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FACTORIZATION OF THE POMERON SECTOR AND CURRENTS
IN THE DUAL RESONANCE MODEL

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ABSTRACT

The Pomeron sector of the dual resonance model is exhibited by an explicit factorization of the non-planar orientable one-loop graph. A Pomeron propagator and a Pomeron-Reggeon vertex are obtained. The Pomeron propagator is shown to be essentially the same as in the Shapiro-Virasoro model. Gauge properties of the Pomeron-Reggeon vertex allow us to construct conserved or partially conserved vector and tensor currents of arbitrary rank and to exhibit a ghost-killing mechanism.

1. INTRODUCTION

An irritating problem in dual resonance models has been the appearance in the non-planar orientable one-loop graph of a singularity violating perturbative unitarity ¹⁾⁻³⁾. Since, on the other hand, this singularity appears in channels with zero isospin, and gives rise to a Regge cut of slope $\frac{1}{2}$, it has been associated with the Pomeron ¹⁾⁻³⁾. It was shown by Lovelace ⁴⁾ that in the case where the Reggeon intercept, $\alpha(0)$ is 1, this singularity is factorizable; further if we work with a 26-dimensional space-time ($D=26$) and assume that two sets of oscillators are cancelled by gauge conditions ($E=2$), we obtain a factorizable series of poles, so that perturbative unitarity is no longer violated. Remarkably enough, it was shown ^{5),6)} that for $\alpha(0)=1$, $D \leq 26$, the conventional dual resonance model (DRM) is ghost free, while for $D < 26$, $E=1$ indeed at $D=26$, one obtains $E=2$. The "magical number" 26 is model dependent, since in the Neveu-Schwarz model, it becomes 10 ⁶⁾⁻⁸⁾. So it is not unreasonable to think that one could obtain a model where the hadronic symmetries would be included, and where D would turn out to be 4. Meanwhile, one has to study the internal consistency of the existing theories, in the hope that the general features would survive in better models. A new problem arising in the conventional model is to factorize simultaneously Reggeon and Pomeron poles [for $D=26$, $\alpha(0)=1$] so as to be able to give to the Pomeron poles a particle interpretation.

In this paper, we exhibit the Pomeron sector by this double factorization. The Pomeron sector looks quite similar to the Shapiro-Virasoro model ^{9),10)} and the degeneracy grows exponentially with the mass, like in the Born term. It is remarkable that a new sector of particles is produced in the computation of a one-loop diagram. A new kind of duality (Pomeron particles/loops) appears, essentially due to the two-dimensional character of the Riemann surfaces associated with the graphs of DRM.

A related question is the presence or absence of ghosts in the Pomeron sector. Although we exhibit a gauge mechanism of the Virasoro type, a scalar ghost is found on the second level. The Pomeron propagator is the same as in the Shapiro-Virasoro model, except for an integration measure which creates the scalar ghost. One obtains an infinite number of gauges $\mathcal{L}_n$ which decouple some Pomeron states from physical Reggeon states. They are combinations of the usual gauges associated with the two kinds of oscillators which generate the Pomeron sector.

From the factorization, we can construct an infinite number of currents/off-shell amplitudes in the Reggeon sector. The lowest of them have been constructed. One finds a scalar current identical to the one studied in Refs. 11),12).

For such a current, multicurrent amplitudes have also been constructed, and have been shown to be free of the unphysical singularities which have plagued former models of currents in the DRM. On the next level, we obtain conserved symmetric and antisymmetric conserved tensor currents. The symmetric tensor can be considered as a candidate for the gravitational current.

These currents all have vacuum quantum number. It is, however, possible to restrict the integration region, as done in Ref. 11), to obtain currents with quantum number. In this way, one can obtain an infinite number of vector currents with non-vanishing form factors. However, it seems that one can build only one conserved vector current of such a type, if we require a "minimal" coupling.

A complete factorization of the Pomeron sector would involve a similar operation on multiloop graphs. In particular, from the graph containing two non-planar orientable loops, one could extract a triple Pomeron vertex, but it involves a complicated mathematical problem, which is still unsolved. So it is not obvious that a factorization of the Pomeron consistent with N loop diagrams does exist.

The article is organized as follows : in Section 2 the Jacobi transformation is used to express the non-planar orientable one-loop amplitude in terms of new integration variables. In Section 3, we factorize the Pomeron singularity in Reggeon space. In Section 4, we obtain the Pomeron-Reggeon vertex and the Pomeron propagator. In Section 5, we study the currents on the first and second level. The gauge properties of the Reggeon-Pomeron vertex are discussed in Section 6, where gauges are related to partial conservation laws for currents. Finally, it is briefly discussed in Section 7 how these techniques can be applied to the problem of regularizing the planar loop for $\alpha_0 = 1$, $D = 26$.

2. NON-PLANAR ORIENTABLE AMPLITUDE IN THE POMERON VARIABLES

We start with the expression of a non-planar orientable diagram in the case of a loop with M (M even) adjacent twists, as given in Ref. 1) :

$$F_{N,M} = (2\pi)^{13} g^{N+M} \int \prod_{i=1}^N dv_i \prod_{j=1}^M du_j w^{-2} \frac{f(w)^{-24}}{(-\ln w)^{13}} \prod_{i < j} [\Psi(C_{ji}) \text{ or } \Psi_{\tau}(C_{ji})]^{-P_i \cdot P_j} \quad (2.1)$$

Ψ is used when an even number of twisted lines are included between p_i and p_j , and Ψ_T is used for an odd number. C_{ji} is the product of the integration variables of the propagators between p_i and p_j , and $w = \prod_i u_i \prod_j v_j$ (see Fig.1).

The integration measure has been written for the Virasoro case ¹³⁾, i.e., by choosing the Reggeon intercept $\alpha(0)=1$. Further, we have chosen the case where the space-time dimension D is 26, and two sets of oscillators are decoupled ($E=2$). In this case we are going to show that the Pomeron singularity turns out to be a pole, rather than a cut. However, all the algebraic manipulations of this paper, if they depend crucially on the choice $\alpha(0)=1$, will not depend on the choice $D=26$, $E=2$; this choice will only serve to make the Pomeron a pole, but the properties of factorization are independent of this.

The integration region on the variables u_i, v_j is given by the restrictions :

$$\begin{aligned} 0 &\leq u_j u_{j+1} \leq 1 \\ 0 &\leq v_i \leq 1 \\ 0 &\leq u_n v_1 \dots v_N u_1 \leq 1 \end{aligned} \quad (2.2)$$

The functions Ψ and Ψ_T are defined as :

$$\Psi(x) = -\ln w \theta_1\left(\frac{\ln x}{\ln w} \middle| \frac{-2\pi i}{\ln w}\right) / \theta_1'(0 \middle| \frac{-2\pi i}{\ln w}) \quad (2.3)$$

$$\Psi_T(x) = -\ln w \theta_4\left(\frac{\ln x}{\ln w} \middle| \frac{-2\pi i}{\ln w}\right) / \theta_4'(0 \middle| \frac{-2\pi i}{\ln w}) \quad (2.4)$$

where θ_1, θ_4 are Jacobi θ functions in Bateman's notation. In this paper we will make use of two fundamental properties of the above integrand.

The first one is the Jacobi transformation of θ functions. This transformation has been very helpful in studying the renormalization problem of planar loops ¹⁾⁻³⁾, the nature of the Pomeron singularity, and more recently to generate currents ^{11), 12)}. It has been generalized on a Riemann surface, so that it is a general property of dual loops. In our case, this property is that Ψ and Ψ_T can be re-expressed in the following form :

$$\Psi(x) = -\frac{\ln w}{\pi} \sin \pi \frac{\ln x}{\ln w} \prod_{n=1}^{\infty} \left[\frac{1 - 2q^{2n} \cos 2\pi \frac{\ln x}{\ln w} + q^{4n}}{(1 - q^{2n})^2} \right] \quad (2.5)$$

$$\Psi_T(x) = -\frac{1}{2} \ln w \cdot q^{-\frac{1}{4}} \prod_{n=1}^{\infty} \frac{(1 - 2 q^{2n-1} \cos 2\pi \frac{\ln x}{\ln w} + q^{4n-2})}{(1 - q^{2n})^2} \quad (2.6)$$

where

$$q = \exp \frac{2\pi^2}{\ln w} \quad (2.7)$$

The first form of the Ψ functions enables us to study the unitarity singularities arising from $w \sim 0$, while with the second we can study the Pomeron singularities arising from $q \sim 0$ ($w \sim 1$).

The second property of which we will make use is that we can express the partition function $f(w)$ in terms of the variable q by use of the Hardy-Ramanujan formula

$$\frac{(2\pi)^{12}}{(\ln w)^{12}} f(w)^{-24} = w q^{-2} f(q^2)^{-24} \quad (2.8)$$

Although this formula is derived from the theory of elliptic functions no generalization of it is known in the case of the multiloop graphs for the generalized "partition function" which appear in the integrand. This is basically because the partition function is defined as a functional integral on a surface rather than as an Abelian differential on it. So the possibility of generalizing this work to multiloops raises an interesting mathematical problem.

