Archives

EXCLUSIVE CROSS-SECTIONS AND THE THERMODYNAMIC MODEL

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A B S T R A C T

The formalism of generating functions is applied to obtain expressions for the exclusive n particle cross-section $\sigma_n(s,w)$ from the thermodynamic model for single particle spectra and many-particle correlations.

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The thermodynamic model (TM) for particle production ¹⁾ has been formulated such as to determine inclusive single particle spectra

$$\rho^{(1)}(\vec{p}) = \frac{E}{\sigma_T} \frac{d^3\sigma}{d^3p}, \quad (1)$$

which are related to the average multiplicity $\langle n \rangle$ by

$$\langle n \rangle = \int \frac{d^3p}{E} \rho^{(1)}(\vec{p}) = F^{(1)}(s). \quad (2)$$

The particles produced result from the decay of fireballs.

To extend the TM to two-particle distributions ²⁾ which are related to the second moment $\langle n(n-1) \rangle$,

$$\langle n(n-1) \rangle = \int \frac{d^3p_1}{E_1} \frac{d^3p_2}{E_2} \frac{d^6\sigma}{\frac{d^3p_1}{E_1} \frac{d^3p_2}{E_2}} \frac{1}{\sigma_T} = F^{(2)}(s), \quad (3)$$

for identical particles 1 and 2, it was necessary to specify the production mechanism of the fireballs. The average masses and number of fireballs might increase with the collision energy and depend also on the fireball rapidity. A general form for this relation was discussed in Ref. 2). The two extreme limiting cases of this relation were found to lead to results equivalent to the multiperipheral model on the one hand and to the diffractive excitation model on the other.

The parameter w describes the dependence of the fireball mass M_F on the total energy; its possible dependence on the fireball rapidity η can be described by a parameter v . In Ref. 2), two possible choices were discussed:

$$(i) \quad M_F^{(i)}(s, \eta) = M_c \left(\frac{\sqrt{s}}{M_c} \right)^w \left(e^{-|\eta|} \right)^v, \\ 0 \leq w, v \leq 1, \quad \text{for} \quad M_F^{\min} < M_F(s, \eta) < M_F^{\max}, \quad (4)$$

and

(ii)

$$M_F^{(ii)}(s, \eta) = M_c \ln \left[\frac{\sqrt{s}}{M_c} \cdot e^{-|\eta|v} \right]. \quad (5)$$

In Ref. 2), for (ii), it was used $v = 1$.

Equations (4) and (5) are possible choices for M_F ; they are certainly not the only choices. For the choice in Eq. (4), it was shown in Ref. 2) that the transition between different production pictures of the fireballs is obtained by the parameter w changing in the range $0 \leq w \leq 1$. The parameter v is responsible for less important changes in two-particle rapidity correlations. Therefore we discuss in the following the two ansatzes (4) and (5) in the simple case $v = 0$.

Should the choice (4) describe indeed correctly the energy dependence of the fireball mass, then one parameter w describes all higher moments $F^{(n)}(s)$, $n \geq 2$, and all the other kinds of correlations such as, e.g., rapidity correlations of several particles. Therefore, adopting the choice (i), Eq. (4), the parameter w can be determined from experiment and might furnish complete information on all possible inclusive distributions. Similarly, the choice (ii), Eq. (5), allows to compute all inclusive distributions without further assumptions.

According to the formalism of generating functions ^{3),4)}, the complete knowledge on all inclusive distributions is equivalent to the knowledge of all exclusive cross-sections. Therefore we can use the knowledge of all the moments in the TM to determine exclusive quantities like, e.g., topological cross-sections or n particle cross-sections $\sigma_n(s, w)$. These will be functions of s , n and, in the choice (i), of the parameter w . In this way the TM can be applied to exclusive processes.

Here we only line out the method and discuss moments and integrated partial cross-sections σ_n as function of s , n and w . We leave to later work the extraction of more detailed quantities.

