CM-P00058835

Ref.TH.1489-CERN

Archives

HOW TO TEST THE ALGEBRA OF BILOCAL CURRENTS ?

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A B S T R A C T

We discuss the possibility of testing the algebra of bilocal currents as abstracted from the quark model by use of deep inelastic electroproduction of massive muon pairs.

Ref.TH.1489-CERN

4 May 1972

One way of interpreting the results of the experiments in the deep inelastic region is to assume a very simple algebraic structure of the commutator of two currents when the space-time distance involved approaches the light cone ¹⁾. It was proposed that this algebraic structure might be isomorphic to the algebraic structure realized by the free quark model ²⁾. It can be shown that this algebraic structure is maintained in quark field theory models with neutral gluons, using formal manipulation of operators and not conventional renormalization theory ²⁾⁻⁴⁾. The strongest light cone singularity of a product of two currents is characterized by a bilocal current $\mathcal{F}_\mu(x,y)$, in the formal quark model just given by $\mathcal{F}_\mu(x,y) \sim : \bar{q}(x) \gamma_\mu q(y) :$. Following the quark model these operators generate a closed algebra with respect to the highest singularity on the light cone. We obtain for example the relation ²⁾

$$\begin{aligned} [\mathcal{F}_{i\mu}(x,u), \mathcal{F}_{j\nu}(y,v)] \cong \\ \frac{1}{4\pi} \mathcal{I}^S(\varepsilon(x_0 - v_0) \delta[(x-y)^2]) (i\delta_{ijk} - d_{ijk}) (s_{\mu\nu\rho\sigma} \mathcal{F}_k^\sigma(x,u) + i\varepsilon_{\mu\nu\rho\sigma} \mathcal{F}_k^{5\sigma}(x,u)) \\ + \frac{1}{4\pi} \mathcal{I}^S(\varepsilon(u_0 - y_0) \delta[(u-y)^2]) (i\delta_{ijk} + d_{ijk}) (s_{\mu\nu\rho\sigma} \mathcal{F}_k^\sigma(x,v) - i\varepsilon_{\mu\nu\rho\sigma} \mathcal{F}_k^{5\sigma}(x,v)) \end{aligned}$$

where we consider the connected part of the bilocal current commutator only. The tensor $s_{\mu\nu\rho\sigma}$ is given by

$$s_{\mu\nu\rho\sigma} = g_{\mu\rho} g_{\nu\sigma} + g_{\nu\rho} g_{\mu\sigma} - g_{\mu\sigma} g_{\nu\rho}$$

In the following analysis we shall investigate the possibility to test this algebra, as already indicated in Ref. 5). Since the bilocal currents are defined algebraically by the product of two local currents the only way to test their algebra is to consider a product of four currents, e.g., a product of four electromagnetic currents. Such a test would be important for two reasons :

- 1) it would be possible to verify at least parts of the algebra of bilocal currents ;
- 2) the test would give us information about the detailed SU_3 structure of the hadronic electromagnetic current, e.g., one can check, if the fractional charges of the underlying quark triplet are correct.

We consider an inclusive production process involving two currents as shown in Fig. 1. Summing up all hadronic final states we can describe the corresponding inclusive cross-section by the following amplitude :

$$M_{r\nu\rho\sigma} \delta^*(p+k-k'-n) = \frac{1}{(2\pi)^4} \sum_n \int dx dy \langle p | T_{j_\mu(x) j_\nu(y)} | n \rangle e^{i(kx - k'y)} \cdot \int dz dt \langle n | T_{j_\rho(z) j_\sigma(t)} | p \rangle e^{-i(kz - k't)} \quad (1)$$

Note that we consider spin-averaged amplitudes only. Using completeness we obtain :

$$M_{r\nu\rho\sigma} = \int dx dy dz e^{-ik(x-z) + ik'y} \langle p | (T_{j_\mu(x) j_\nu(y)}) (T_{j_\rho(z) j_\sigma(0)}) | p \rangle \quad (2)$$

We describe the process by the kinematical variables $k^2, k'^2, 2p \cdot k, 2p \cdot k', k \cdot k'$. We shall also use the variables t and v , defined by : $t = (k - k')^2$, $v = 2p(k - k')$. Especially we will be interested in the following limit

$$K^2, K'^2, v, 2p \cdot k, 2p \cdot k', -t \longrightarrow \infty$$

$$\eta = \frac{k^2}{2p \cdot k}, \quad \eta' = \frac{k'^2}{2p \cdot k'}, \quad \xi = -\frac{t}{v}, \quad \frac{2p \cdot k}{2p \cdot k'} =: \varphi \text{ finite} \quad (3)$$

