PHOTON AND HADRON INTERACTIONS IN NUCLEI

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1. INTRODUCTION

The subject of nucleus-particle interactions is exceedingly large, and I shall not even attempt to allude to most of it. I shall confine myself to those interactions of high-energy particles with nuclei which, hopefully, shed light on fundamental interactions. The converse aspect, wherein one uses energetic particles to study nuclear structure, will be largely ignored.

By high energy, I shall mean energies such that the wavelengths of both the incident and produced particles are short compared not only to nuclear, but also to nucleonic dimensions. This is true if $p \gg 1 \text{ GeV/c}$. At such short wavelengths the theoretical problem simplifies enormously, and a surprisingly accurate theoretical description of nuclear scattering has come into being.

Let me briefly sketch some of the unique features that nuclei offer when they are used as targets, features that are not available if one restricts oneself to hydrogen.

i) Variability of target quantum numbers

One can have targets with $I = 0$ and/or $J = 0$. This permits elegant tests of certain symmetry principles, and the isolation of specific production mechanisms. Thus in a production experiment on an $I = J = 0$ target, most exchange mechanisms that carry quantum numbers are forbidden if the target remains in its ground state, while other exchanges are selected if the final nucleus is excited to a state with known and specific quantum numbers.

ii) Spatial proximity between production and rescattering

It is this feature which permits measurement of scattering amplitudes of highly unstable systems by nucleons.
iii) Variability of target size

This feature is of crucial importance in all processes wherein the produced system bears the same quantum number as the projectile, because the production amplitude for the nucleus is then a coherent superposition of amplitudes from the separate nucleons. As a result copious beams of (unstable) secondaries can be produced, and their decay modes and scattering amplitudes can be more readily measured. Furthermore, it may be that at very high energies hadronic and electromagnetic interactions proceed over longitudinal distances which vastly exceed the lengths characteristic of elementary particles (i.e. \( \sim 1 \text{ fm} \)). Distances of this magnitude can only be studied with nuclear targets.

iv) Availability of strong Coulomb fields free of hadronic contamination

With high-energy projectiles -- either photons or hadrons -- collisions off the virtual photons in the Coulomb field become both feasible and highly intriguing. By means of such collisions one can measure the \( \eta^0 \) and \( \eta \) lifetime (Primakoff effect), and in the future perhaps such remarkable quantities as the total cross-section for \( \gamma + \gamma \rightarrow \text{hadrons} \).

Before I turn to an elaboration of some of these points, I should like to make another remark of a general nature. With the advent of SU(3) symmetry we learned that there is nothing fundamental that distinguishes the stable particles from at least some of their unstable kinfolk. No one believes any longer that \( \Omega^- \) is more "elementary" than \( \Delta \), or \( K \) than \( \eta \). By studying the collision amplitudes of objects such as \( \rho \) or \( \Sigma^* \), we are therefore enriching our knowledge of how "elementary" particles scatter, and not delving into something that is intrinsically less fundamental than \( \pi p \) or \( pp \) scattering. Furthermore, deep inelastic electron scattering, and other high-energy phenomena, seem to tell us that hadrons are complex structures with a very large number of degrees of freedom, perhaps no more "elementary" than nuclei\(^1\)). If this is actually true, highly excited hadronic states may not be very different from the stable particles, and the scattering properties of such states, which can only be studied in nuclei, would then be of great importance in unravelling the dynamics and structure of hadrons.
2. COHERENT, INCOHERENT AND DIFFRACTIVE PROCESSES

The most exhaustively studied and in many ways the most interesting nuclear production processes are coherent and diffractive. As these terms will occur frequently in these lectures, I shall, as a prelude to the more detailed theory, define these terms and describe their significance in a qualitative manner.

2.1 Coherent versus incoherent processes: definitions and examples

Consider a target composed of constituents, or at least having a size such that we can think of it as being composed of sub-units. If the amplitude for a process is the sum of amplitudes for the process occurring in the sub-units, it is said to be coherent. If, on the other hand, the intensities are added, it is said to be incoherent. Life is not quite so simple, of course, because there is partial coherence, etc.; we shall see these complications presently.

As a concrete and familiar example, consider a $\mu$ scattering elastically from an atom. Provided the energy is high enough and $Z$ not too large, the elastic amplitude is

$$A_{00}(q) = \left\langle \psi_0 \left| \int d^3r \ e^{i\mathbf{q} \cdot \mathbf{r}} \sum_{i=1}^{Z} \frac{e^{2}}{\mathbf{r} - \mathbf{r}_i} \right| \psi_0 \right\rangle, \quad (1)$$

where $\mathbf{q}$ is the momentum transfer, and $\psi_0$ the atom’s ground state. By simple manipulation

$$A_{00}(q) = \frac{4\pi e^2}{q^2} \ Z\Phi(q), \quad (2)$$

where

$$\Phi(q) = \frac{1}{Z} \left\langle \psi \left| \sum_i e^{i\mathbf{q} \cdot \mathbf{r}_i} \right| \psi \right\rangle \quad (3)$$

$$= \frac{1}{Z} \int n(r) e^{i\mathbf{q} \cdot \mathbf{r}} d^3r.$$
and \( n(r) \) is the electronic charge density normalized to \( Z \). The form factor \( F(q) \) is unity at \( q = 0 \); here one has full coherence as Eq. (2) is just \( Z \) times the Rutherford amplitude. For \( qR \gg 1 \), \( F \to 0 \) rapidly, where \( R \) is the atomic size. Thus coherence is lost if the momentum transfer is too large: the bigger the target, the more stringent this condition, but the more dramatic the coherence at \( q = 0 \).