It is our aim to give an example that exclusive quantities can be obtained from the TM. For this we choose the partial cross-sections $\sigma_n(s, w)$ which can be obtained from the thermodynamic moments, the first of which were given in Ref. 2) at high s . For simplicity we limit ourselves

mentioned in Ref. 1), 2).

values of w in choice (i) and for choice (ii)

$$F^{(1)}(j) = AB \ln \left(\frac{j}{M_i^2} \right) + C_1 \quad (6)$$

$H \equiv 1$, which we shall use in this work.

from the same fireball. This is displayed in the Figure.

For the integrated $1, 2, \dots, n$ particle distributions we have

$$\begin{aligned} F^{(1)}(s) &= K^{(1)}(s), \\ F^{(2)}(s) &= K^{(2)}(s) + [K^{(1)}(s)]^2, \\ F^{(3)}(s) &= K^{(3)}(s) + 3 K^{(2)}(s) K^{(1)}(s) + [K^{(1)}(s)]^3, \\ F^{(4)}(s) &= K^{(4)}(s) + 4 K^{(3)}(s) K^{(1)}(s) + 3 [K^{(2)}(s)]^2 \\ &\quad + 6 K^{(2)}(s) [K^{(1)}(s)]^2 + [K^{(1)}(s)]^4, \\ &\dots \dots \dots \end{aligned} \quad (7)$$

The factors are those of the cluster expansion ³⁾.

With $K^{(n)}(s)$ we denote the contribution from n particles emerging from one fireball. For these we take the expressions obtained from the TM in the cases (i) and (ii) above. So far, these quantities have been determined only up to leading order in s ; the exact values also in non-leading terms can only be obtained from a non-approximate treatment of the kinematics and of the integrand in the thermodynamic n particle spectra.

To take into account only leading terms of the inclusive moments means that for $\sigma_n(s, w)$ further terms are missing which could be rather important. Therefore our treatment should only be considered as a first attempt. Our results could eventually be modified in various respect by a more detailed knowledge of the integrated inclusive distributions.

For the functions $K^{(n)}(s)$ we have :

- in case (i) with $0 \leq w < 1$:

$$K^{(n)}(s) = AB \ln\left(\frac{s}{M_c^2}\right) \left[B\left(\frac{\sqrt{s}}{M_c}\right)^w\right]^{n-1}, \quad (8)$$

with

$$n \geq 1,$$

$$A = (1-w) \frac{m}{q_0 M_c} C = (1-w) \bar{A}, \quad (9)$$

and

$$B = \frac{Vz}{(2\pi)^2} 2m^2 T_0 \frac{q_0 M_c}{m} \underline{P}. \quad (10)$$

The $K^{(n)}(s)$ have a factor $\ln(s/M_c^2)$ due to our choice $v = 0$; besides this, they are consistent with those given in Ref. 2). The factors A and B are due to the integration of the thermodynamic n particle spectra as worked out for the first few $K^{(n)}(s)$ in Ref. 2). In B the factor $\underline{P}$ is due to the $dy_{\perp}$ integration. All other factors have been defined in Ref. 2); only the leading terms are taken into account.

- in case (ii), Eq. (5) with $v = 0$:

$$K^{(n)}(s) = \bar{A} \left[\bar{B} \ln\left(\frac{s}{M_c^2}\right)\right]^n, \quad n \geq 1, \quad (11)$$

with

$$\bar{A} = 2 \bar{\bar{A}}, \quad \bar{B} = B/2. \quad (12)$$

The correlations between 2,3,4... particles are, respectively

$$\begin{aligned} f^{(2)}(s) &= F^{(2)}(s) - [F^{(1)}(s)]^2 \\ f^{(3)}(s) &= F^{(3)}(s) - 3 F^{(1)}(s) F^{(2)}(s) + 2 [F^{(1)}(s)]^3 \\ f^{(4)}(s) &= F^{(4)}(s) - 4 F^{(1)}(s) F^{(3)}(s) - 3 [F^{(2)}(s)]^2 + \\ &\quad + 12 [F^{(1)}(s)]^2 F^{(2)}(s) - 6 [F^{(1)}(s)]^4 \\ &\dots \dots \dots \end{aligned} \quad (13)$$

Often, as in Ref. 2), $f^{(2)}(s)$ is denoted by $R^{(2)}(s)$.