To study now the Pomeron singularity arising from $q \sim 0$ we make the following change of variables :

$$\begin{aligned} \theta_2 &= \pi \frac{\ln v_2}{\ln w} \quad \dots \quad \theta_N = \pi \frac{\ln v_2 \dots v_N}{\ln w} \\ \theta_{N+1} &= \pi \frac{\ln v_2 \dots v_N u_1}{\ln w} \quad \dots \quad \theta_{N+M} = \pi \frac{\ln v_2 \dots v_M u_1 \dots u_M}{\ln w} \quad (2.9) \\ \theta_1 &= 0 \quad \quad \quad q = \exp \frac{2\pi^2}{\ln w} \end{aligned}$$

The Jacobian is :

$$\prod_i du_i \prod_j dv_j = \left(-\frac{\ln w}{\pi}\right)^{M+N-1} \frac{2\pi^2 w}{\ln^2 q} \frac{dq}{q} \prod_{i=2}^{M+N} d\theta_i$$

Defining

$$\bar{\Psi}(\theta) = \sin \theta \prod_{n=1}^{\infty} \frac{(1 - 2q^{2n} \cos 2\theta + q^{4n})}{(1 - q^{2n})^2} \quad (2.10)$$

$$\bar{\Psi}_T(\theta) = \prod_{n=1}^{\infty} \frac{(1 - 2q^{2n-1} \cos 2\theta + q^{4n-2})}{(1 - q^{2n})^2} \quad (2.11)$$

and using Eqs. (2.5), (2.6) and (2.8), we re-express $F_{N,M}$ in the following form

$$F_{N,M} = g^{N+M} \left(\frac{\pi}{2}\right)^{p^2} \int_0^1 dq q^{-1-\alpha_p(p^2)} f(q)^{-24} \prod_{i=2}^{M+N} d\theta_i \prod_{i,j} [\bar{\Psi}(\theta_i - \theta_j) \text{ or } \bar{\Psi}_T(\theta_i - \theta_j)]^{-R \cdot B} \quad (2.12)$$

where $\alpha_p(p^2) = 2 + \frac{1}{4}p^2$ (we work with the convention $\alpha' = \frac{1}{2}$). In this expression we see explicitly that from $q=0$ new singularities arise, in this case poles on a trajectory having half-slope of the ordinary Regge trajectory $\alpha(s) = 1 + \frac{1}{2}s$. If we had not been working with the choice $D=26$, $E=2$, the only difference would be in the Pomeron intercept and in the appearance of an additional term $\mu(q)$ in the integration measure containing logarithms of q . Therefore the Pomeron would not be a pole, but a cut.

The integration region is defined by :

$$0 \leq \theta_2 \leq \theta_3 \dots \leq \theta_N \leq \theta_{N+2} \leq \theta_{N+4} \leq \dots \leq \theta_{N+M}$$

$$\theta_{N+1} \leq \theta_{N+3} \leq \dots \leq \theta_{N+M-1} \leq \pi + \theta_{N+1}$$

Redefining

$$\theta'_{N+2i-1} = \theta_{N+2i-1} - \theta \quad \theta'_{N+1} = 0$$

$$\theta = \theta_{N+1}$$

we obtain $0 \leq \theta'_{N+1} \leq \theta'_{N+3} \leq \dots \leq \theta'_{N+M-1} \leq \pi$, θ unconstrained.

Thus we see that the integration over the angle θ counts the same contribution an infinite number of times since the integrand is periodic with period π in the variable θ . This is the periodicity problem met previously, and it is solved by restricting the integration over θ on a fundamental period, for instance $[0, \pi]$.

We relabel now the momenta belonging to one boundary as $p_1, p_2, \dots, p_r$, and to the other as $k_1, k_2, \dots, k_s$, and change the angles associated with $k_1, \dots, k_s$ into their complement with respect to π so that we obtain the following expression for the non-planar loop in the configuration of Fig. 2 :

$$F_{r,s} = g^{r+s} \left(\frac{\pi}{2}\right)^{p^2} \int_0^1 dq \, q^{-1-\alpha_p(p^2)} f(q^2)^{-24} \int_0^\pi d\theta \int_{i=2}^r d\theta_i \prod_{e=2}^s d\varphi_e$$

$$\prod_{i < j} \overline{\Psi}(\theta_j - \theta_i)^{-p_i \cdot p_j} \prod_{e < m} \overline{\Psi}(\varphi_m - \varphi_e)^{-k_m \cdot k_e} \quad (2.13)$$

$$\prod_{i,e} \overline{\Psi}_T(\theta + \theta_i + \varphi_e)^{-p_i \cdot k_e}$$

with the ordering :

$$0 = \theta_1 \leq \theta_2 \leq \dots \leq \theta_r = \pi$$

$$0 = \varphi_1 \leq \varphi_2 \leq \dots \leq \varphi_s = \pi$$

We shall now proceed to refactorize this expression firstly in the space of the usual harmonic oscillators. Of course, a self-energy non-planar operator reproducing $F_{r,s}$ has already been obtained, however, it is not useful for our purpose here since it is expressed in terms of variables w, x instead of q, θ . The operator which we are going to obtain will be equivalent, up to Virasoro gauges, to the one derived in Ref. 14), but will make the factorization properties of the Pomeron more obvious.

3. REFACTORIZATION IN THE REGGEON SPACE

We look now for an operator $\sum_T(p)$ defined in the space of the usual Reggeon harmonic oscillators $a_n, a_n^\dagger$ such that :

$$F_{r,s} = \langle \text{tree}(k_e) | \sum_T(p) | \text{tree}(p_i) \rangle \quad (3.1)$$

where :

$$|tree(p_i)\rangle = g^{r-1} \int \prod_{i=2}^r dy_i \prod_{i < j} (y_j - y_i)^{-p_i \cdot p_j} \left| \sum_{i=1}^r p_i [y_i] \right\rangle$$

$$\langle tree(k_e) | = g^{s-1} \int \prod_{e=2}^s dx_e \prod_{e < m} (x_m - x_e)^{-k_m \cdot k_e} \left(\sum_{e=1}^s k_e [x_e] \right) \quad (3.2)$$

with the ordering :

$$0 = y_1 \leq y_2 \leq \dots \leq y_r \leq 1$$

$$0 = x_1 \leq x_2 \leq \dots \leq x_s \leq 1$$

and where the notation $|p[x]\rangle$ denotes a coherent state of component in the n th mode $|p(x^n/\sqrt{n})\rangle$. Note that these trees are always defined as ending with a propagator.

We look for a $\sum_T$ operator of the following form :

$$\sum_T(p) = \frac{g^2}{\pi^2} 2^{-p^2} \int_0^1 dq q^{-1-\alpha_p(p)} f(q)^{-24} \int_0^\pi d\theta [\bar{\Psi}_T(\theta)]^{p^2} \exp \sigma(a, a^\dagger) \quad (3.3)$$

where the operatorial part $\sigma(a, a^\dagger)$ is such that :

$$\begin{aligned} \langle tree(k_e) | : \exp \sigma(a, a^\dagger) : | tree(p_e) \rangle &= \exp \{ -p^2 \ln \Psi_T(\theta) \\ &- \sum_{i,e} p_i k_e \ln \bar{\Psi}_T(\theta + \pi(y_i + x_e)) - \frac{1}{2} \sum_{i,j} p_i \cdot p_j \ln \left[\frac{\bar{\Psi}(\pi(y_i - y_j))}{\pi(y_i - y_j)} \right] \\ &- \frac{1}{2} \sum_{e,m} k_e k_m \ln \left[\frac{\bar{\Psi}(\pi(x_e - x_m))}{\pi(x_e - x_m)} \right] \end{aligned} \quad (3.4)$$

It is then easy to show that $\sum_T$ satisfies Eq. (3.1). The operatorial form of σ is the following :

$$\begin{aligned} \sigma(a, a^\dagger) &= (a | E | a) + (a^\dagger | E | a^\dagger) \\ &+ (a^\dagger | (E_T - I) | a) + (p | F_T [| a \rangle - | a^\dagger \rangle] \end{aligned} \quad (3.5)$$

The operators E, E_T, F_T are defined by their matrix elements between coherent states :

$$(k[x] | E_T | [y] p) = k \cdot p \ln \left[\frac{\bar{\Psi}_T(\theta + \pi x) \bar{\Psi}_T(\theta + \pi y)}{\bar{\Psi}_T(\theta + \pi(x+y)) \bar{\Psi}_T(\theta)} \right] \quad (3.6)$$

$$(k_1[x_1] | E | [x_2] k_2) = \frac{1}{2} k_1 \cdot k_2 \ln \left[\frac{\pi(x_1 - x_2) \bar{\Psi}(\pi x_1) \bar{\Psi}(\pi x_2)}{\bar{\Psi}(\pi(x_1 - x_2)) \pi x_1 \pi x_2} \right] \quad (3.7)$$

$$(p | F_T | [x] k) = p \cdot k \ln \left[\frac{\bar{\Psi}_T(\theta) \bar{\Psi}(\pi x)}{\bar{\Psi}_T(\theta + \pi x) \pi x} \right] \quad (3.8)$$

$$p = - \sum_i p_i = \sum_e k_e \quad (3.9)$$

The reader will notice the analogy with the $\sum_T$ operator of Ref. 14).

