since the same mechanism is responsible for both (see Fig. 12).

Fig. 12

The rise in the height of the plateau as a function of energy

The height of the plateau rises by an amount

$$\Delta h / h = n_0 \epsilon / 2c \quad (6.8)$$

over the ISR range ⁹⁾, $\Delta h/h$ is approximately 30%. Setting $c=1$ and $\epsilon=0.1$, this suggests a value $n_0 \simeq 6$.

Since the above mechanism leads to a Pomeron pole with an intercept above unity, as the energy increases the effects of s channel unitarity must eventually become overwhelmingly important. However, the energy above which this is the case can be as large as $s=10^7 \text{ GeV}^2$ ¹⁰⁾. One way of seeing that s channel iterations are relatively small in the forward direction at present energies is the position $t=t_{\text{dip}}$ of the so-called diffractive dip in pp scattering and the height of the secondary maximum. At ISR energies, $t_{\text{dip}} \simeq -1.4 \text{ GeV}^2$ and $(d\sigma/dt)_{\text{secondary max}} / (d\sigma/dt)_{t=0} \simeq 10^{-6}$ ¹¹⁾. In our description of the Pomeronchuk mechanism, this dip arises from the interference between the Pomeron pole and s channel generated Pomeron cuts. It is clear from the position of the dip and the height of the secondary maximum, that the level of the cut contributions is at least two orders of magnitude smaller than the pole in the ISR region. However, they rise as a function of energy relative to the pole by virtue of the pole intercept being above unity. It is interesting to note in passing how fast the dip moves inwards as a function of energy. Over the ISR region $\Delta t_{\text{dip}} \simeq 0.1 \text{ GeV}^2$. In Ref. 5) we obtained a Pomeron trajectory, which has a large curvature, i.e.,

$$\alpha_P(t) = 1 + \epsilon + \alpha_P' t + \alpha_P'' t^2 + \dots \quad (6.9)$$

with

$$\alpha_P' \simeq 0.25 \text{ GeV}^{-2} \quad \text{and} \quad \alpha_P'' \simeq 0.5 \text{ GeV}^{-4}$$

Since, in our description of the Pomeron, the trajectory does not correspond to any crossed channel resonances, we expect that

$$\alpha_P(t) = 1 - \epsilon \leq \kappa' \quad \text{for } t > 0 \quad \text{with} \quad \epsilon + \kappa' < 1$$

and assuming $\alpha_P(t)$ is monotonic, by analyticity

$$\alpha_P(t) = 1 - \epsilon \rightarrow -\kappa \quad \text{as } t \rightarrow -\infty$$

If we assume that

$$\alpha_P(t) - 1 - \epsilon \simeq -\kappa \quad \text{for } t \lesssim -0.5 \text{ GeV}^2 \quad (6.10)$$

then the two-Pomeron cut-pole interference gives approximately

$$t_{\text{dip}} \simeq \frac{-4(\epsilon - \kappa) \log s/s_1}{b_0} + \text{Constant} \quad (6.11)$$

where b_0 is the constant part of the forward slope parameter, which, in part, is related to the hadronic form factor. For pp elastic scattering $b_0 \simeq 7 \text{ GeV}^2$, so that $\Delta t_{\text{dip}} \simeq 0.1 \text{ GeV}^2$ for s between 300 and 1500 GeV^2 corresponds to $\epsilon - \kappa \simeq 0.1$, i.e., $\kappa \simeq 0$. It is interesting to note that for $\kappa = 0$, $\Delta t_{\text{dip}}/t_{\text{dip}} \simeq \Delta \sigma_{\text{tot}}/\sigma_{\text{tot}}$ which would be implied, for example, by geometrical scaling.

The smallness of the slope parameter $\alpha' \simeq 0.25 \text{ GeV}^2$ is related to the fact that the transverse momentum of the final state particles in the central region is only conserved within each large central cluster, i.e., non-locally in rapidity. This was shown in Section 2 where we saw that, within each $q\bar{q}$ system, the rapidity and transverse momentum distributions are uncorrelated. This connection was recently pointed out by Weber¹²⁾ from more general considerations, and is explicitly realized in the present description of the Pomeronchuk mechanism. A further consequence of this property is that the rise in the height of the central plateau should be independent of the transverse momentum, which is confirmed by the recent ISR data⁹⁾.