Other old examples of coherent processes are pair production and Delbrück (i.e. photon) scattering in the nuclear Coulomb field.

An example of an incoherent process is the photoeffect by hard X-rays. Here the photoelectron's position at the time of absorption is, in essence, measured (recall Heisenberg's famous \( \gamma \)-ray microscope). By the general principles of quantum mechanics the amplitude can then not be coherent over the whole atom, and so it is the intensity which is integrated over the atom.

Another example of an incoherent cross-section is provided by atomic electron scattering where the final state of the atom is undetermined. This is usually the case at high energies. The cross-section is then

\[
\frac{d\sigma_{\text{sc}}}{d\Omega} = \int_{E_{\text{min}}'}^{E_{\text{max}}'} \sum_n |A_{0n}(q)|^2 \delta(E + E_0 - E_n - E') dE',
\]

where \( E, E' \) are the incident and final electron energies, and \( E_0, E_n \) the atom's energies. If \( E \gg E_n - E_0 \) for all \( n \) that are significantly excited, we can set \( E'_{\text{max}} \to \infty \), remove the \( \delta \)-function by \( E' \)-integration, and do the sum over \( n \) because \( \sum_n \langle \psi_n | \langle \psi_n | = 1 \). This is called the closure approximation -- approximation because of setting \( E'_{\text{max}} \to \infty \).

If \( d\sigma_0/d\Omega \) is the point Rutherford cross-section, Eq. (4) then becomes

\[
\frac{d\sigma_{\text{sc}}}{d\Omega} = \frac{d\sigma_0}{d\Omega} \left\langle \sum_{ij} e^{i\mathbf{q} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right\rangle_0
\]

and the inelastic cross-section is

\[
\frac{d\sigma_{\text{in}}}{d\Omega} = \frac{d\sigma_0}{d\Omega} \sum_{ij} \left[ \langle e^{i\mathbf{q} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \rangle_0 - \langle e^{i\mathbf{q} \cdot \mathbf{r}_i} \rangle_0 \langle e^{-i\mathbf{q} \cdot \mathbf{r}_j} \rangle_0 \right].
\]
For a first orientation, assume that the atom's wave function is a product of one-electron functions. Then

$$\sum_{ij} \langle e^{i\mathbf{q} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \rangle_0 = Z + \sum_{i \neq j} \langle e^{i\mathbf{q} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \rangle_0$$

$$= Z + Z(z - 1)|F(q)|^2$$

and

$$\frac{d\sigma_{in}}{d\Omega} = \frac{d\sigma_0}{d\Omega} Z[1 - |F(q)|^2] . \tag{7}$$

Thus the total inelastic cross-section (from the point of view of the atom, the inclusive cross-section) is incoherent -- proportional to $Z$. For $qR \gg 1$ it is just $Z$ times the elementary cross-section. Furthermore, it vanishes in the forward direction where the coherent cross-section has its peak.

### 2.2 Simple model of coherent nuclear processes

Let $A$ represent a nucleus of mass number $A$, $a$ the projectile and $b$ the produced system:

$$a + A \rightarrow b + A . \tag{8}$$

Also let $f_{ab}^{(i)}(\theta)$ be the amplitude for $a + b$ on nucleon $i$, with $\theta$ the production angle. For pedagogic purposes we consider a simple model patterned on the atomic example; it purposely ignores all multiple scattering. The amplitude for reaction (8) is then

$$A_{ab}(\theta) = \sum_i \langle \psi_0 | e^{i\mathbf{q} \cdot \mathbf{r}_i} f_{ab}^{(i)}(\theta) | \psi_0 \rangle , \tag{9}$$

where $\psi_0$ is $A$'s wave function, and $\mathbf{q} = \mathbf{p}_b - \mathbf{p}_a$ is again the momentum transfer.

In contrast to elastic scattering, there is now an essential complication because $q \neq 0$ at $\theta = 0$. In all our applications it is true that $p_a^2 \gg m_a^2$, $p_b^2 \gg m_b^2$, and $\theta^2 \ll 1$. When this is so $p_b \approx p_a = p$. At $\theta = 0$, $\mathbf{q}$ is along the incident ($z$) direction, and its value is...
\[ q_t = \frac{m_b^2 - m_A^2}{2p}. \]  

(10)

For \( \theta \neq 0 \), but \( \theta^2 \ll 1 \), \( \mathbf{q} \) has a transverse component \( q^t \), \( q^t = p \theta \), and \( q_t \) remains (10). In terms of \( t \)

\[-t = q^t + q^2.\]