The matrix elements $(E)_{mn}$, $(E_T)_{mn}$, $(F_T)_m$ can be computed using Eqs. (3.6)-(3.8) :

$$(E_T)_{mn} = - \frac{\sqrt{mn}}{m!n!} \left(\frac{\partial}{\partial x} \right)^m \left(\frac{\partial}{\partial y} \right)^n \ln \left[\bar{\Psi}_T(\theta + \pi(x+y)) \right]_{x=y=0} \quad (3.10)$$

$$(E)_{mn} = - \frac{1}{2} \frac{\sqrt{mn}}{m!n!} \left(\frac{\partial}{\partial x} \right)^m \left(\frac{\partial}{\partial y} \right)^n \ln \left[\frac{\bar{\Psi}(\pi(x-y))}{\pi(x-y)} \right]_{x=y=0} \quad (3.11)$$

$$\{(p | F_T\}_m = - \frac{\sqrt{m}}{m!} \left(\frac{\partial}{\partial x} \right)^m \ln \left[\frac{\bar{\Psi}_T(\theta + \pi x) \pi x}{\bar{\Psi}(\pi x)} \right]_{x=0} \quad (3.12)$$

The computation of these matrix elements rests on the two following identities :

$$\ln \bar{\Psi}_T(\alpha) = - \sum_{r=1}^{\infty} \frac{q^r}{r(1-q^{2r})} \left[e^{2ir\alpha} + e^{-2ir\alpha} - 2 q^r \right] \quad (3.13)$$

$$\ln \Psi(\alpha) = \ln \sin \alpha - \sum_{r=1}^{\infty} \frac{q^{2r}}{r(1-q^{2r})} (e^{2ir\alpha} + e^{-2ir\alpha} - 2) \quad (3.14)$$

So that we can carry out the derivations with respect to x and y and obtain, for instance :

$$(E_T)_{mn} = \sum_{r=1}^{\infty} \frac{q^r}{r(1-q^{2r})} \frac{\sqrt{mn}}{m!n!} \left[e^{2ir\theta} (2i\pi r)^m (2i\pi r)^n + e^{-2ir\theta} (-2i\pi r)^m (-2i\pi r)^n \right] \quad (3.15)$$

It is then convenient to define the following complex current operators :

$$J_\mu^{(r)}(a) = \sum_{m=1}^{\infty} \frac{\sqrt{m}}{m!} (2i\pi r)^m a_{m,\mu} \quad (3.16)$$

$$J_\mu^{(-r)}(a) = \sum_{m=1}^{\infty} \frac{\sqrt{m}}{m!} (-2i\pi r)^m a_{m,\mu} \quad (3.17)$$

In terms of these currents $(a^\dagger | E_T | a)$ reads :

$$(a^\dagger | E_T | a) = \sum_{r=1}^{\infty} \frac{q^r}{r(1-q^{2r})} \left[e^{2ir\theta} J^{(r)}(a^\dagger) J^{(r)}(a) + e^{-2ir\theta} J^{(-r)}(a^\dagger) J^{(-r)}(a) \right] \quad (3.18)$$

and similarly :

$$(a | E | a) = (a | \tilde{\Delta} | a) + \sum_{r=1}^{\infty} \frac{q^{2r}}{r(1-q^{2r})} J^{(r)}(a) \cdot J^{(-r)}(a) \quad (3.19)$$

$$(p | F_T | a) = p \cdot (a | \tilde{F}_T) + p \cdot \sum_{r=1}^{\infty} \frac{q^r}{r(1-q^{2r})} \left[e^{2i\theta r} J^{(r)}(a) + e^{-2i\theta r} J^{(-r)}(a) \right] - p \cdot \sum_{r=1}^{\infty} \frac{q^{2r}}{r(1-q^{2r})} \left[J^{(r)}(a) + J^{(-r)}(a) \right] \quad (3.20)$$

The operators $\tilde{\Delta}$, $\tilde{F}_T$ are independent of θ , q and come from the first term in Eq. (3.14). They have been used in Ref. 11) to construct currents, and their form is :

$$\begin{aligned}\tilde{\Delta} &= \frac{1}{2} T (\mathcal{O} - I) \quad \mathcal{O} = \sum_{n \neq 0} P^n \quad P = M_- [-1] \\ P | [x] \rangle &= | [x+1] \rangle - | 1 \rangle \\ | \tilde{F}_T \rangle &= T (\mathcal{O} - I) | \infty \rangle\end{aligned}\tag{3.21}$$

A last trick to simplify the expression of $\sigma(a, a^\dagger)$ is to shift the currents $J^{(r)}$, defining :

$$j_r^{(r)}(a, p) = J_r^{(r)}(a) - P_r \quad r \neq 0 \tag{3.22}$$

$$j_r^{(r)}(a^\dagger, p) = J_r^{(r)}(a^\dagger) + P_r \tag{3.23}$$

In the following we will forget the label p in j_r , it being understood that p_μ is associated with a , $-p_\mu$ with $a^\dagger$. $\sigma(a, a^\dagger)$ takes then the following form :

$$\begin{aligned}\sigma(a, a^\dagger) &= (a | \tilde{\Delta} | a) + p (a | \tilde{F}_T) + (a^\dagger | \tilde{\Delta} | a^\dagger) - p (a^\dagger | \tilde{F}_T) \\ &+ \sum_{r=1}^{\infty} \frac{q^r}{r(1-q^{2r})} \left[e^{2ir\theta} j^{(r)}(a^\dagger) j^{(r)}(a) + e^{-2ir\theta} j^{(r)}(a^\dagger) j^{(r)}(a) \right] \\ &+ \sum_{r=1}^{\infty} \frac{q^{2r}}{r(1-q^{2r})} \left[j^{(r)}(a) j^{(r)}(a) + j^{(r)}(a^\dagger) j^{(r)}(a^\dagger) \right] \\ &- p^2 \ln \overline{\Psi}_T(\theta) - (a^\dagger | a)\end{aligned}\tag{3.24}$$

Introducing the operator :

$$t(a) = \pi^{-\alpha(p^1)} \exp \left[(a | \tilde{\Delta} | a) + p (a | \tilde{F}_T) \right] \tag{3.25}$$

We finally rewrite $\sum_T(p)$ in the form :

$$\sum_T(p) = q^2 \left(\frac{\pi}{2} \right)^{p^2} t(a^\dagger) \left[\int_0^1 dq q^{-1-\alpha_p(p^1)} f(q^1)^{-24} \int_0^\pi d\theta : \exp F(\theta, q) : \right] t(a)$$

where

$$F(\theta, q) = - (a^\dagger | a) + \sum_{r=1}^{\infty} \frac{q^r}{r(1-q^{2r})} \left\{ e^{2ir\theta} j^{(r)}(a^\dagger) j^{(r)}(a) + e^{-2ir\theta} j^{(-r)}(a^\dagger) j^{(-r)}(a) \right. \\ \left. + q^r [j^{(r)}(a) j^{(-r)}(a) + j^{(r)}(a^\dagger) j^{(-r)}(a^\dagger)] \right\} \quad (3.26)$$

The advantage of the above expression is that the parts which do not depend on q and θ have been factored out from the integrand. Further, in F , the dependence on the oscillators $a, a^\dagger$ is factorized.

If we expand in powers of q and keep only the zero order term, we obtain :

$$\sum_T(p) \simeq t(a^\dagger) |0_a\rangle g^2 \left(\frac{\pi}{2}\right)^{p^2} \frac{-1}{\alpha_p(p)} \langle 0_a | t(a) \quad (3.27)$$

The matrix elements $\langle 0_a | t(a) | \text{tree} \rangle$ are the scalar currents extensively studied in Ref. 11).

We have achieved our goal of obtaining an operator reproducing the non-planar orientable loop after the Jacobi transformation has been done on it, and on which the pole structure of the Pomeron is manifest. We have not, however, exhibited the particle structure of the Pomeron poles, which we will do in the following section.

Before that, let us make a few remarks on the Jacobi transformation. It uses two changes of variables :

$$1) \quad \ln q^2 \ln w = 4\pi^2$$

$$2) \quad z = \exp \left[2i\pi \frac{\ln x}{\ln w} \right] = \exp 2i\theta$$

It would be interesting to factorize directly, using the variables z instead of θ for the Reggeon space. In particular, this would rid us of the terms $t(a^\dagger)$ and $t(a)$ in $\sum_T(p)$ which essentially realize the transformation 2). But although it is possible to reproduce the integrand of the non-planar loop in such a factorization, it does not seem possible to recover the correct integration region.

However, if we do not insist to factorize the Reggeon poles simultaneously with the Pomeron poles, then it is possible to do such a factorization ⁴⁾. This

phenomenon has already been seen in the problem of defining off shell extrapolation for dual amplitudes and this choice has been shown to produce unphysical singularities in multicurrent amplitudes ^{11),12)}.

4. FACTORIZATION OF THE POMERON SINGULARITY

We will now write the operator $\sum_T(p)$ in the following form :

$$\sum_T(p) = \langle 0_{bc} | V(a^+, b, c) \Delta(p) V(a, b^+, c^+) | 0_{bc} \rangle \quad (4.1)$$

$\Delta(p)$ will be the Pomeron propagator defined in a space spanned by two sets of harmonic oscillators $b_{n,\mu}$; $c_{n,\mu}$, $n=1,2,\dots,\infty$; $\mu=1,2,\dots,26$

$$[b_{n\mu}, b_{m\nu}^+] = [c_{n\mu}, c_{m\nu}^+] = \delta_{mn} g_{\mu\nu} \quad (4.2)$$

The b and c oscillators commute with each other. V will be a vertex of transition between the Reggeon space and the Pomeron space.