In the case of meson-baryon scattering, the central component corresponds, in lowest order, to the diagram in Fig. 13a. However, in this case we also have a mechanism corresponding to diffractive dissociation. This is shown in Fig. 13b, and introduces a separate component in the multiplicity distribution, corresponding to particle emission from a simple $q\bar{q}$ system

Fig. 13

- a) Basic mechanism for the central region in meson-baryon scattering
- b) The diffractive dissociation mechanism

The shaded blob in Fig. 13b corresponds to a variety of mechanisms, all involving vacuum exchange and a scaling-like behaviour. For the present discussion, we only need to note that this component corresponds to a mean multiplicity increasing at high energies like

$$\begin{aligned}\bar{n}_d &= \bar{n}_q = c \log s/s_0 \\ &\approx \frac{1}{2} \bar{n}_c\end{aligned}\tag{6.12}$$

If we define σ_d and σ_c to be the total cross-sections for, respectively, the diffractive and central components, then the over-all multiplicity distribution is given by

$$P(n, \bar{n}) = \frac{\sigma_c}{\sigma_c + \sigma_d} P_c(n, \bar{n}_c) + \frac{\sigma_d}{\sigma_c + \sigma_d} P_d(n, \bar{n}_d)\tag{6.13}$$

At very high energies, $P(n, \bar{n})$ will have two peaks centred around $n = \bar{n}_d$ and $n = \bar{n}_c$, respectively, because both $P_c(n, \bar{n}_c)$ and $P_d(n, \bar{n}_d)$ tend asymptotically to Poisson distributions. However, at lower energies these peaks will overlap strongly, resulting in a broad distribution of the form shown in Fig. 14.

Fig. 14

Two-component multiplicity distributions at intermediate energies

If we calculate the mean multiplicity and dispersion from (6.13), inserting $\bar{n}_d = \frac{1}{2} \bar{n}_c$, we obtain

$$\bar{n} = \bar{n}_c \left[1 - \frac{1}{2} \frac{\sigma_d}{\sigma_{tot}} \right]\tag{6.14}$$

and

$$D^2 = \bar{n}_c^2 \left[1 - \frac{3}{4} \frac{\sigma_d}{\sigma_{tot}} \right] - \bar{n}^2$$

i.e.,

$$D \approx \frac{\bar{n}}{2} \sqrt{\frac{\sigma_d}{\sigma_{tot}}} \quad \text{for } \sigma_d \ll \sigma_{tot} \quad (6.15)$$

If we allow, in addition, for the distortion of the individual components at low mean multiplicities from a Poisson distribution, then we would expect an even larger dispersion than that given in (6.15). In particular, we expect important contributions to $P_d(n, \bar{n}_d)$ coming from the decay of diffractively produced baryon resonances. Further, although (6.13) does not lead to KNO scaling¹³⁾, it could well be approximated by it at lower energies because $D^2 \propto \bar{n}^2$ and the distribution could be symmetric around $n = \bar{n}$ due to the distortion of $P_d(n, \bar{n}_d)$ at low multiplicities. In such a case, the scaling will be broken asymptotically as the components separate for large $\bar{n}$. However, since for each component the dispersion D is proportional to $\bar{n}^{\frac{1}{2}} \sim (\log s/s_0)^{\frac{1}{2}}$, the effect will be rather slow and would need a large energy range to be seen.

For baryon-baryon scattering, the discussion is complicated by the presence of a double diffractive dissociation component which, at lower energies, will overlap strongly with the central region. Nevertheless, we expect in the baryon-baryon case to have the same sort of features as for meson-baryon scattering, except for a further broadening of the multiplicity distribution due to the additional component.

The crucial tests of the kind of dynamics presented here will lie in a detailed comparison of the correlations predicted in the final states with those observed experimentally. However, we see already some promising features. From the study of the two-particle distribution emerging from the super excited $q\bar{q}$ bag systems, we saw that there exist short-range correlations in rapidity similar to those predicted by a multiperipheral model and observed experimentally. Further, because of the generality of the Goldhaber effect¹⁴⁾, we will also expect the observed very short-ranged correlations for like pions¹⁵⁾ (i.e., due to Bose-Einstein statistics). There will also be long-range correlations which will depend on the detailed nature of the production mechanism and the distribution of quarks in the hadrons. A study of these correlations will ultimately provide valuable

tests of aspects of the dynamics not covered in the present discussion. The existence of two components in the multiplicity distributions for meson-baryon and baryon-baryon scattering - one of which (diffractive dissociation), having asymptotically half the mean multiplicity of the other (the central component), is a further promising feature of the hadron dynamics presented here. This results in a broad distribution with $D \sim \bar{n}$, which seems to be the case empirically. Finally, we saw that the one-particle distribution in the central region should increase over the ISR range in which the total cross-section is rising, the two effects being intimately linked. Furthermore, this effect is independent of the transverse momentum $k_{\perp}$ for small $k_{\perp}$, which seems to be the case experimentally.