We describe the limit (3) in the following way. We choose a fixed lightlike vector e and timelike vectors a, b and write the current momenta k and k' as

$$K = ue + \alpha$$

$$K' = ve + b$$

where the variables u and v are allowed to approach ∞ . Then for the fixed hadron momentum p we have

$$\begin{aligned} 2k \cdot p &\longrightarrow 2ue \cdot p & K^2 &\longrightarrow 2u(e \cdot \alpha) \\ 2k' \cdot p &\longrightarrow 2ve \cdot p & K'^2 &\longrightarrow 2v(e \cdot b) \\ v &\longrightarrow 2(u-v)e \cdot p & t &\longrightarrow 2(u-v)e \cdot (\alpha - b) \end{aligned} \quad (4)$$

and the ratios are finite as $u \rightarrow \infty, v \rightarrow \infty, u/v \rightarrow \text{const.}$ We write in (2)

$$e^{-ik(x-t) + ik'y} = e^{-iue(x-t) + iv e \cdot y} e^{-i\alpha(x-t) + iby}$$

In the limit $u \rightarrow \infty, v \rightarrow \infty$, we probe lightlike planes $E(e,0)$ and $E(e,z)$ defined by the vector and the points 0 and z respectively. So x and z are together on lightlike planes parallel to $E(e,0)$. Now only the space-time region where the four-current product has its strongest singularities in the restricted domain parallel to $E(e,0)$ determines the amplitude in the limit (3). This is the line where the lightlike plane $E(e,0)$ touches the light-cone, which is just the space-time region where one might apply the algebra of the bilocal currents ^{2),3)}.

If we use the algebra of Ref. 2), we can work out the most singular part of the four-current product above. The singularity structure of this product is the same as in the formal diagram of Fig. 2 where the solid line denote formal massless quarks.

The leading singularity of the four-current product (connected part) is described by a bilocal current which might be written formally as :

$$\tilde{F}_r(x, y) \sim : \bar{q}(x) Q^* \gamma_r q(y) : - : \bar{q}(y) Q^* \gamma_r q(x) : \quad (5)$$

where Q is the charge matrix of the quarks. Also the corresponding bilocal axial vector current appears. But its contribution drops out if one averages over the spins.

We describe the proton expectation values of the bilocal current (5) by a function $A(\tau)$, defined as follows :

$$\langle p | \tilde{F}_\sigma(x, y) | p \rangle = 2ip_\sigma \int_{-1}^1 e^{i\tau p(x-y)} A(\tau) d\tau \quad (6)$$

One can easily work out the contribution of the strongest singularity on the light-cone to the amplitude (1). Taking the spin- and polarization average of the amplitude (1) we can use the Klein-Nishina formula generalized for off-shell photons according to the formal diagram in Fig. 2 which is given by the following expression ⁶⁾

$$M_{(p, k, k')}^{KN} = \text{tr} (\hat{p} + \hat{k} - \hat{k}') Q^{\mu\nu} \hat{p} \bar{Q}_{\mu\nu} \quad (7)$$

$$Q_{\mu\nu} = \frac{1}{(p+k)^2} \gamma^\mu (\hat{p} + \hat{k}) \gamma^\nu + \frac{1}{(p-k')^2} \gamma^\nu (\hat{p} - \hat{k}') \gamma^\mu$$

and

$$M_{(p, k, k')}^{KN} = \frac{1}{(k^2 + 2pk)} \left[\left(\frac{1}{(k^2 + 2pk)^2} + \frac{1}{(k'^2 + 2pk')^2} \right) (2pk(2pk' + k^2 - k'^2) - k^2 v) \right. \\ \left. + \frac{v(k^2 + k'^2)}{(k^2 + 2pk)(k'^2 + 2pk')} \right] \delta(v+t) \quad (8)$$

We get the wanted result if we replace $M_{(p,k,k')}^{KN}$ by $M_{(\tau p,k,k')}^{KN}$, integrate over τ using the function $A(\tau)$ as a weight function and neglect terms which are less leading in the limit (3) :

$$M_{(p,k,k')} \longrightarrow \int_{-1}^1 d\tau A(\tau) \frac{1}{(k^2 + \tau \cdot 2pk)} \delta(\tau v + t)$$

$$\left[\left(\frac{1}{(k^2 + \tau \cdot 2pk)^2} + \frac{1}{(k'^2 - \tau \cdot 2pk')^2} \right) \left(\tau \cdot 2pk (\tau \cdot 2pk' + k^2 - k'^2) - k^2 \tau \cdot v \right) \right. \\ \left. + \frac{\tau v (k^2 + k'^2)}{(k^2 + \tau \cdot 2pk)(k'^2 - \tau \cdot 2pk')} \right] \quad (9)$$