Let us introduce

$$\mathcal{V}(a^+, b, c) = g \left(\frac{\pi}{2}\right)^{\frac{p^2}{2}} t(a^+) \exp \sum_{r=1}^{\infty} \left[\frac{b_r}{\sqrt{r}} j^{(r)}(a^+) + \frac{c_r}{\sqrt{r}} j^{(r)}(a^+) \right] | 0_a \rangle \quad (4.3)$$

$$\mathcal{V}(a, b^+, c^+) = g \left(\frac{\pi}{2}\right)^{\frac{p^2}{2}} \langle 0_a | \exp \left[\sum_{r=1}^{\infty} \left[\frac{b_r}{\sqrt{r}} j^{(r)}(a) + \frac{c_r}{\sqrt{r}} j^{(r)}(a) \right] \right] t(a) \quad (4.4)$$

Then one checks that

$$\sum_T(p) = \langle 0_{bc} | \mathcal{V}(a^+, b, c) \mathcal{D}(p) \mathcal{V}(a, b^+, c^+) | 0_{bc} \rangle \quad (4.5)$$

where

$$\mathcal{D}(p) = \int_0^1 dq q^{-1-\alpha_p(p^2)} f(q^2)^{-24} \int_0^\pi d\theta \prod_{r=1}^{\infty} \exp \frac{q^{2r}}{1-q^{2r}} b_r^+ c_r^+ \left[\frac{q^r e^{2ir\theta}}{1-q^{2r}} \right] b_r^+ b_r \left[\frac{q^r e^{-2ir\theta}}{1-q^{2r}} \right] c_r^+ c_r \exp \frac{q^{2r}}{1-q^{2r}} b_r c_r \quad (4.6)$$

Notice that $\mathcal{D}$ has two kinds of singularities : the Pomeron singularity, arising from $q=0$, but also singularities from $q=1$. These are the unitarity cuts ($w \approx 0$) contained in the self-energy graph. Therefore $\mathcal{D}$ cannot be considered as a satisfactory propagator for the Pomeron.

Using the equation :

$$\exp \alpha b^+ x^{b^+ b} = x^{b^+ b} \exp \frac{\alpha}{x} b^+$$

we can rewrite :

$$\begin{aligned} X_r &= \exp \left[\frac{q^{2r}}{(1-q^{2r})} b_r^+ c_r^+ \right] \left[\frac{q^r e^{2i\theta}}{1-q^{2r}} \right]^{b_r^+ b_r} \left[\frac{q^r e^{-2i\theta}}{1-q^{2r}} \right]^{c_r^+ c_r} \exp \left[\frac{q^{2r}}{(1-q^{2r})} b_r c_r \right] \\ &= \left(\frac{q^r e^{2i\theta}}{1-q^{2r}} \right)^{b_r^+ b_r} \left(\frac{q^r e^{-2i\theta}}{1-q^{2r}} \right)^{c_r^+ c_r} \exp b_r^+ c_r^+ (1-q^{2r}) \exp \frac{q^{2r}}{1-q^{2r}} b_r c_r \quad (4.7) \end{aligned}$$

Using now the following identity :

$$\exp \alpha b^+ c^+ \exp \beta b \cdot c = (1 - \alpha \beta)^D \exp \left[\frac{\beta b c}{(1 - \alpha \beta)} \right] (1 - \alpha \beta)^{b^+ b + c^+ c} \quad (4.8)$$

$$\exp \frac{\alpha}{1 - \alpha \beta} b^+ c^+$$

where D is the dimension of space time (here 26), we get

$$X_r = (1 - q^{2r})^{26} \exp b_r^+ c_r^+ (q e^{2i\theta})^{b_r^+ b_r} (q e^{-2i\theta})^{c_r^+ c_r} \exp b_r^+ c_r^+ \quad (4.9)$$

Introducing

$$\begin{aligned} H_b &= \sum_{r=1}^{\infty} r b_r^+ \cdot b_r \\ H_c &= \sum_{r=1}^{\infty} r c_r^+ c_r \end{aligned} \quad (4.10)$$

we obtain

$$\mathcal{D}(p) = \exp (b|c) \Delta(p) \exp (b^+|c^+) \quad (4.11)$$

$$\Delta(p) = \int_0^1 dq q^{-1 - \alpha_p(p^2) + H_b + H_c} [f(q^2)]^2 \int_0^\pi d\theta e^{2i\theta (H_b - H_c)} \quad (4.12)$$

We recognize that apart from the term in $f(q^2)^2$, this propagator is the same as in the Shapiro-Virasoro model ^{15),16)}. In particular, we have the condition :

$$H_b = H_c$$

The factors $\exp bc$ and $\exp b^+c^+$ can then be put in the vertex for the Reggeon-Pomeron transition and obtain

$$V(a^+, b, c) = \exp(bc) \mathcal{V}(a^+, b, c) \quad (4.13)$$

$$V(a, b^+, c^+) = \mathcal{V}(a, b^+, c^+) \exp(b^+c^+)$$

We see that the Pomeron vertex is quadratic both in the a and b, c oscillators, which reflects the loop structure from which we have started ^{*)}. In Eq. (4.12) we see that the convergent factor $f(q^2)^2$, which comes from the two cancelled sets of oscillators, plays an analogous role to the $(1-x)^{\alpha_0-1}$ in the propagator of the conventional model. Therefore we have obtained the important results that all the Pomeron poles are factorizable, and hence can receive a particle interpretation. Further, the degeneracy on the Pomeron daughters increases like in the Virasoro-Shapiro model.

We remind again the reader that if we had started from a different set for D, E , other than 26 and 2, we would have obtained the same result, except for a factor $\mu(q)$ in the integrand of $\Delta(p)$ which would contain $(\log q)$ factors. So it seems that the Pomeron singularities are always factorizable if $\alpha_0=1$, but that we need to go to the critical dimension only to make them poles rather than cuts.

So we have proved that a self-energy graph can always be replaced by a Reggeon-Pomeron transition. (See Fig. 3, where the thick line represents the Reggeons, the wavy line the Pomeron particles.) This is a new form of duality, first conjectured by Lovelace. If this property extends to multiloop graphs, which raises interesting mathematical questions, one could from the two loop non-planar orientable deduce the triple Pomeron vertex (Fig.4). Further many loops can be replaced by Pomeron. For instance, a tadpole can be viewed as a vacuum Pomeron transition :

*) As in the Shapiro-Virasoro model ¹⁶⁾ we can, if we want, include the part

$$\int_0^\pi d\theta \exp 2i\theta (H_b - H_c)$$

in the vertex. This is done essentially by the following change

$$V(a, b, c) \rightarrow \int_0^\pi d\theta V(a, b, c, \theta)$$

where $V(a, b, c, \theta)$ is obtained from $V(a, b, c)$ by the following replacement

$$b_r \rightarrow b_r e^{2ir\theta} ; c_r \rightarrow c_r e^{-2ir\theta}$$

$$T = \langle O_a | \Sigma_T (p=0) \rangle$$

(see Fig.5).

We note, however, that this new "duality" property is not peculiar to the DRM alone, but is found also in the two-dimensional Ising model ¹⁷⁾ where thermodynamic quantities can be expressed by expansions for both high and low temperatures. In the DRM, the parameter T defined by $q = e^{-T}$ ($w = e^{-2\pi^2/T}$) plays the role of a fictitious temperature and the integrand of the self-energy graph is also expressible in function of T or of $1/T$. The origin of this "duality" in both cases is the two-dimensional character of the underlying mathematical structure.

Knowing now that the Pomeron singularities are factorizable, we will look at the problem of ghosts for the Pomeron particles. In the next section, we shall study the first two levels and the currents associated with them.

5. CURRENTS ON THE FIRST AND SECOND LEVELS

We are going to consider $\langle \text{tree}(k) | \Sigma_T | \text{tree}(p) \rangle$ and expand the integrand of Σ_T in successive powers of q . We will then obtain an expansion of the type

$$\langle \text{tree}(k) | \Sigma_T | \text{tree}(p) \rangle = \sum_{N=0}^{\infty} \frac{1}{2N - \alpha_p(p')} R_N(p_i, k_j, p^2) \quad (5.1)$$

Since we have shown the factorization property of Σ_T [Eq. (4.1)], we know that only a finite number of Pomeron states contribute to the N^{th} level.

Hence :

$$R_N(p_i, k_j, p^2) = \sum_{\alpha=1}^{d(N)} \epsilon_{\alpha} T_{\alpha}(p_i, p^2) \cdot T_{\alpha}(k_i, p^2) \quad (5.2)$$

where the T_{α} are Lorentz tensors of rank smaller or equal to $2N$. $\epsilon_{\alpha} = \pm 1$. $d(N)$ is the degeneracy of the Pomeron on the N^{th} level.

In the case where we have no internal symmetry, $\epsilon_{\alpha} = +1$ corresponds to a positive norm state, while $\epsilon_{\alpha} = -1$ would be a ghost. If we have an internal symmetry, the product of ϵ_{α} by the relevant Chan-Paton ¹⁸⁾ factors has to be positive for a positive norm state.

Before we start looking at the first levels, we have to modify a little the factorization of Eq. (4.1) to make the positivity conditions more transparent. This is due to the fact that some Reggeon-Pomeron couplings could be, in principle, imaginary since the currents $j^{(r)}(a)$ are complex fields :

$$j^{(r)}(a) = j_+^{(r)}(a) + i j_-^{(r)}(a) \quad (5.3)$$

$$j^{(r)}(a) = j_+^{(r)}(a) - i j_-^{(r)}(a) = [j^{(r)}(a)]^*$$

where the * denotes the operation of complex conjugation.

$$j_+^{(r)}(a) = \sum_{m=1}^{\infty} \frac{\sqrt{2m}}{2m!} (-1)^m (2\pi r)^{2m} a_{2m} \quad (5.4)$$

$$j_-^{(r)}(a) = \sum_{m=1}^{\infty} \frac{\sqrt{2m+1}}{(2m+1)!} (-1)^m (2\pi r)^{2m+1} a_{2m+1}$$

(the Lorentz index has been omitted for simplicity.)