There is another important complication not present in atomic Coulomb scattering, because we have two types of target constituents, and their spin orientation can be of crucial significance in production processes involving particles with spin. Let \( \alpha \) indicate the four kinds of nucleons (n, p, spin up and down), and let \( n_\alpha \) be their densities normalized so that

\[ \int d^3r \, n_\alpha(r) = N_\alpha; \quad \sum_\alpha N_\alpha = A. \]

There are then four form factors

\[ F_\alpha(q) = \frac{1}{N_\alpha} \int n_\alpha(r) \, e^{iqr} \, d^3r, \]  

(11)

and in terms of them Eq. (9) reads

\[ A_{ab}(\theta) = \sum_\alpha N_\alpha \, f_{\alpha}^a(\theta) \, F_\alpha(q). \]  

(12)

Now we can see what is needed for full coherence:

i) \( q^t \) and \( q^2 \) must be small compared to \( 1/R \). In the case of \( q^t \) this is achieved by setting \( \theta \to 0 \). For \( q^2 \) this means the machine energy \( p \) must be raised, or, conversely, the mass range \( m_b \) that can be reached is restricted. Thus even at \( \theta \), and for fixed \( p \), \( m_A \), and \( m_b \), the coherence will decrease as \( R \) (or, equivalently \( A \)) is increased.

ii) Scattering from \( p \) and \( n \) should not cancel, nor that from different spin orientation. In general, \( f_{ab}^{(i)} \) can be written as

\[ f_{ab}^{i} = f_{ab}^{A} + c_3^i f_{ab}^{B} + \gamma_3^i f_{ab}^{C} + c_3^i \gamma_3^i f_{ab}^{D}, \]  

(13)
where $\sigma^i$ and $t^i_3$ are the usual spin and isospin matrices for the $i^{th}$ nucleon. Then, to the extent that nuclei have $J = 0$ (except for deuterium, $J \ll A$), $\tilde{f}^B$ and $\tilde{f}^D$ cannot contribute. Furthermore, $\tilde{f}^C$ will have coefficient $(Z - N)$. In the cross-section this contribution is $(Z - A)^2$ as compared to $A^2$. Thus only the first term of Eq. (13) is fully coherent.

iii) It could still happen that $\tilde{f}^A_{ab}(\theta) \to 0$ as $\theta \to 0$, and this would spoil the coherence as $\tilde{f}^A$ would not become appreciable inside the angular region where the form factors $F_{\alpha}$ are appreciable.

### 2.3 $s$-channel selection rules for coherent processes

Clearly we must establish under what circumstances an amplitude will have a coherent piece, i.e. the first term of Eq. (13), and when $\tilde{f}$ is not found to vanish as $\theta \to 0$. This will be done on general grounds, not on the basis of production models.

For this purpose we may treat the nucleus as a $0^+$ particle. At times it is convenient to describe $a$ and $b$ by their helicity, $\lambda_a$ and $\lambda_b$. Recall that helicity is a pseudoscalar, and so it changes sign under space reflection.

Angular momentum conservation, all by itself, only provides restrictions at $\theta = 0$ because cylindrical symmetry prevails there -- otherwise there is no rotational symmetry in the configuration of a collision. Thus at $\theta = 0$ we have (recall that $A$ has $\lambda_A = 0$)

$$\lambda_a = \lambda_b,$$

and the amplitude vanishes at $\theta = 0$ if Eq. (14) is violated. As an application, consider photoproduction. Here $\lambda_a = \pm 1$ only, and $\lambda_b = 0$ is forbidden at $\theta = 0$. Thus $\pi^0$ photoproduction must vanish in the forward direction if $A$ is to remain in its ground state.

*) This is somewhat over simplified. Other terms, involving say $\sigma_1$ or $\tau_2$, can also occur in Eq. (13). The $\tau_{1,2}$ terms cannot contribute to Eq. (9) because $\psi_0$ is an eigenstate of $I_3$, and $\sigma_{1,2}$ cannot contribute if the nuclear spin $J = 0$. Departures from $J = 0$ are always small except in the lightest nuclei ($J \ll A$).
Space reflection requires\(^2\) the amplitudes to satisfy
\[
\langle \lambda_a | f(s,t) | \lambda_b \rangle = \nu_a \nu_b (-1)^{s_a - s_b} \langle -\lambda_a | f(s,t) | -\lambda_b \rangle ,
\]
(15)

where \(\nu_a\) is the naturality \((\pm 1)\) of a particle, i.e. \(\nu_a = (-1)^{s_a} P_a\)
where \(s_a\) and \(P_a\) are its spin and parity. Unless both \(\lambda_a\) and \(\lambda_b\) vanish, Eq. (15) only provides a relation between otherwise independent amplitudes, and parity therefore cannot force a \(\lambda \neq 0\) amplitude to vanish.

However, for \(\lambda_a = \lambda_b = 0\)
\[
\langle 0 | f | 0 \rangle = \nu_a \nu_b \langle 0 | f | 0 \rangle ,
\]
and therefore
\[
\nu_a = \nu_b
\]
(16)
is required in this case. This is the only rigorous restriction parity can provide; obviously it has nothing to say for baryon processes because \(\lambda = 0\) is impossible there.