The negative terms on the right-hand side of Eq. (13) exactly cancel those terms which are due to contributions of more than one fireball. In our scheme, the n particle correlation is related to the contribution to $F^{(n)}(s)$ of one fireball decaying into n particles. This simplicity, which is due to our choice $H \equiv 1$, again justifies that we have weighted each topology with the appropriate factor, as shown in the Figure. In fact we assume

$$f^{(n)}(s) \simeq K^{(n)}(s). \quad (14)$$

The generating function $\Psi(s, w, h)$ can be constructed ^{3), 4)} from the inclusive n particle correlations $f^{(n)}(s, w)$ by

$$\Psi(s, w, h) = \exp \left[\sum_{n=1}^{\infty} \frac{h^n}{n!} f^{(n)}(s, w) \right]. \quad (15)$$

On the other hand, it is

$$\Psi(s, w, h) = \sum_{n=2}^{\infty} (1+h)^n \psi_n(s, w), \quad (16)$$

where $\psi_n(s, w)$ are the exclusive n particle cross-sections, normalized by σ_T ,

$$\begin{aligned} \psi_n(s, w) &= \frac{\sigma_n}{\sigma_T} = \frac{1}{n!} \left. \frac{\partial^n \Psi(s, w, h)}{\partial h^n} \right|_{h=-1} = \\ &= \frac{1}{2\pi i} \oint \frac{dh}{(1+h)^{n+1}} \Psi(s, w, h). \end{aligned} \quad (17)$$

Knowing the inclusive correlations $f^{(n)}(s, w)$, we can deduce the exclusive cross-sections $\psi_n(s, w)$ and study their behaviour as functions of w , n and s .

From the definition (15) and from the expressions (8) and (11), we find :

- in case (i) with $w = 0$

$$\Psi(s, 0, h) = \left[\frac{s}{M_c^2} \right]^A [\exp(hB) - 1] \quad (18)$$

This is a special case of the multiperipheral case treated by Mueller³⁾; we therefore do not discuss it further here.

- in case (i) with $0 < w < 1$

$$\Psi(s, w, h) = \exp \left\{ A \ln \left(\frac{s}{M_c^2} \right) \cdot \left(\frac{\sqrt{s}}{M_c} \right)^{-w} \left[\exp \left\{ B h \left(\frac{\sqrt{s}}{M_c} \right)^w \right\} - 1 \right] \right\} \quad (19)$$

- in case (ii)

$$\Psi(s, h) = \exp \left\{ \bar{A} \left[\left(\frac{s}{M_c^2} \right)^{\bar{B} h} - 1 \right] \right\} \quad (20)$$

According to Eq. (17), we obtain the exclusive cross-sections $\psi_n(s, w)$ by differentiation of $\Psi(s, w, h)$ at $h = -1$.

- Case (i), $0 \leq w \leq 1$

Keeping only the leading term in s , we find for the $2 \rightarrow n$ particle cross-section $\psi_n(s, w)$

$$\psi_n(s, w) = \frac{\sigma_n(s, w)}{\sigma_T} \approx \frac{AB}{n!} \ln\left(\frac{s}{M_c^2}\right) \left[B \left(\frac{\sqrt{s}}{M_c}\right)^w\right]^{n-1} \cdot \exp\left[-B \left(\frac{\sqrt{s}}{M_c}\right)^w\right], \quad (21)$$

$n < B \left(\frac{\sqrt{s}}{M_c}\right)^w.$

For $n > B(\sqrt{s}/M_c)^w$, that is in the tail of the multiplicity distribution, the n particle cross-section can be estimated by the saddle-point method, leading to

$$\psi_n(s, w) = \frac{\sigma_n(s, w)}{\sigma_T} \approx \left[2\pi B (n+1) \left(\frac{\sqrt{s}}{M_c}\right)^w\right]^{-1/2} \cdot \exp\left\{-\frac{n+1}{B \left(\frac{\sqrt{s}}{M_c}\right)^w} \left[\ln\left(\frac{n+1}{AB \ln\left(\frac{s}{M_c^2}\right)}\right) - 1\right]\right\} \quad (22)$$

for $n > B(\sqrt{s}/M_c)^w$.