And we do not have $V(a, b^+, c^+) = [V(a^+, b, c)]^\dagger$ where $\dagger$ denotes the Hermitian conjugate (complex conjugation + transposition, changing a, b, c into a^+, b^+, c^+).

We can see that this problem is connected with the fact that the Pomeron singularity carries both even and odd twist parity C_μ . In a four-point function, the cyclic symmetry on the p 's and k 's guarantees that $C_\mu = -1$ states are absent. However, in a six-point function for instance, this cyclic symmetry is not identical to twisting and therefore $C_\mu = -1$ states are present. Decomposing :

$$|tree(p_i)\rangle = \frac{1}{2} [|P_1 \dots P_r\rangle + |P_r \dots P_1\rangle] + \frac{1}{2} [|P_r \dots P_1\rangle - |P_1 \dots P_r\rangle] \quad (5.5)$$

we notice that :

$$\langle tree(k) | \sum_T | p_1 \dots p_r \rangle$$

is given by Eq. (2.13), but with the ordering :

$$0 = \theta_r \leq \theta_{N+1} \leq \dots \leq \theta_1 \leq \pi$$

Making the change of variables $\theta_1' = \pi - \theta_1$ we see that $\langle tree | \sum_T | p_1, \dots, p_r \rangle$ is given by the expression (2.13) with the same ordering on the angles, except that now $\bar{\Psi}_T(\theta + \theta_i + \varphi_\ell)^{-p_i k_\ell}$ has to be replaced by $\bar{\Psi}_T(\theta + \varphi_\ell - \theta_i)^{-p_i k_\ell}$. So

$$\langle \text{tree} | \Sigma_T | p_1 \dots p_r \rangle = \langle \text{tree} | \Sigma'_T | p_r \dots p_1 \rangle$$

where Σ'_T is obtained from Σ_T by replacing $j^{(r)}(a)$ by $j^{(-r)}(a)$ keeping $j^{(r)}(a^\dagger)$, unchanged. This can be checked also directly by computing the action of the twisting operator Ω on $V(a, b^+, c^+)$.

We can rewrite

$$\begin{aligned} \Sigma_T(p) &= \frac{1}{2} \langle 0_{bc} | V(a^+, b, c) \Delta(p) [V(a, b^+, c^+) + V^*(a, b^+, c^+)] | 0_{bc} \rangle \\ &+ \frac{1}{2} \langle 0_{bc} | V(a^+, b, c) \Delta [V(a, b^+, c^+) - V^*(a, b^+, c^+)] | 0_{bc} \rangle \end{aligned} \quad (5.6)$$

(* is the complex conjugation). Since Σ_T itself is real, we obtain :

$$\begin{aligned} \Sigma_T(p) &= \Sigma_T^{(+)}(p) - \Sigma_T^{(-)}(p) \\ \Sigma_T^{(+)}(p) &= \langle 0_{bc} | \frac{[V(a^+, b, c) + V^*(a^+, b, c)]}{2} \Delta(p) \frac{[V(a, b^+, c^+) + V^*(a, b^+, c^+)]}{2} | 0_{bc} \rangle \\ \Sigma_T^{(-)}(p) &= \langle 0_{bc} | \frac{[V(a^+, b, c) - V^*(a^+, b, c)]}{2i} \Delta(p) \frac{[V(a, b^+, c^+) - V^*(a, b^+, c^+)]}{2i} | 0_{bc} \rangle \end{aligned} \quad (5.7)$$

The first term is even under twisting of the external lines, while the second is odd. The vertices for the Reggeon-Pomeron transition appearing on the left and on the right of the propagator $\Delta(p)$ are now real and Hermitian conjugate of each other, so that the usual reasoning that ghosts come only from the time excitations of the b, c operators applies.

Let us firstly consider the dual model without internal symmetry. Then when we sum over the non-cyclic permutations of external particles, the contribution of the second term vanishes. Therefore the only ghost particles the Pomeron may contain come from the metric $g^{\mu\nu}$, like in the ordinary theory. In particular, the Pomeron "photon" which we are going to exhibit is not coupled.

In the case of an internal symmetry, like $SU(2)$, G parity is conserved. In the theory where one assigns G parity $+1$ and -1 to the isoscalar and isovector ground states of the Reggeon, we assign G parity of $(-1)^t C_\mu$ to intermediate states of isospin t and twist parity C_μ . Since the Pomeron carries only $t=0$, the first term in Eq. (5.7) contributes only between states having $G=+1$, and the second between $G=-1$ states, so that their contributions are disconnected.

What is now required by unitarity to be positive is the amplitude where $r=s$, $k_i = -p_i$, and where the i particles on both sides are antiparticles of each other. The Chan-Paton factor which multiplies $\langle -p_1, \dots, -p_n | \sum_T | p_r, \dots, p_1 \rangle$ is therefore $\text{Tr}[\tau_{a_1} \tau_{a_2} \dots \tau_{a_r}] \text{Tr}[\tau_{a_1}^+ \tau_{a_2}^+ \dots \tau_{a_r}^+]$ where the τ matrices are $(1, \tau_0, \tau_+, \tau_-)$. Using the fact that

$$\text{Tr}[\tau_{a_1} \dots \tau_{a_r}] = (-1)^G \text{Tr}[\tau_{a_r} \dots \tau_{a_1}]$$

this Chan-Paton factor is

$$(-1)^G |\text{Tr}(\tau_{a_1} \dots \tau_{a_r})|^2$$

Therefore we see that the positivity condition implies that both $\langle \text{tree} | \sum_T^{(+)} | \text{tree} \rangle$ and $\langle \text{tree} | \sum_T^{(-)} | \text{tree} \rangle$ have positive E_α coefficients on each pole. We examine now the first two levels of the Pomeron sector.

1) Ground state

The ground state of the Pomeron is a tachyon with $(\text{mass})^2 = -8$. Introducing $|0\rangle \langle 0|$ as an intermediate state, we get

$$\langle \text{tree}(k) | \sum_T | \text{tree}(p) \rangle = \langle \text{tree}(k) | \sum_T^{(+)} | \text{tree}(p) \rangle \quad (5.8)$$

$$\approx \frac{-1}{d_p(p^2)} q^2 \left(\frac{\pi}{2}\right)^{p^2} S(p_i) S(k_j) \quad (5.9)$$

the scalar currents $S(p_i) = \int d\omega$, where

$$d\omega = q^{r-1} d\theta_1 \dots d\theta_r \prod_{1 \leq i < j \leq r} \exp(\theta_j - \theta_i)^{-p_i \cdot p_j} \quad (5.10)$$

$$[\theta_1 = 0 \leq \theta_2 \leq \dots \leq \theta_r \leq \pi]$$

have been studied in Refs. 11) and 12). As a matter of principle, if we study the residue on each Pomeron pole, we have to set p^2 on its mass shell. However, if we expand the Pomeron integrand in series of q^2 , we can study the currents $S(p_i)$ also away from $p^2 = -8$ since they have acceptable analytic properties as function of p^2 , and obey factorization and duality by themselves. We can indeed, when a current has been obtained, forget about the Pomeron from which it has been derived and study it by itself, since it is defined in the Reggeon space.

2) Second level

We look now at $\alpha_p(p^2)=2$, i.e., $p^2=0$. Since we have to expand $f(q^2)^2$, and since the condition $H_b=H_c$ has to be met, we insert

$$|0\rangle\langle 0| + b_{1\mu}^+ c_{1\nu}^+ |0\rangle\langle 0| b_{1\rho} c_{1\sigma} g^{\mu\rho} g^{\nu\sigma} \quad (5.11)$$

α) contribution of $\sum_T^{(+)}$

We obtain

$$\langle \text{tree}(k) | \sum_T^{(+)} | \text{tree}(p) \rangle = \frac{1}{2 - \alpha_p(p^2)} g^{\frac{1}{2}(\frac{\pi}{2})} p^2 \left[-2 S(p_i) S(k_j) + Z_{\mu\nu}(p_i) Z^{\mu\nu}(k_j) \right] \quad (5.12)$$

S is the previous scalar current and comes from expanding the partition function remaining in the integrand, and

$$\begin{aligned} Z_{\mu\nu}(p_i) &= \langle 0_a | \left[g_{\mu\nu} + \frac{1}{2} \left(j_{\mu}^{(+)}(a) j_{\nu}^{(-)}(a) + j_{\mu}^{(-)}(a) j_{\nu}^{(+)}(a) \right) \right] t(a) | \text{tree}(p_i) \rangle \\ &= \int d\omega \left[g_{\mu\nu} + \sum_{i,j} P_{i,\mu} P_{j,\nu} \cos 2(\theta_i - \theta_j) \right] \end{aligned} \quad (5.13)$$

This tensor is symmetric, and yields a spin two current which we may relate to the coupling of a graviton to the hadronic matter.