As an illustration of Eq. (16), let \(a = \pi, b = \rho; \nu_a = -1, \nu_b = 1\).
Then \(\lambda_\rho = 0\) cannot be produced at any \(\theta\). Hence \(f\) vanishes for all \(\lambda_\rho\) at \(\theta = 0\), because angular momentum forbids \(\lambda_\rho = 1\) at \(\theta = 0\).

2.4 t-channel character of coherent processes.
Selection rules for diffraction dissociation\(^3\)

At high \(s\) and low \(t\), it is the t-channel that characterizes the production mechanism. We already know that coherence requires the absence of \(\tau_3\) terms in Eq. (13). Said another way, the t-channel isospin, \(I_t\), must vanish for coherence.

Coherent processes themselves naturally break up into two types: those that have vacuum quantum numbers, i.e. \(C_t = G_t = I_t = J_t = P_t = 0\), and those that do not. The former are closely akin to shadow or diffraction scattering, and such processes are called diffraction dissociation. An example of a coherent process that does carry quantum numbers is \(\omega\)-exchange (\(C_t = G_t = -1\)). Note that \(C\) and \(G\) need not be \(+1\) for nuclear coherence because nuclei are not close to being eigenstates of \(G\) and \(C\). An essential distinction between shadow or diffractive processes is that they are \(s\)-independent, or nearly so, whereas other coherent processes, such as \(\omega\)-exchange, drop as a power of \(1/s\).
In a coherent process, the naturality $\nu_t$ of the t-channel is necessarily $+1$ because we are combining two $0^+$ particles, and so the overall parity of the t-channel state is that of the orbital angular momentum. The $ab$ state must then also have $N_t = 1$.

\[
\begin{array}{c}
a \\
\downarrow \ \\
J_t, \nu_t \\
\uparrow \\
0^+ \ \\
0^+ \\
\end{array}
\]

If $a$ and $b$ are fermions, no restrictions emerge because all helicities are non-zero. Hence for baryons all the natural states $0^+, 1^-, 2^+ \ldots$ can always be formed from $ab$ and diffraction production is always a possibility for any $s_a$ and $s_b$.

For bosons consider first the examples $s_a = 0$, $s_b = 1$. By combining $s_a + s_b + L = J_t$, we construct the following families of states

\[ P_a P_b = \pm 1: \quad 3S_1^-, \quad 3P_0,1,2, \quad 3D_{i,2,3}^-, \ldots \]  \hspace{1cm} (17)

Here the subscript is $J_t$, the letter gives $L$, and the superscript $\pm$ the parity. We note that for $J_t > 0$, states of natural spin-parity ($\nu_t = +1$) are always present, and so coherent scattering is always possible. For $J_t = 0$ the situation is different, however, because the natural $0^+$ state is missing if $P_a = P_b$. Hence diffraction production does not contribute to $0^- \rightarrow 1^-$ processes, e.g. $\pi \rightarrow \rho$, or $K \rightarrow K^*$.

This result is easily generalized to the case where $s_b$ has any value ($s_a$ still zero):

\[ P_a \nu_b = 1. \]  \hspace{1cm} (18)

Thus for incident $\pi$ or $K$ beams, $b$ must have unnatural spin-parity if it is to be diffractively produced.
Finally, some consequences of $C$ and $G$. If $C_\pi = +1$, and $a = \pi$, $b$ can only be an odd number of $\pi$'s. Even numbers of $\pi$'s can be produced coherently (via $\omega$-exchange) but not diffractively. From $C_\pi = +1$ it follows that photoproduction of $f^0$ and $A$ cannot be diffractive, whereas $\rho$, $\omega$ and $\phi$ can be photoproduced diffractively.

3. **MULTIPLE SCATTERING THEORY**

The foregoing discussion was based, in part, on a simple model that ignores multiple scattering. Not everything was based on this simplifying assumption, of course. In particular, the arguments leading to the selection rules do not make reference to any details of the production mechanism. Furthermore, the qualitative features of the angular distribution, and the dependence on the minimum momentum transfer $q_n$, do not stem from the approximations inherent in Eq. (9). But if we hope to extract amplitudes for the elastic scattering of the produced state $b$ by nucleons, we must surely learn how to handle the multiple scattering. Multiple scattering effects are also of very considerable importance in determining the $A$-dependence of the cross-section because hadronic mean free paths in nuclear matter are of order $2-4$ fm. Hence a nucleus is fairly opaque to hadrons, and even when the coherence conditions are satisfied, the amplitude falls well below the naïve estimate of $A$ times the one-nucleon amplitude.

There is another important reason why we need a multiple scattering theory. Until now I have virtually ignored incoherent effects. This would not matter if we could establish, by measurement, that the nucleus had stayed in its ground state. In practice this is usually impossible. Hence we need a theoretical understanding of incoherent processes.

The modern theory of nuclear multiple scattering is due to Glauber\(^3\), and I shall now summarize its most important facets.