Carrying out the integration we obtain the following behaviour of the amplitude (1) in the limit (3) ($0 \leq \eta \leq 1$) :

$$M_{(p,k,k')} \longrightarrow \frac{A(\eta)}{v(k^2 + \eta \cdot 2pk)}$$

$$\left[\left(\frac{1}{(k^2 + \eta \cdot 2pk)^2} + \frac{1}{(k'^2 - \eta \cdot 2pk')^2} \right) \left(2pk \cdot \eta (\eta \cdot 2pk' + k^2 - k'^2) - k^2 \cdot \eta \cdot v \right) \right. \\ \left. + \frac{\eta \cdot v (k^2 + k'^2)}{(k^2 + \eta \cdot 2pk)(k'^2 - \eta \cdot 2pk')} \right] \quad (10)$$

We also write down the formal cross-section for this process putting in the usual flux factor :

$$\frac{d^2 \sigma}{dt dv} (k^2, k'^2, s = k^2 + 2pk) = \frac{-2\pi \alpha^2}{(k^2 + 2pk)} M_{(p,k,k')} \quad (11)$$

and it behaves in the scaling region scale-invariant like

$$\frac{d^2\sigma}{dt dv} \sim \frac{1}{v^3} F(\eta, \eta', \varphi, \varphi)$$

where F is a function of the dimensionless ratios (3) only. In (10) the scaling function $A(\varphi)$ given by (4) appears. It is important to note that this function is completely determined by the electroproduction and neutrino production scaling functions according to the decomposition

$$Q^* = \frac{1}{3^4} \begin{pmatrix} 16 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \frac{1}{3^4} \left(6\sqrt{\frac{3}{2}} \lambda_0 + \frac{5\sqrt{3}}{2} \lambda_8 + \frac{15}{2} \lambda_3 \right) \quad (12)$$

Using the representations of the electro and neutrino production scaling functions ²⁾, we obtain :

$$\begin{aligned} A(\varphi) = & \frac{1}{18\varphi} \left[6 (F_2^{en}(\varphi) + F_2^{ep}(\varphi)) - (F_2^{\nu n}(\varphi) + F_2^{\nu p}(\varphi)) \right] \\ & + \frac{5}{18\varphi} \left[F_2^{ep}(\varphi) - F_2^{en}(\varphi) \right] \\ & + \frac{5}{3^4\varphi} \left[(F_2^{\nu n}(\varphi) + F_2^{\nu p}(\varphi)) - 3(F_2^{en}(\varphi) + F_2^{ep}(\varphi)) \right] \end{aligned} \quad (13)$$

An explicit check of relation (13) by experiment would verify the fractional charges of the underlying quark triplet. In other words, it would give us information about the SU_3 structure of the electromagnetic current. If the electromagnetic current would have a SU_3 singlet component as it does for example in the Han-Nambu model ⁷⁾ our result (13) would be modified. In the Han-Nambu model which introduces as any three-triplet model a new SU_3 spin the singlet component of the electromagnetic current transforms as a pure octet component in the new SU_3 spin. Therefore the electromagnetic current leads to transitions of ordinary hadron states (singlets under the new SU_3 spin) to charmed states which transform as an octet in the new SU_3 spin.

Recently it was stressed by Lipkin ⁸⁾ that the usual quark model and the Han-Nambu model give completely the same results as long as the production of charmed states is not observed. We point out that this is no longer true in our case, where we are dealing with inclusive experiments induced by two electromagnetic currents. This can be seen in the following way. The transition amplitude

$$\langle p | j_\mu(x) j_\nu(y) | n \rangle$$

where $|p\rangle$ and $|n\rangle$ are both singlets under the new SU_3 group receives contributions from the charm part of the electromagnetic current, since the product of two octets in the new SU_3 space contains a singlet component which has matrix elements between ordinary hadron states. The diagram of Fig. 1 describes the electroproduction of a lepton pair plus anything. But it is not the only diagram which contributes to that process. We also could have contributions corresponding to the diagrams of Fig. 3. In all these additional diagrams only one current couples to the hadron. If we sum up all final states we obtain terms which involve the electroproduction amplitude in the scaling region which is known, but also terms coming from the interference of the processes in Fig. 2 and the process in Fig. 1. The latter involve three current amplitudes, which can also be calculated in the light-cone approach. The interference terms are determined by the expectation value of the following bilocal current written in formal quark fields : $\langle p | \bar{q}(x) \gamma_\mu Q^3 q(y) - h.c. | p \rangle$. In such a way we can calculate the additional processes (Fig. 2) as well.