It is shown in the Appendix that indeed $Z_{\mu\nu}$ is conserved up to a gradient, and this independent of p^2 (even off shell) :

$$p^\mu Z_{\mu\nu}(p_i) = - p_\nu S(p_i) \quad (5.14)$$

We can then decompose $Z_{\mu\nu}$ into a traceless conserved current, a transverse part and a longitudinal part :

$$Z_{\mu\nu} = \Theta_{\mu\nu} + A \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) + B \frac{p_\mu p_\nu}{p^2} \quad (5.15)$$

$\Theta_{\mu}^{\mu} = 0$ and $p^\mu \Theta_{\mu\nu} = 0$ imply : $B = -1$, $A = (Z_{\mu}^{\mu} + S)/(D-1)$, where D is the space time dimension (here 26).

Using this decomposition, one gets :

$$Z_{\mu\nu}(p_i) Z^{\mu\nu}(k_j) = \Theta_{\mu\nu}(p_i) \Theta^{\mu\nu}(k_j) + \frac{1}{2S} [(Z_{\mu}^{\mu} + S)(Z_{\nu}^{\nu} + S)] + S(p_i) S(k_j) \quad (5.16)$$

so that :

$$\langle \text{tree}(p_i) | \Sigma_T^{(+)} | \text{tree}(k_j) \rangle = \frac{1}{2-\alpha_p(p)} g^2 \left(\frac{\pi}{2}\right)^2 \left\{ \Theta_{\mu\nu}(p_i) \Theta^{\mu\nu}(k_j) + \frac{1}{2S} [Z_{\mu}^{\mu} + S](p_i) [Z_{\nu}^{\nu} + S](k_j) - S(p_i) S(k_j) \right\} \quad (5.17)$$

Therefore three-states contribute at this level : a spin two symmetric tensor coupled through a traceless, conserved tensor. Two scalar currents one of which is a ghost. There is unfortunately no relationship between $S(p_i)$ and $Z_{\mu}^{\mu}(p_i) + S(p_i)$. Indeed, on a two-particle states :

$$S(p_1, p_2) = g \int_0^{\pi} d\theta \sin \theta^{-1-\alpha(p')} = g B\left(\frac{1}{2}; -\alpha\left(\frac{p'}{2}\right)\right) = 0$$

$$Z_{\mu}^{\mu}(p_1, p_2) = g \int_0^{\pi} d\theta \sin \theta^{-1-\alpha(p')} [D + p^2 - 4(p_1 \cdot p_2) \sin^2 \theta] = -8 B\left(\frac{1}{2}, \frac{1}{2}\right) \neq 0$$

and $S(p_i)$ is not identically zero at $p^2 = 0$ [$S(p_1, p_2, p_3) \neq 0$].

The ghost state we obtain comes only from the integration measure, so that we can believe it is model dependent. Indeed, if we had $E=1$ instead of 2, we see that the scalar ghost would decouple. So, there is a ghost killing mechanism working in the Pomeron sector ($Z_{\mu\nu}$ a priori may not have been conserved up to a gradient), although some ghosts are still present.

(β) contribution of $\Sigma_T^{(-)}$

$$\langle \text{tree}(k_i) | \Sigma_T^{(-)} | \text{tree}(p_j) \rangle = \frac{1}{2-\alpha_p(p')} g^2 \left(\frac{\pi}{2}\right)^{p^2} F_{\mu\nu}(k_i) F^{\mu\nu}(p_j) \quad (5.18)$$

$$F_{\mu\nu}(p_i) = \langle 0_a | \frac{1}{2i} (j_\mu^{(1)}(a) j_\nu^{(1)}(a) - j_\nu^{(1)}(a) j_\mu^{(1)}(a)) t(a) | t_{tree}(p_i) \rangle$$

$$= \int d\omega \sum_{i,j} p_{i\mu} p_{j\nu} \sin 2(\theta_i - \theta_j) \quad (5.19)$$

$F_{\mu\nu}(p_i)$ is an antisymmetric tensor in μ and ν ; again we see that some gauge mechanism works for the Pomeron since we obtain

$$p^\mu F_{\mu\nu}(p_i) = 0 \quad (5.20)$$

(see the Appendix). This conservation holds also away from the mass shell ($p^2 \neq 0$). $F_{\mu\nu}(p_i)$ itself has a pole at the photon mass ($p^2 = 0$) so that we are looking here really at a triple pole in the variable p^2 . We can exhibit the photon pole by writing

$$F_{\mu\nu} = 4 \int d\omega \sum_{i,j} p_{i\mu} p_{j\nu} \sin \theta_i \sin \theta_j \sin(\theta_i - \theta_j) + [p_\mu V_\nu - p_\nu V_\mu] \quad (5.21)$$

The first term has poles in p^2 starting at $\alpha(p^2) = 3$ while the second has the structure of the electromagnetic tensor, with

$$V_\mu(p_i) = \int d\omega \sum_i p_{i\mu} \sin 2\theta_i \quad (5.22)$$

This current has been introduced in Ref. 11). It contains the photon pole, and is conserved even off shell :

$$p^\mu V_\mu(p_i) = 0$$

We conclude this section by noting that the analysis of the Pomeron form factors on the first and second levels already yields a rich spectrum of conserved currents. It is tempting to assume that by expanding further the Pomeron, we will get an infinite number of conserved currents.

6. GAUGE PROPERTIES OF THE POMERON-REGGEON VERTEX AND CURRENTS

1) Gauge properties of the Pomeron vertex

We can now ask ourselves the following questions :

- a) are some Pomeron states decoupled from the physical Reggeon states ?
- b) are the spurious Reggeon states decoupled from the Pomeron states ?

We can answer the first question if we find some gauge operators which annihilate Pomeron states coupled to a physical Reggeon state. The factorization of the Pomeron sector has shown that it is quite similar to the Shapiro-Virasoro model, in which it has been shown by Yoshimura ¹⁶⁾ that there exist two sets of gauge operators associated with the two modes. It seems therefore reasonable to look for some relations of the following type

$$\begin{aligned} \langle 0_{bc} | V(a^+, b, c) \left(\sum_n \beta_n L_n^b \left(\frac{p}{2} \right) + \gamma_n L_n^c \left(\frac{p}{2} \right) \right) \\ = \sum_{n=0}^{\infty} \alpha_n L_{-n}^a(p) \langle 0_{bc} | V(a^+, b, c) \end{aligned} \quad (6.1)$$

We can determine the coefficients $\beta_n, \gamma_n, \alpha_n$ by sandwiching the Eq. (6.1) between coherent states in a, b and c modes. This is straightforward but rather lengthy, and we shall not give the details.

Let us define as usual

$$W_{-n}^a = L_{-n}^a(p) - L_0^a(p) + 1 \quad (6.2)$$

which are the gauges for off-shell physical states in the Reggeon sector.

We define also

$$\mathcal{L}_m = L_m^b \left(\frac{p}{2} \right) - L_{-m}^c \left(\frac{p}{2} \right) \quad (6.3)$$

We then have the following set of identities for an integer positive or negative :

$$\begin{aligned}
 & \langle 0_{bc} | V(a^+, b, c) (\mathcal{L}_m - \mathcal{L}_0 + m) \\
 &= -\frac{1}{2i\pi} \sum_{n=1}^{\infty} \frac{(2i\pi m)^{n+1}}{(n+1)!} W_{-n}^a(p) \langle 0_{bc} | V(a^+, b, c)
 \end{aligned} \tag{6.4}$$

so that when applying it on a Reggeon tree

$$\langle \text{tree} | \langle 0_{bc} | V(a^+, b, c) (\mathcal{L}_m - \mathcal{L}_0 + m) = 0 \tag{6.5}$$

It is easy to show that the $\mathcal{L}_m$'s satisfy the algebra

$$[\mathcal{L}_m, \mathcal{L}_n] = (m-n) \mathcal{L}_{m+n} \tag{6.6}$$

(let us note that there is no c number).

In order to know what these relations imply for on-shell Pomeron physical states, we must first remember that in the factorization only the states satisfying

$$\mathcal{L}_0 |\phi\rangle = 0 \tag{6.7}$$

occur. Since

$$P_0 = \frac{1}{\pi} \int_0^\pi d\theta e^{2i\theta \mathcal{L}_0} \tag{6.8}$$

is the projector on these states, we must find in fact the properties of

$$\langle 0_{bc} | V(a^+, b, c) P_0$$

From (6.6), $\mathcal{L}_m$ is a rising operator for the eigenvalue of $\mathcal{L}_0$.

Then

$$P_0 \mathcal{L}_m \mathcal{L}_{-m} = \mathcal{L}_m \mathcal{L}_{-m} P_0 \tag{6.9}$$

and since

$$\mathcal{L}_0 P_0 = 0$$

$$\begin{aligned}
 P_0 \mathcal{L}_m \mathcal{L}_{-m} &= (\mathcal{L}_m \mathcal{L}_{-m} - \mathcal{L}_{-m} \mathcal{L}_0) P_0 \\
 &= (\mathcal{L}_m - \mathcal{L}_0 + m) \mathcal{L}_{-m} P_0
 \end{aligned}$$

and therefore

$$\langle \text{tree} | \langle 0_{bc} | V(a^+, b, c) P_0 \mathcal{L}_m \mathcal{L}_{-m} = 0 \quad (6.10)$$

This means that all the states $\mathcal{L}_m \mathcal{L}_{-m} |\phi\rangle$ where $|\phi\rangle$ satisfies (6.7) are decoupled; but we see that the situation of defining physical states is not as clear as in the Reggeon sector because these operators $\mathcal{L}_m \mathcal{L}_{-m}$ mix states with different eigenvalues of $R_b = R_c$.

This situation has been illustrated in the previous section (we shall see in the next section that the conservation law used one consequence of these gauge properties) at the level $N=2$.