3.1 **High-energy potential scattering**

For purposes of orientation, consider non-relativistic scattering by a static potential $V$. If the wavelength $1/k$ is small compared to all dimensions characterizing $V$, the particle will be weakly reflected and
its wave function will depart only adiabatically from the incident wave \( e^{ikz} \). We therefore write the wave function as

\[
\psi(\mathbf{r}) = e^{ikz} \phi(r),
\]  

(19)

where \( \phi \) is assumed to vary slowly. With this latter assumption the three-dimensional Schrödinger equation is reduced to the first-order one-dimensional equation

\[
\left( \frac{d}{dz} + \frac{i}{v} V \right) \phi = 0,
\]  

(20)

where \( v = k/m \), the incident velocity. Hence

\[
\phi(\mathbf{r}) = \exp \left[ -\frac{i}{v} \int_{-\infty}^{z} V(bz') \, dz' \right],
\]  

(21)

where \( \mathbf{b} \) is the component of \( \mathbf{r} \) transverse to the incident direction \( z \); \( \mathbf{b} \) is the impact parameter. For \( z \) beyond the range of the potential

\[
\psi(\mathbf{r}) \rightarrow e^{ikz} S(\mathbf{b}),
\]  

(22)

where the S-matrix element is

\[
S(\mathbf{b}) = \exp \left[ -\frac{i}{v} \int_{-\infty}^{z} V(bz) \, dz \right].
\]  

(23)

Note that \(|S(\mathbf{b})| = 1\), which is unitarity when there are no open channels.

Equation (22) cannot be the correct form for all \( z \) because it does not possess an outgoing spherical wave. In fact, Eq. (22) -- the so-called eikonal wave function -- only is correct for\(^5\)

\[
z \lesssim ka^2,
\]  

(24)

where \( a \) is the range of \( V \). Equation (24) defines the domain where geometrical optics is valid. For \( z \approx ka^2 \) diffraction effects become important, and for \( z \gg ka^2 \) the familiar asymptotic form

\[
\psi \rightarrow e^{ikz} + \frac{e^{ikr}}{r} f(k, \rho)
\]  

(25)
holds. At high energies, in view of Eq. (24), the eikonal form can be valid over distances of order nuclear dimensions even if $a$ is the size of a hadron.

To calculate the scattering amplitude $f$ from Eq. (21) we must use a formula that only requires a knowledge of $\psi$ at small distances. This is

$$f(k_xk_i) = -\frac{m}{2\pi} \int e^{-ik_xr} V(r) \psi_{ki}(r) d^3r$$

$$= -\frac{m}{2\pi} \int e^{iq'z} \frac{\partial}{\partial z} \exp \left[ -\frac{i}{\nu} \int_{-\infty}^{r} V(bz') \, dz' \right]$$

$$= \frac{ik}{2\pi} \int e^{iq'b}[1-S(b)] d^3b . \tag{26}$$

Introduce the profile function $\Gamma(b)$ as the departure of $S(b)$ from 1:

$$\Gamma(b) = 1 - S(b) . \tag{27}$$

Then

$$f(kq) = \frac{ik}{2\pi} \int e^{iq'b} \Gamma(b) \, d^3b . \tag{28}$$

Note the important inversion formula

$$\Gamma(b) = \frac{1}{2\pi i k} \int e^{-iq'b} f(kq) \, d^2q . \tag{29}$$

3.2 Examples of profile functions

Although our preceding discussion was couched in the language of potential theory, the simple form

$$\psi(bz) = [1 - \Gamma(b)] e^{ikz}$$

for the wave function also applies to an enormous class of phenomena where inelastic scattering and particle production are important. In the case of a potential, $1 - \Gamma$ was found to be a phase factor of modulus one, and this simply expresses the fact that when the scattering is due to an inert potential the incident wave cannot be depleted in over-all intensity. When inelastic scattering is possible, on the other hand,
the probability of finding the projectile with the incident energy following a collision must be less than unity. From the viewpoint of the projectile wave function, this appears as an absorption of the incident wave. The potential $U$ then becomes complex, and $S$ is no longer a mere phase. Nevertheless, we always have the restriction

$$|S(b)| \leq 1,$$  \hspace{1cm} (30)

because the incident wave can only be depleted as it feeds the inelastic channels.

At this point it is useful to record some formulae for integrated cross-sections. If the scattering is confined to small angles, $d\Omega = \theta d\theta d\phi = d^2q/k^2$. From Eq. (28) the total elastic cross-section is

$$\sigma_{el} = \int d\Omega |f|^2 = \int |\Gamma(b)|^2 d^2b.$$  \hspace{1cm} (31)

The total cross-section is given by the optical theorem

$$\sigma_{tot} = \frac{4\pi}{k} \text{Im} f(q = 0) = 2 \int \text{Re} \Gamma(b) d^2b.$$  \hspace{1cm} (31')

To gain some familiarity with profile functions, we study two specific examples. In both cases $\Gamma$ shall be taken to be real, which means that the interaction is purely absorptive. Although 100% absorptive interactions do not exist in nature, this is a far more realistic description of high-energy phenomena than the opposite limit described by a real potential. Our first example is a completely absorbing sphere

$$\Gamma(b) = \begin{cases} 1, & b \leq R \\ 0, & b > R \end{cases}.$$  \hspace{1cm} (32)

This type of profile bears a reasonably close resemblance to the profile of a large nucleus as seen by hadrons, although in reality the edge is somewhat diffuse, and no nucleus is totally opaque. When $\Gamma$ is given by Eq. (32), Eq. (28) is easily evaluated:

$$f(k,\theta) = ikR^2 \frac{J_1(qR)}{qR}.$$
This is the familiar Fraunhofer diffraction pattern of a black disc. The first diffraction minimum occurs at

\[ q \approx \frac{3.8 R}{k} \].