- Case (ii)

Up to leading order in s we have from Eqs. (20) and (17)

$$\psi_n(s) = \frac{\sigma_n(s)}{\sigma_T} \approx \frac{\bar{A}}{n!} e^{-\bar{A}} \left[\bar{B} \ln\left(\frac{s}{M_c^2}\right)\right]^n \left(\frac{s}{M_c^2}\right)^{-\bar{B}}, \quad (23)$$

for $n < \bar{B} \ln\left(\frac{s}{M_c^2}\right).$

In the tail of the multiplicity distribution, that is for large n , we find from the saddle-point method, for $n > \bar{B} \ln(s/M_c^2)$:

$$\psi_n(s) = \frac{\sigma_n(s)}{\sigma_T} \approx \left[2\pi \bar{B} (n+1) \ln\left(\frac{s}{M_c^2}\right)\right]^{-1/2} \cdot e^{-\bar{A}} \cdot \exp\left\{-\frac{n+1}{\bar{B} \ln\left(\frac{s}{M_c^2}\right)} \left[\ln\left(\frac{n+1}{\bar{A} \bar{B} \ln\left(\frac{s}{M_c^2}\right)}\right) - 1\right]\right\}. \quad (24)$$

We now discuss the s and n dependence of our result Ψ_n in the case (i), $0 < w < 1$, Eqs. (21) and (22), and in the case (ii), Eqs. (23) and (24).

- s dependence

To study the s dependence of $\Psi_{\bar{n}}(s, w)$ in the case (i), $0 < w < 1$, for a fixed multiplicity $\bar{n}$, we write Eq. (21) in the form :

$$\Psi_{\bar{n}}(s, w) \approx \frac{AB}{\bar{n}!} \ln\left(\frac{s}{M_c^2}\right) \cdot \exp\left\{(\bar{n}-1) \ln\left[B\left(\frac{\sqrt{s}}{M_c}\right)^w\right] - B\left(\frac{\sqrt{s}}{M_c}\right)^w\right\}. \quad (25)$$

In our treatment we find $\Psi_{\bar{n}} \rightarrow 0$ for $s \rightarrow \infty$. We expect that this behaviour will be modified by taking leading particle effects - i.e., diffraction - into account.

The exponential function gives a rise with increasing s which becomes steeper for larger multiplicity $\bar{n}$. At large s , the rate of decrease is approximately the same for all multiplicities $\bar{n}$. The maximum in s of $\Psi_{\bar{n}}(s, w)$ lies approximately at

$$\frac{\hat{s}}{M_c^2} \approx \left(\frac{\bar{n}-1}{B}\right)^{\frac{2}{w}} \quad (26)$$

and shifts therefore to higher s with increasing multiplicity. This shift can be rather fast for small w .

The height of this maximum decreases for higher multiplicities $\bar{n}$, as seen from inserting Eq. (26) into Eq. (25)

$$\Psi_{\bar{n}}(\hat{s}, w) \approx \exp\left\{(\bar{n}-1) \ln(\bar{n}-1) - (\bar{n}+\frac{1}{2}) \ln(\bar{n}+1)\right\} \simeq \bar{n}^{-\frac{3}{2}}. \quad (27)$$

At increasing values of w , the maximum becomes sharper and sharper.

In case (ii) our result, Eq. (23), respects the upper bound for $\sigma_n(s)$

$$\sigma_n(s) < \text{const.} \cdot s^{2\alpha-2} \frac{\left[K \ln\left(\frac{s}{m_c^2}\right)\right]^{n-2}}{(n-2)!}, \quad (K = \text{const.}) \quad (28)$$

that has been derived from a general multiperipheral scheme ⁵⁾. This is indeed evidence that the case (ii) belongs to a general multiperipheral scheme.