We would like to end this discussion with the following reminder. Although we have discussed here the purely hadronic aspects of the description of hadrons and their interactions in terms of permanently confined quark degrees of freedom formulated in I, the successes of the MQM [see Ref. 4)] indicate we have also a viable description of current induced interactions. Further, the ability of bag models in describing the spectrum of hadrons and their low-energy parameters ¹⁶⁾ gives us confidence that this kind of framework will embrace all aspects of hadronic phenomena.

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APPENDIX

$$\text{THE ASYMPTOTIC WIDTH PARAMETER } \gamma = \lim_{N \rightarrow \infty} M_N \Gamma_N$$

The main decay mechanism for a given super excited $q\bar{q}$ bag state, with spin J and $N=n+J/2$, is the cascade mechanism discussed in Section 1, and the total width can be calculated from the diagram in Fig. 15.

Fig. 15

Diagram given total width Γ_N

Referring to I and Section 1 for notation, we can write $\gamma(M_N) = M_N \Gamma_N$ in the form

$$\begin{aligned} \gamma(M_N) = & \frac{1}{2J+1} \sum_m \int \frac{d^3 k}{(2\pi)^3 2E_k} \int \frac{d^4 q}{(2\pi)^4} \frac{d^4 q'}{(2\pi)^4} g^2 N_J D_{m0}^J(\Omega_q) D_{m0}^{J*}(\Omega_{q'}) \\ & \times \frac{g^2}{4\pi} (\mu^2 R^2 \pi^{-1})^6 \prod_{i=1}^3 \delta_{R2}[\sigma_i] \delta_{R2}[\sigma'_i] \\ & \times 2S' \sigma_{q\bar{q}} f_{R1}[t'] \end{aligned} \quad (A.1)$$

where $S' = (P - k)^2$, $t' = (q - q')^2$ and $q = (2\pi)^2 (2\sqrt{2\pi}) / (\mu Z^{\frac{1}{2}} R^2)$.

After carrying out the integrations over $dq_0 q^2 dq$ and $dq'_0 q'^2 dq'$ in the rest frame $P = (M_N, \vec{0})$, using the δ_{R2} functions $\delta_{R2}[\sigma_1]$, $\delta_{R2}[\sigma_3]$ and $\delta_{R2}[\sigma'_1]$, $\delta_{R2}[\sigma'_3]$, respectively, we obtain

$$\gamma(M) = c I_J(M^2)$$

with

$$C = (2\pi)^{-8} \frac{\sigma_{q\bar{q}}}{(2\pi)^3} \frac{g^4}{(4\pi)^2} (\mu^2 R^2 \pi^{-1})^6$$

and

$$\begin{aligned} I_J(M) = & \int \frac{d^3 k}{E_k} \frac{s'}{16} \int d\cos\theta_q d\varphi_q d\cos\theta_{q'} d\varphi_{q'} P_J(\cos\Theta) f_{R_1}[t'] \\ & \times \delta_{R^2} \left[\frac{1}{4} \{ M^2 - s' - m_\pi^2 - 2Mk \cos\theta_q \} \right] \\ & \times \delta_{R^2} \left[\frac{1}{4} \{ M^2 - s' - m_\pi^2 - 2Mk \cos\theta_{q'} \} \right] \end{aligned}$$

where

$$s' = M^2 - 2ME_k + m_\pi^2$$

$$t' = -\frac{M^2}{2} [1 - \cos\Theta]$$

$$\cos\Theta = \cos\theta_q \cos\theta_{q'} + \sin\theta_q \sin\theta_{q'} \cos(\varphi_q - \varphi_{q'})$$

(A.2)

We can simplify (A.2) by using the small angle approximations

$$q_\perp = \frac{M}{2} \sin\theta_q \approx \frac{M}{2} \theta_q$$

$$q'_\perp = \frac{M}{2} \sin\theta_{q'} \approx \frac{M}{2} \theta_{q'}$$

and

$$\lim_{M^2 \rightarrow \infty} P_J \left[1 - 4 \frac{(\vec{q}_\perp - \vec{q}'_\perp)^2}{M^2} \right] \Big|_{J=\frac{M}{2}b} = J_0(b|\vec{q}_\perp - \vec{q}'_\perp|)$$

(A.3)

Defining

$$I(b, M^2) = \lim_{M^2 \rightarrow \infty} I_J(M^2) \Big|_J = bM/2$$

we obtain

$$I(b, M^2) = \pi \int_0^1 dx x(1-x) \int d^2 \vec{q}_\perp d^2 \vec{q}'_\perp J_0(b|\vec{q}_\perp - \vec{q}'_\perp|) f_{R_\perp}[-(\vec{q}_\perp - \vec{q}'_\perp)^2] \\ \times \delta_{R^2}[\frac{1-x}{2} q_\perp^2] \delta_{R^2}[\frac{1-x}{2} q'^2_\perp] \quad (A.4)$$

where $x = 2E_k/M$.