Observe that at fixed momentum transfers \( q \) (not \( \theta \)), \( f \) grows linearly with \( k \). This is intimately connected with the fact that the total cross-section of an opaque disc is independent of the wavelength (provided \( kR \gg 1 \), of course). To see this we employ the optical theorem Eq. (31'); thus the total cross-section of the disc is

\[ \sigma_{\text{tot}} = 2\pi R^2. \] (33)

This is twice the area of the disc, for reasons that are too well known to be repeated here.

The square profile is a reasonably good description of a large nucleus, but is highly unrealistic for collisions between "elementary" hadrons. At momenta above several GeV/c, all hadron–hadron elastic differential cross-sections for moderate momentum transfers \( q \lesssim 1 \) GeV/c have the form

\[ \frac{d\sigma}{dq^2} = A e^{-Bq^2}. \] (34)

As the Fourier transform of a Gaussian is also a Gaussian, it is clear that Eq. (34) implies a Gaussian profile

\[ \Gamma(b) = \Gamma_0 e^{-b^2/2B}, \] (35)

and the relationship between \( A \) and \( \Gamma_0 \) is \( A = B^2 |\Gamma_0|^2 \). If \( \text{Re } \Gamma_0 = 0 \), the optical theorem states that \( \sigma_{\text{tot}} = 4\pi B\Gamma_0 \). But \( |\Gamma_0| \lesssim 1 \), and therefore

\[ \frac{\sigma_{\text{tot}}}{4\pi B} \leq 1, \] (36)

if \( \Gamma_0 \) is purely imaginary. In this case it is also convenient to write the amplitude as

\[ f = \frac{i\sigma_{\text{tot}} k}{4\pi} e^{-Bq^2}. \] (37)
3.3 Elastic scattering by deuterium

The deuteron is a very loosely bound system, and its constituents are usually outside each other's force range. When a high-energy projectile passes through the system, we may treat the scattering as successive scattering by the two instantaneously stationary nucleons. Let particle 1 be at \( \left( \frac{1}{2} \vec{y}, \frac{1}{2} z \right) \), particle 2 at \( \left( -\frac{1}{2} \vec{y}, -\frac{1}{2} z \right) \), with \( \vec{y} \) transverse to the incident direction, and the origin at the centre of mass.

![Diagram of deuteron scattering](image)

We assume \( k \) is large enough to allow the use of geometrical optics over the whole deuteron \((k \geq 1 \text{ GeV/c})\). After the wave has swept past particle 2 the wave function is

\[
S_2(\vec{b} + \frac{1}{2} \vec{y}) e^{ikz}.
\]

This wave is then incident on particle 1, which distorts it once more with its own \( S \)-matrix, so that the wave after emerging from the whole system is

\[
\psi = S_1(\vec{b} - \frac{1}{2} \vec{y}) S_2(\vec{b} + \frac{1}{2} \vec{y}) e^{ikz}.
\]  \( \text{(38)} \)

The \( S \)-matrix for the deuteron is just \( e^{-ikz} \psi \), and the deuteron's instantaneous profile is therefore
\[ \Gamma_{1} (\mathbf{b} - \frac{1}{2} \hat{y}) = 1 - S_{1} (\mathbf{b} - \frac{1}{2} \hat{y}) S_{1} (\mathbf{b} + \frac{1}{2} \hat{y}) \]
\[ = \Gamma_{1} (\mathbf{b} - \frac{1}{2} \hat{y}) + \Gamma_{2} (\mathbf{b} + \frac{1}{2} \hat{y}) - \Gamma_{1} (\mathbf{b} - \frac{1}{2} \hat{y}) \Gamma_{2} (\mathbf{b} + \frac{1}{2} \hat{y}) . \]  

(39)

The scattering amplitude when the deuteron is in the instantaneous configuration \((\mathbf{y}, z)\) is then given by the Fourier transform (28) of the profile. The observed amplitude is obtained therefrom by averaging over the ground-state wave function \(\psi_{d}(\mathbf{r})\)
\[ f_{d}(\mathbf{q}) = \frac{i k}{2\pi} \int d^{3}\mathbf{b} \int d^{2}y \ dz |\psi_{d}(\mathbf{y}, z)|^{2} e^{i \mathbf{q} \cdot \mathbf{b}} \Gamma_{d}(\mathbf{b}) . \]  

(40)

From Eq. (39) we see that the amplitude breaks up naturally into three pieces
\[ f_{d}(\mathbf{q}) = f^{(1)}_{d}(\mathbf{q}) + f^{(2)}_{d}(\mathbf{q}) + f^{(3)}_{d}(\mathbf{q}) . \]  

(41)

The first and second term describe, respectively, scattering by particle 1 or 2 individually, while the last term accounts for successive or double scattering. There is no triple (or higher) scattering amplitude because in our approximation the wave function always moves forward. (This is a superb approximation at the energies of concern to us.)