Comparing case (i) with case (ii), the main differences are :

- the bump of $\Psi_{\bar{n}}(s,w)$ as a function of s is more pronounced in the first case ;
- the shift of the maximum $\hat{s}$ of $\Psi_{\bar{n}}(s,w)$ for increasing values of $\bar{n}$ is much faster in the second case. In fact, for the case (ii), the equation corresponding to Eq. (26) is :

$$\frac{\hat{s}}{M_c^2} \approx \exp \left(\frac{\bar{n}}{\bar{B}} \right) . \quad (29)$$

The s behaviour we have found is in qualitative agreement with the expected behaviour of the integrated partial cross-sections.

- n dependence

To study the n dependence of $\Psi_n(s,w)$ we consider first case (ii) and write Eq. (23) in the form :

$$\Psi_n(\bar{s}) \approx \text{const.} \exp \left\{ n \ln \left[\bar{B} \ln \left(\frac{\bar{s}}{M_c^2} \right) \right] - \left(n + \frac{1}{2} \right) \ln (n+1) + (n+1) \right\} . \quad (30)$$

At large fixed $\bar{s}$, $\Psi_n(\bar{s})$ shows a rise due to the first term in the exponential function. The rise becomes steeper at increasing s . The maximum of the multiplicity distribution is found approximately at

$$\hat{n} \approx \bar{B} \ln \left(\frac{\bar{s}}{M_c^2} \right) \quad (31)$$

and increases in the same way as the average multiplicity with $\ln(s/M_c^2)$.

Before discussing the complete n dependence of $\Psi_n(\bar{s},w)$ in case (i) according to Eqs. (21) and (22), the influence of non-leading terms should be studied in a more detailed way.

The tail of the multiplicity distribution, in the cases we have treated here, is much shorter than in the diffractive excitation model, which gives ⁶⁾

$$\sigma_n \simeq \eta^{-2} . \quad (32)$$

On the other hand we obtain distributions broader than Poisson, as can be seen from Eqs. (22) and (24), since our $f^{(2)}(s)$ are large and positive. This feature is in qualitative agreement with the results on $\langle n \rangle$ and $f^{(2)}(s)$ found at 200 GeV/c ⁷⁾.

In view of the asymptotic treatment we have given here, our results are rather encouraging. They show that the TM indeed can make qualitative predictions for exclusive cross-sections.

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FIGURE CAPTION

Contributions from different fireball configurations to the integrated inclusive distributions.

$$F^{(1)}(s) = \bullet \text{---} 1$$

$$F^{(2)}(s) = \bullet \text{=}= 1_2 \quad + \quad \begin{matrix} \bullet \text{---} 1 \\ \bullet \text{---} 2 \end{matrix}$$

$$F^{(3)}(s) = \bullet \text{=}= 1_3 \quad + \quad \left\{ \begin{matrix} \bullet \text{=}= 1_2 \\ \bullet \text{---} 3 \end{matrix} \right. + \begin{matrix} \bullet \text{=}= 1_3 \\ \bullet \text{---} 2 \end{matrix} + \left. \begin{matrix} \bullet \text{---} 1 \\ \bullet \text{=}= 2_3 \end{matrix} \right\} + \begin{matrix} \bullet \text{---} 1 \\ \bullet \text{---} 2 \\ \bullet \text{---} 3 \end{matrix}$$

$$F^{(4)}(s) = \bullet \text{=}= 1_4 \quad + \quad 3 \left\{ \begin{matrix} \bullet \text{=}= \\ \bullet \text{---} \end{matrix} \right\} + 4 \left\{ \begin{matrix} \bullet \text{=}= \\ \bullet \text{=}= \end{matrix} \right\} + 6 \left\{ \begin{matrix} \bullet \text{=}= \\ \bullet \text{---} \\ \bullet \text{---} \end{matrix} \right\} + \begin{matrix} \bullet \text{---} 1 \\ \bullet \text{---} 2 \\ \bullet \text{---} 3 \\ \bullet \text{---} 4 \end{matrix}$$

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