What happens if we do not work in the limit (3) but consider instead the case where we are dealing with photons on their mass shell ? Here one has the advantage that the additional diagrams (Fig. 2) are not present. Certainly one cannot prove light-cone dominance in this case, and one has to make additional assumptions in order to make statements about this process ⁹⁾. Let us go back to Eq. (10) and consider the case ($k^2 = 0, k'^2 = 0$). Just taking the contribution coming from the most singular part on the light-cone we obtain in the limit

$$t, v, 2p \cdot k \longrightarrow \infty, \quad k_1^2 = 0, \quad k_2^2 = 0$$

$$\xi = -\frac{t}{v}, \quad \frac{2p \cdot k}{2p \cdot k'} = \xi \quad \text{fixed} \quad (14)$$

$$\frac{d^2\sigma}{dt dv} \rightarrow - \frac{2\pi\alpha^2}{(2pk)^2 \xi} A(\xi) \left(\frac{\xi^2+1}{\xi} \right) \quad (15)$$

But how do we know if in this case the lower lying singularities do not contribute? E.g., one could expect that the vector meson pole gives some contribution in the limit (14). In order to clarify the situation we consider ordinary deep inelastic scattering which might be described by the formal diagram of Fig. 4. We take ξp as the four-momentum of the formal quark. The scaling phenomenon is a consequence of the fact that the formal quark propagator (singularity function in the light-cone approach) provides us with a δ function :

$$\delta[(\xi p + q)^2 - m^2] = \frac{1}{|2p\xi|} \delta\left[\frac{\xi^2}{2p\xi} + \xi + \frac{\xi^2 p^2 - m^2}{2p\xi}\right] \quad (16)$$

The scaling function in the deep inelastic region is given by :

$$F(\xi) = \int F(\tau) \delta\left(\frac{\xi^2}{2p\xi} + \xi\right) d\tau \quad (17)$$

where $F(\xi)$ is the Fourier transform of the expectation value of the bilocal operator (in a formal parton approach the momentum distribution function). It is easy to see in which limit only the leading singularity on the light-cone contributes. This would be the case where one can neglect the term $(\xi^2 p^2 - m^2)/(2p\xi)$ in (16), that means : in the ordinary Bjorken limit $-q^2 \rightarrow \infty$, $2pq \rightarrow \infty$, $-(q^2/2pq)$ fixed. But also in the Regge limit q^2 fixed, $2pq \rightarrow \infty$, the influence of this term is killed, and in the limit the amplitude is given completely by the function $F(\xi)$ at $\xi = 0$ provided $F(\xi)$ is well behaved at $\xi = 0$. From the electroproduction experiments in the deep inelastic region we know that the scaling function $F_1(\xi)$ [scaling limit of $W_1(q^2, \nu)$], which is relevant for our considerations here, is not well behaved at $\xi \rightarrow 0$, since it appears to diverge linearly. We stress that it is just this divergent behaviour at small ξ which forbids us to apply the light-cone considerations also in the Regge limit, where we work at fixed external masses. It can be shown that also the vector meson poles at timelike q^2 are generated by the small ξ region.

In our case [see Eq. (9)] appears the δ function

$$\delta\left(\frac{t}{v} + \tau\right)$$

from the quark propagator, and in the limit (13) we do not pick up contributions from the region $\tau \approx 0$, since we stay at fixed $\tau = -t/v$ in the limit (14). So we expect naively that contributions of poles in the current channels like vector meson poles do not appear in the limit (14). However, photons on their mass-shell act like hadrons, and one can apply the idea of limiting fragmentation to our process¹⁰⁾. In the limit (14) one can arrive just at the case where the outgoing photon is a fragment of the incoming proton so we have clearly a large momentum transfer between incoming and outgoing photon as required by (14), and one is working in the Feynman scaling region. Our scaling variable ξ is related to the Feynman scaling variable x by : $\xi = (x/1-x)$. On the other hand the cross-section in the Feynman scaling region does not show the naive scale invariant behaviour as in (15), and we have clearly a violation of (15) in the Feynman scaling region, where the transverse momentum of the outgoing photon is small. Within the light-cone language the amplitude in the Feynman scaling region is determined by a superposition of many lower-lying singularities which cannot be described just by a formal quark mass but have a complicated origin. So it seems to be that the simple test proposed in Ref. 9) does not work at small transverse momentum of the outgoing photon (or lepton pair at small invariant mass). One way out would be to consider only events with large transverse momenta. In this case the behaviour of the cross-section might be described by Eq. (15) and might provide us with a simple test of the bilocal algebra. Similar considerations also apply the processes where one of the photons is very far off-shell, while the other one stays on or near the mass shell¹¹⁾.

We would like to thank M. Gell-Mann, J. Prentki and P. Roy for discussions on this subject and W. Bardeen and C. Jarlskog for reading the manuscript.

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Fig.1

Fig.2

Fig.3

FIG.4