Consider \(f^{(1)}_{d}\):
\[ f^{(1)}_{d}(\mathbf{q}) = \frac{i k}{2\pi} \int d^{3}\mathbf{b} \int d^{2}y \ dz e^{i \mathbf{q} \cdot (\mathbf{b} - \frac{1}{2} \hat{y})} \Gamma_{1} (\mathbf{b} - \frac{1}{2} \hat{y}) e^{i \mathbf{q} \cdot \mathbf{y}} |\psi_{d}(\mathbf{y}, z)|^{2} \]
\[ = f_{1}(\mathbf{q}) F_{d}(\frac{1}{2} \mathbf{q}) , \]  

(42)

where \(f_{1}\) is the amplitude for scattering by particle 1 as if it were free, and \(F_{d}(\mathbf{k})\) is the deuteron's form factor
\[ F_{d}(\mathbf{k}) = \int d^{3}\mathbf{r} e^{i \mathbf{k} \cdot \mathbf{r}} |\psi_{d}(\mathbf{r})|^{2} . \]

For \(q^{2}\) not too large a reasonable representation of \(F_{d}\) is provided by
\[ F_{d}(\mathbf{q}) = e^{-Aq^{2}} , \]  

(43)

with \(A = 32 \text{ GeV}^{-2}\). This rapid fall-off in \(f^{(1)}_{d}\) merely reflects the fact that this amplitude describes the process where only one nucleon is struck
by the projectile, and that the most likely outcome of such an occurrence, when \( f \) is large, is deuteron break-up. \(|F_d(\frac{1}{2} q)|^2\) is actually the probability that the deuteron can hold together if one of its constituents receives a blow \( \vec{q} \), and this drops rapidly with increasing \( \vec{q} \).

We now turn to the double scattering term

\[
F_d^{(12)}(\vec{q}) = - \frac{i k}{2 \pi} \int e^{i \frac{1}{2} \vec{q} \cdot (\vec{b} - \vec{y})} \Gamma_1(\vec{b} - \frac{1}{2} \vec{y})

\times e^{i \frac{1}{2} \vec{y} \cdot (\vec{b} + \frac{1}{2} \vec{b})} \Gamma_3(\vec{b} + \frac{1}{2} \vec{y}) \left| \psi_d(\vec{y} z) \right|^2 d^2 b d^2 y dz .
\]

Because of the large deuteron radius, a reasonably good approximation to this expression can be obtained by taking advantage of the rapid variation of \( \Gamma_1 \) compared to \( \psi_d \). That is to say, double scattering can only occur if the nucleons are approximately lined up, or \( \vec{y} \approx 0 \). We therefore replace the slowly varying function \( \psi_d \) by its value at \( \vec{y} = 0 \), and this immediately gives us

\[
F_d^{(12)}(q) = - \frac{2 \pi}{i k} f_{1}(\frac{1}{2} q) f_{2}(\frac{1}{2} q) \int dz \left| \psi_d(0 z) \right|^2 .
\]

But \( \psi_d \) is spherically symmetric, and so

\[
\int dz \left| \psi_d(0 z) \right|^2 = \frac{2}{4 \pi} \int d^3 r \frac{1}{r^2} \left| \psi_d(0 z) \right|^2

= \frac{1}{2 \pi} \left< \frac{1}{r^2} \right>_d .
\]

Our final result therefore has a very simple and elegant form (originally due to Glauber, 1955):

\[
f_d(q) = \left[ f_1(\vec{q}) + f_2(\vec{q}) \right] F_0(\frac{1}{2} \vec{q})

- \frac{1}{i k} f_1(\frac{1}{2} q) f_2(\frac{1}{2} q) \left< \frac{1}{r^2} \right>_d .
\]  

(44)

Note that the double scattering amplitude does not have any \( \vec{q} \) dependence arising from the deuteron's wave function. This is because in double scattering each particle, on the average, receives 50% of the momentum transfer, and following the collision both particles therefore move off with the same momentum.
From Eq. (44) we can immediately calculate the total cross-section by using the optical theorem

\[ \sigma_{\text{tot}}^d = \sigma_{\text{tot}}^{(1)} + \sigma_{\text{tot}}^{(2)} - \frac{1}{4\pi} \left( \frac{1}{r^2} \right)_d \sigma_{\text{tot}}^{(1)} \sigma_{\text{tot}}^{(2)} \xi, \]

(45)

where

\[ \xi = 1 - \frac{\text{Re} f_1(0)}{\text{Im} f_1(0)} \frac{\text{Re} f_2(0)}{\text{Im} f_2(0)} , \]

All the present evidence points to \( \xi \approx 1 \) in the multi-GeV region.

The physical significance of double scattering is particularly clear in Eq. (45). The last term describes the eclipsing of one nucleon by the other. That is, when particle 1 is in front of 2, 2 cannot be struck by the projectile, and no reactions can therefore originate from 2 under this circumstance. The probability that this happens is just given by the solid angle subtended by 1 at 2 (or vice versa), and this is just the effective area \( \sigma_{\text{tot}} \) divided by the full solid angle \( 4\pi r^2 \).