Using the properties of the δ_{R^2} functions (see I), and remembering $R_\perp^2 \gg R^2$, we obtain after integration

$$I(b, M^2) = \pi^3 \frac{R^2}{R_\perp^2} \theta(R_\perp - b) \quad \text{for } b < R_\perp$$

Hence, for $b < R_\perp$, i.e., $J < M/2(R_\perp)$

$$\gamma = \lim_{N \rightarrow \infty} M_N \Gamma_N = 2 (4\mu^2 R^2)^3 \frac{2\pi}{R^2} \quad (A.5)$$

We have set $Z=1$ in (A.5) for simplicity. From the arguments in I, we see that $\gamma > 2\Delta M^2$.

Full Green's function

We recall that we construct the $q\bar{q}$ Green's function by summing over all intermediate states of the above type. This is represented in Fig. 16

Fig. 16

Diagram for full $q\bar{q}$ Green's function

The dependence on the transverse momentum transfer $q_{\perp} (t = -q_{\perp}^2)$ comes from the factor

$$\sum_{J=0}^{\frac{1}{2}MR_1} \sum_m N_J^2 D_{m0}^J(\theta, \varphi) D_{m0}^{J*}(\theta', \varphi') = \sum_{J=0}^{\frac{1}{2}MR_1} (2J+1) P_J(\cos \Theta)$$

$$\xrightarrow[\substack{M^2 \rightarrow \infty \\ J = \frac{M}{2}b}]{\substack{R_1 \\ \frac{M^2}{2} \int_0^{R_1} db b J_0(b q_{\perp})}} = \frac{M^2}{4} R_{\perp}^2 f_{R_{\perp}}[-q_{\perp}^2]$$

(A.6)

This will also factor out from corresponding expressions for non-forward inclusive distributions, hence the relation (2.7) in Section 2.

The kernel in Eq. (2.5) is given by

$$K[x, k_{\perp}, l_{\perp}, l'_{\perp}, q_{\perp}] = f_{R_1}(l_{\perp} + q_{\perp}/2) f_{R_1}(l'_{\perp} - q_{\perp}/2)$$

$$\times \delta_{R_{\pi}^2}^2 \left[\frac{1-x}{x} (k_{\perp}^2 + m_{\pi}^2) - (l_{\perp} - k_{\perp})^2 \right]$$

$$\times \delta_{R_{\pi}^2}^2 \left[\frac{1-x}{x} (k_{\perp}^2 + m_{\pi}^2) - (l'_{\perp} - k_{\perp})^2 \right]$$

(A.7)

from which the properties (i) and (ii) in Section 2 can readily be deduced using relation (2.21) for the asymptotic behaviour of the δ_{R^2} functions.

REFERENCES

- 1) G. Preparata and N.S. Craigie, CERN Preprint TH.2038 (1975).
- 2) N.S. Craigie and K. Rothe, Nuclear Phys. B82, 370, 381 (1974);
N.S. Craigie and T.F. Walsh, Ann. Phys. (in print).
- 3) D. Amati, S. Fubini and A. Stanghellini, Phys. Letters 1, 29 (1962);
G. Chew and A. Pignotti, Phys. Rev. 181, 1914 (1969).
- 4) G. Preparata, Proceedings of the International School of Subnuclear
Physics, "Ettore Majorana" 12th Course: on Lepton and Hadron
Structure, Erice (1974).
- 5) N.S. Craigie and G. Preparata, Phys. Letters 52B, 84 (1974).
- 6) R. Hagedorn and I. Montvay, Nuclear Phys. B59, 45 (1973).
- 7) B.R. Weber, Nuclear Phys. B87, 269 (1975).
- 8) See, for example, G. Kramer, J. Uretsky and T.F. Walsh, Phys. Rev. D3,
719 (1971).
- 9) K. Böckman, Proceedings of the International Conference on High
Energy Physics, Palermo (1975).
- 10) P.D.B. Collins, F.D. Gault and A. Martin, Phys. Letters 47B, 171 (1973).
- 11) V. Barger, Proceedings of the XVII International Conference on High-
Energy Physics, London (1974).
- 12) B.R. Weber, Invited Paper, Proceedings of the International
Conference on High-Energy Physics, Palermo (1975).
- 13) Z. Koba, H.B. Nielsen and P. Olesen, Nuclear Phys. B40, 317 (1972).
- 14) G. Goldhaber, S. Goldhaber, W. Lee and A. Pais, Phys. Rev. 120, 300
(1960).
- 15) G. Ranft and J. Ranft, Preprint Leipzig KMU HEP 7509 (1975).
- 16) K. Johnson and C.B. Thorn, Proceedings of the XVII International
Conference on High-Energy Physics, London (1974).