For the symmetric part, the fact that there exists a relation between the divergence of the tensor and the off-shell scalar occurring at the level $N=0$, allows us to decompose this tensor in conserved traceless parts, and two scalar parts. A simple counting shows that it has in fact reduced the number of states really present. Moreover, one of the two scalar parts is the one occurring at the level $N=0$, and this allows a partial cancellation of the ghost states coming from the partition function contained in the propagator. This is also true for the antisymmetric part since this tensor is conserved.

Then these gauges decouple some states occurring in the factorization [i.e., satisfying

$$R_a |\phi\rangle = R_b |\phi\rangle = \frac{N}{2} |\phi\rangle \quad]$$

but we have been unable to find some specific relations satisfied by these states as in the Reggeon sector.

In order to answer the second question, we must know what are the gauges in the pure Pomeron sector. This problem is very difficult, but we do not expect that the present factorization exhibits further gauge relations than the condition

$$\mathcal{L}_0 |\phi\rangle = 0$$

and this because of the presence of the partition function in the Pomeron propagator.

However, we can prove the following relation :

$$\langle 0_{bc} | V(a^+, b, c) \mathcal{L}_0 = \frac{1}{2i\pi} L_{+1}^a(p) \langle 0_{bc} | V(a^+, b, c) \quad (6.11)$$

Then the spurious Reggeon states created by $L_{+1}^a(p)$ (on the left) are decoupled from the Pomeron sector via the relation (6.7). But we have not been able to exhibit such a relation with $L_2^a(p)$.

2) Conservation laws for currents

As we have seen in the previous section, the factorization of the Pomeron seems to be the most suitable way of constructing currents in the dual model. That has already been done for scalar and vector currents ^{11),12)}. This can be generalized by defining the following operator tensor currents

$$T_{\mu_1 \dots \mu_n ; \nu_1 \dots \nu_m}^{\alpha_1 \dots \alpha_n ; \beta_1 \dots \beta_m}(a) = \langle 0_{abc} | V(a, b, c) b_{\mu_1}^{+\alpha_1} \dots b_{\mu_n}^{+\alpha_n} c_{\nu_1}^{+\beta_1} \dots c_{\nu_m}^{+\beta_m} | 0_{bc} \rangle \quad (6.12)$$

The conservation law of these currents can be deduced from the identity (6.4) that satisfy the vertex Pomeron-Reggeon. Let us show this explicitly for the vector and tensor currents introduced previously

$$V_\mu(a) = \frac{1}{2} \langle 0_{abc} | V(a, b, c) (b_\mu^{+(1)} - c_\mu^{+(1)}) | 0_{bc} \rangle \quad (6.13)$$

Then

$$\begin{aligned} p^\mu V_\mu(a) &= \langle 0_{abc} | V(a, b, c) [L_{-1}^b(\frac{p}{2}) - L_{-1}^c(\frac{p}{2})] | 0_{bc} \rangle \\ &= \langle 0_{abc} | V(a, b, c) (\alpha_{-1}^b + \alpha_{-1}^c) | 0_{bc} \rangle \\ &= \langle 0_{abc} | V(a, b, c) [(\alpha_{-1}^b - \alpha_0^b - 1) + (\alpha_{-1}^c - \alpha_0^c + 1)] | 0_{bc} \rangle \end{aligned}$$

Then from (6.4)

$$p^\mu V_\mu(a) | \text{tree} \rangle = 0 \quad (6.14)$$

Introducing also :

$$A_\mu(a) = \frac{1}{2} \langle 0_{abc} | V(a, b, c) (b_\mu^{+(1)} + c_\mu^{+(1)}) \rangle \quad (6.15)$$

Then

$$p^\mu A_\mu(a) = \langle 0_{abc} | V(a, b, c) [(\alpha_{-1}^b - \alpha_0^b - 1) - (\alpha_{-1}^c - \alpha_0^c + 1) + 2] | 0_{bc} \rangle$$

so that

$$p^\mu A_\mu(a) |tree\rangle = 2 \langle 0 | t(a) | tree \rangle \quad (6.16)$$

Let us now study the two tensor currents introduced previously, $F_{\mu\nu}$ and $Z_{\mu\nu}$

$$Z_{\mu\nu} = \frac{1}{2} \langle 0 | V(a,b,c) (b_\mu^{(+)} c_\nu^{(+)} + c_\mu^{(+)} b_\nu^{(+)}) | 0_{bc} \rangle | tree \rangle \quad (6.17)$$

Then

$$p^\mu Z_{\mu\nu} = \langle 0 | V(a,b,c) (c_\nu^{(+)}(\mathcal{L}_- - \mathcal{L}_0) - b_\nu^{(+)}(\mathcal{L}_+ - \mathcal{L}_0)) | 0_{bc} \rangle | tree \rangle \quad (6.18)$$

We can now use the following relations for commuting $b_\mu^{(n)}$ or $c_\mu^{(m)}$ with a $\mathcal{L}_n$

$$\begin{aligned} b_\nu^{(+m)}(\mathcal{L}_n - \mathcal{L}_0) &= (\mathcal{L}_n - \mathcal{L}_0 + n) b_\nu^{(+m)} + \frac{p_\nu}{2} \delta_{nm} \sqrt{n} + (m-n) b_\nu^{(+m)} \\ &\quad - \sqrt{m(m-n)} \Theta(m-n-1) b_\nu^{+(m-n)} + \sqrt{m(n-m)} \Theta(n-m-1) b_\nu^{+(n-m)} \end{aligned} \quad (6.19)$$

$$b_\nu^{(+m)}(\mathcal{L}_- - \mathcal{L}_0) = (\mathcal{L}_- - \mathcal{L}_0 + m) b_\nu^{(+m)} - \sqrt{m(m-n)} b_\nu^{+(m+n)} \quad (6.20)$$

and a similar relation for $c_\nu^{(+m)}$. We obtain

$$p^\mu Z_{\mu\nu} = \langle 0 | V(a,b,c) [(\mathcal{L}_- - \mathcal{L}_0 - 1) c_\nu^{(+)} - (\mathcal{L}_+ - \mathcal{L}_0 + 1) b_\nu^{(+)} - p_\nu] | 0_{bc} \rangle | tree \rangle \quad (6.21)$$

$$= -p_\nu \langle 0 | t(a) | tree \rangle \quad (6.22)$$

that is, the relation (5.14). Similarly

$$F_{\mu\nu} = \frac{1}{2} \langle 0 | V(a,b,c) (b_\mu^{(+)} c_\nu^{(+)} - b_\nu^{(+)} c_\mu^{(+)}) | 0_{bc} \rangle | tree \rangle \quad (6.23)$$

$$p^\mu F_{\mu\nu} = \langle 0 | V(a,b,c) [b_\nu^{(+)}(\mathcal{L}_- - \mathcal{L}_0) + c_\nu^{(+)}(\mathcal{L}_+ - \mathcal{L}_0)] | 0_{bc} \rangle | tree \rangle$$

$$p^\mu F_{\mu\nu} = \langle 0 | V(a, b, c) [(\mathcal{L}_1 - \mathcal{L}_0 + 1) b_\nu^+ + (\mathcal{L}_{-1} - \mathcal{L}_0 - 1) c_\nu^{+\dagger}] | 0_{bc} \rangle | tree \rangle \quad (6.24)$$

$$p^\mu F_{\mu\nu} = 0 \quad (6.25)$$

That is, the relation (5.20).

Finally, let us note that this procedure of constructing currents does not seem to give us in a simple way other conserved vector currents.

The most obvious way of constructing vector currents is to define

$$V_\mu^{(\pm n)}(a) = \frac{1}{2} \langle 0 | V(a, b, c) (b_\mu^{+\dagger} \pm c_\mu^{+\dagger}) | 0_{bc} \rangle \quad (6.26)$$

We can see easily that these vector currents are not conserved. Their divergence is related to the trace of some tensor currents. For instance, we have

$$p^\mu V_\mu^{(2)}(a) | tree \rangle = \frac{1}{\sqrt{2}} \langle 0 | V(a, b, c) [(\mathcal{L}_{-2} - \mathcal{L}_0 - 2) + \mathcal{L}_2 - \mathcal{L}_0 + 2 + \frac{1}{2} b_\rho^{+\dagger} b_\rho^{+\dagger} + \frac{1}{2} c_\rho^{+\dagger} c_\rho^{+\dagger}] | 0_{bc} \rangle | tree \rangle \quad (6.27)$$

$$p^\mu V_\mu^{(2)}(a) | tree \rangle = \frac{1}{2\sqrt{2}} \langle 0 | V(a, b, c) [b_\rho^{+\dagger} b_\rho^{+\dagger} + c_\rho^{+\dagger} c_\rho^{+\dagger}] | 0_{bc} \rangle | tree \rangle \quad (6.28)$$

In the same way

$$p^\mu V_\mu^{(-2)}(a) | tree \rangle = \frac{1}{\sqrt{2}} \langle 0 | V(a, b, c) [4 + \frac{1}{2} b_\rho^{+\dagger} b_\rho^{+\dagger} - \frac{1}{2} c_\rho^{+\dagger} c_\rho^{+\dagger}] | 0_{bc} \rangle | tree \rangle \quad (6.29)$$

An obvious way of constructing conserved currents is to take the divergence of an antisymmetric tensor. For instance

$$F_{\mu\nu}^{(2)}(a) = \frac{1}{2} \langle 0 | V(a, b, c) [b_\mu^{+\dagger} c_\nu^{+\dagger} - b_\nu^{+\dagger} c_\mu^{+\dagger}] | 0_{bc} \rangle \quad (6.30)$$

$$p^\mu F_{\mu\nu}^{(2)} | tree \rangle = \frac{1}{2\sqrt{2}} \langle 0 | V(a, b, c) [c_\nu^{+\dagger} b_\rho^{+\dagger} b_\rho^{+\dagger} - b_\nu^{+\dagger} c_\rho^{+\dagger} c_\rho^{+\dagger}] | 0_{bc} \rangle | tree \rangle$$

Currents of the type considered above are coupled only to zero isospin channel. If we want to introduce quantum numbers, we can do so like in Ref. 11) by restricting the integration region on the θ angles from $[0, \pi]$ into $[0, \frac{\pi}{2}]$, or, equivalently, by modifying the vertex $V(a, b, c)$ into $V'(a, b, c)$ where π has been replaced by $\pi/2$. The gauges still work and only the $V^{(-1)}$ current is conserved. Currents of the $p_\mu F_{\mu\nu}^{(2)}$ type are non-minimal because they contain the product of three oscillators. So it seems quite possible that there exists only one conserved vector current in the DRM which could be associated with the electromagnetic current.

These gauges allow us also to construct conserved symmetric tensors of arbitrary order as in the parton models or in conformal invariant field theories ^{19), 20)}.

7. APPLICATION TO THE PLANAR LOOP RENORMALIZATION

The problem of regularizing and renormalizing the planar loop has been extensively studied ²¹⁾. It has been found that the counter-term one has to subtract to make the loop finite has the meaning of a coupling constant renormalization.

We are, however, in the $D=26$ $E=2$ case forced to review the renormalization programme, as indicated in Ref. 22), since the subtraction of the counter-term is not enough to make the loop finite.

The planar loop diagram can be written after the Jacobi transformation

$$F_N = g^N \int_0^1 dq q^{-1-\alpha_P(0)} f(q^2)^{-24} \int \prod_{i,j} d\theta_i \pi_{ij}^{-1} (\theta_j - \theta_i)^{-P_i \cdot P_j}$$

It is therefore tempting to consider

$$\begin{aligned} F_N &= \lim_{p \rightarrow 0} g^{-1} \langle 0 | \Sigma_T(p) | P_N \dots P_1 \rangle \\ &= \lim_{p \rightarrow 0} g\left(\frac{\pi}{2}\right)^{p^2} \int_0^1 dq q^{-1-\alpha_P(p^2)} f(q^2)^{-24} \langle 0 | \left[\exp \sum_{r=1}^{\infty} \frac{q^{2r}}{r(1-q^{2r})} j^{(r)}(a) \bar{j}^{(r)}(a) \right] t(a) | t(a) \rangle \end{aligned}$$

The right-hand side is factorized and is finite for $p^2 < -8$. One is then led to define F_N as the analytic continuation of the right-hand side to $p^2 = 0$. The idea to relate the Pomeron to the planar loop in a similar way was firstly suggested by Goddard ²³⁾.

To perform the analytic continuation, we write

$$F_N = \lim_{p \rightarrow 0} g\left(\frac{\pi}{2}\right)^{p^2} \int_0^1 dq q^{-1-\alpha_p(p^2)} \langle 0 | \left\{ f(q^2)^{-24} \exp \left[\sum_{r=1}^{\infty} \frac{q^{2r}}{r(1-q^{2r})} j^{(r)}(a) \tilde{j}^{(r)}(a) \right] \right. \\ \left. - 1 - q^2 (24 + j^{(1)}(a) \tilde{j}^{(1)}(a)) \right\} t(a) | t_{\text{tr}} \rangle \\ + \lim_{p \rightarrow 0} g\left(\frac{\pi}{2}\right)^{p^2} \frac{-1}{\alpha_p(p^2)} \langle 0 | t(a) | t_{\text{tr}} \rangle + g\left(\frac{\pi}{2}\right)^{p^2} \frac{1}{2-\alpha_p(p^2)} \langle 0 | [24 + j^{(1)}(a) \tilde{j}^{(1)}(a)] t(a) | t_{\text{tr}} \rangle$$

We see that if $\alpha_p(0) < 2$, this would define $F^{(N)}$. The first "counter-term" is related to the scalar current, while the second is related to the trace of the tensor current. In our case $\alpha_p(0) = 2$ the last pole has to be subtracted away to define the planar loop.

Since the $\alpha_p(0) = 2$ is obviously unphysical, we will not devote more time to the meaning of this additional renormalization, and conclude that one can still renormalize the planar loop in a way consistent with factorization, duality, and analyticity.

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NOTE ADDED IN PROOF

After completion of this work, we learnt that L. Clavelli, C. Lovelace and J.A. Shapiro²⁴⁾ have obtained similar results on the Pomeron factorization, and have extended it to the Neveu-Schwarz model. However, they do not factorize simultaneously Reggeon and Pomeron poles.

A P P E N D I X

CURRENT CONSERVATION

We consider the scalar density

$$S = \int d\omega \quad (A.1)$$

$$d\omega = d\theta_1 \dots d\theta_r \prod_{1 \leq i < j \leq r} \sin(\theta_j - \theta_i)^{-P_i \cdot P_j} \quad (A.2)$$

$$\theta_1 = 0 \leq \theta_2 \leq \dots \leq \theta_r \leq \pi$$

Let us introduce also

$$V_\mu = \int d\omega \sum_{i=1}^r P_{i\mu} \sin 2\theta_i \quad (A.3)$$

$$A_\mu = \int d\omega \sum_{i=1}^r P_{i\mu} \cos 2\theta_i \quad (A.4)$$

In (A.1) we make the following change of variable, leaving $[0, \pi]$ invariant

$$\theta_i \rightarrow \theta_i + 2\epsilon \sin^2 \theta_i \quad (A.5)$$

Then

$$d\omega \rightarrow d\omega (1 + \epsilon p \sum_i P_i \sin 2\theta_i + O(\epsilon^2)) \quad (A.6)$$

hence

$$p^\mu V_\mu = 0 \quad (A.7)$$

Similarly, making the change of variables in (A.1) :

$$\theta_i \rightarrow \theta_i + \epsilon \sin 2\theta_i \quad (A.8)$$

we get :

$$d\omega \rightarrow d\omega [1 + \epsilon (p \sum_{i=1}^r P_i \cos 2\theta_i - 2) + O(\epsilon^2)] \quad (A.9)$$

Hence

$$p^\mu A_\mu = 2 S$$

We want now to compute the divergence of

$$T_{\mu\nu} = \int d\omega \sum P_{i\mu} P_{j\nu} \cos 2(\theta_i - \theta_j) = T_{\mu\nu}^{(1)} + T_{\mu\nu}^{(2)} \quad (\text{A.10})$$

$$T_{\mu\nu}^{(1)} = \int d\omega \sum P_{i\mu} P_{j\nu} \sin 2\theta_i \sin 2\theta_j \quad (\text{A.11})$$

$$T_{\mu\nu}^{(2)} = \int d\omega \sum P_{i\mu} P_{j\nu} \cos 2\theta_i \cos 2\theta_j \quad (\text{A.12})$$

To compute $p_\mu T_{\mu\nu}^{(1)}$ we make the change of variables (A.5) in Eq. (A.3) :

$$p^\mu T_{\mu\nu}^{(1)} = -4 \int d\omega \sum_i P_{i\nu} \sin^2 \theta_i \cos 2\theta_i \quad (\text{A.13})$$

To compute $p^\mu T_{\mu\nu}^{(2)}$ we start with Eq. (A.4), where we make the change of variables (A.8). We get

$$\begin{aligned} p^\mu T_{\mu\nu}^{(2)} &= \int d\omega \sum_i P_{i\nu} (2 \cos 2\theta_i + 2 \sin^2 2\theta_i) \\ &= - \int d\omega \left(\sum_i P_{i\nu} (4 \sin^2 \theta_i \cos 2\theta_i + 2) \right) \end{aligned} \quad (\text{A.14})$$

$$= -p_\mu T_{\mu\nu}^{(1)} - 2 p_\nu S \quad (\text{A.15})$$

and $p_\mu T^{\mu\nu} = -2S p^\nu$, which proves Eq. (5.14).

Let us now consider

$$F_{\mu\nu} = \int d\omega \sum P_{i\mu} P_{j\nu} \sin 2(\theta_i - \theta_j) = F_{\mu\nu}^{(1)} - F_{\mu\nu}^{(2)}$$

$$F_{\mu\nu}^{(1)} = \int d\omega \left(\sum P_{i\mu} \sin 2\theta_i \right) \left(\sum P_{j\nu} \cos 2\theta_j \right)$$

$$F_{\mu\nu}^{(2)} = F_{\nu\mu}^{(1)}$$

Using the same technique, we get through (A.4) and (A.5) :

$$p^\mu F_{\mu\nu}^{(1)} = \int d\omega \sum_j P_{j\nu} 4 \sin^2 \theta_j \sin 2\theta_j$$

through (A.3) - (A.8) :

$$p^\mu F_{\mu\nu}^{(2)} = \int d\omega \sum_j P_{j\nu} (2 \sin 2\theta_j - \sin 4\theta_j) = p^\mu F_{\mu\nu}^{(2)}$$

Hence $p^\mu F_{\mu\nu} = 0$. Note that we have never used a mass shell condition on p^2 .

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FIG.4

FIG.5