To understand the angular distribution we use the Gaussian two-body amplitude (37) with \( B = 8 \), and Eq. (43)

\[ f_d(q^2) = \frac{ik}{4\pi} \left\{ [\sigma_{\text{tot}}^{(1)} + \sigma_{\text{tot}}^{(2)}] e^{-2q^2} - \frac{\sigma_{\text{tot}}^{(1)} \sigma_{\text{tot}}^{(2)}}{4\pi} \left( \frac{1}{r^2} \right) e^{-2q^2} \right\} . \]

(46)

The single scattering falls off very rapidly with angle, because of the form factor, and according to Eq. (46) there comes a point,

\[ q_0^2 = \frac{1}{10} \ln \left[ \frac{\sigma_{\text{tot}}^{(1)} \sigma_{\text{tot}}^{(2)}}{\sigma_{\text{tot}}^{(1)} + \sigma_{\text{tot}}^{(2)}} \frac{1}{4\pi} \left( \frac{1}{r^2} \right) \right] , \]

where there is exact cancellation between the single and double scattering amplitudes. Beyond that point the double scattering dominates.

The simple theory leading to Eq. (44) does not agree with the data in the vicinity of the interference minimum at \( q_0^2 \). As is almost always the case, in a minimum small effects that are elsewhere negligible can become very important.
First of all, in the interference region, one cannot ignore Re $f_1$. But even realistic*) real parts do not suffice to explain the data, and it was discovered that when the small D-state admixture in the deuteron is taken into account, beautiful agreement is achieved (see Figs. 1-3). The theory then predicts a very interesting behaviour in the interference region if the deuterium target is polarized$^{(b)}$, an effect that will soon be measured.

3.4 Elastic scattering by a large nucleus - the optical model

We now turn to elastic scattering by large nuclei. Crudely speaking, this was already treated in our discussion of the square profile (32). But now we seek a more precise theory that relates the nuclear profile to the underlying two-body profiles, and does not merely replace the whole system by a black disc.

As we saw in our treatment of deuterium, the S-matrix for a two-body scatterer is simply the product of the individual S-matrices. By the same argument we quickly recognize that this result generalizes to an A-body scatterer, provided:

i) the geometrical optics form of the wave function is valid over the whole target, and

ii) the system moves negligibly while the projectile sweeps through it.

Actually both of these provisions are stated in far too strict a fashion; that is, the product formula for $S$ that appears below is correct under considerably weaker conditions. The reason for this is that the conditions, as stated above, depend on the scatterer as a whole, and can always be violated by simply making it large enough. That this is an absurd conclusion can be seen by recalling that in optics it is the index of refraction -- a local property of the system -- that, in essence, determines the scattering, while a global property, the size of the object, determines the angular distribution. It is therefore not surprising that the conditions put forward above can be restated in a weaker

*) In precise calculation one does not assume that $\Gamma$ is far narrower than $\Psi_d$, and one evaluates $f^{(12)}_d$ numerically$^c$.)
form which does not refer to the over-all size of the scattering system, to wit:

i') the geometrical-optics form of the wave function must be valid over distances long compared to the absorption lengths in the scattering system (i.e. $\lesssim 4$ fm);

ii') the projectile's energy must be large compared to the typical excitation energies of the target$^{1,7}$ (i.e. several MeV).

When these conditions are satisfied we have

$$S(\vec{y}_1, \ldots, \vec{y}_A; b) = \prod_{n=1}^{A} S_n(\vec{y}_n - \vec{b})$$

for the S-matrix of an A-body nucleus whose constituents have the instantaneous transverse positions $\vec{y}_1, \ldots, \vec{y}_A$. The profile function of the nucleus is therefore

$$\Gamma(\vec{y}_1, \ldots, \vec{y}_A; b) = 1 - S(\vec{y}_1, \ldots, \vec{y}_A; b)$$

$$= 1 - \prod_{n=1}^{A} [1 - \Gamma_n(\vec{y}_n - \vec{b})]$$

$$= \sum_n \Gamma_n - \sum_{n>m} \Gamma_n \Gamma_m + \sum_{n>m\ell} \Gamma_n \Gamma_m \Gamma_{m\ell} - \ldots .$$

(47)

Here we see the multiple-scattering series in its full glory. The next problem is to evaluate it in a convenient and treatable fashion.

We restrict ourselves for now to the very important case of elastic scattering. In that case an important simplification occurs because we are only concerned with the degrees of freedom of the projectile; the nucleus, in a manner of speaking, is inert, its degrees of freedom do not manifest themselves explicitly. The action of the nucleus on the projectile, in elastic scattering, can therefore be described by a potential, the so-called optical potential.

As we saw at the outset, the profile function for a potential is

$$\Gamma(b) = 1 - e^{-i\Delta(b)}$$

(48)

where $\Delta(b)$ is an integral through the potential at impact parameter b.
In true potential scattering $\Delta$ is real. This can no longer be the case when the system (and its constituents) can be excited.

To determine $V$ we shall use our "microscopic" formula (47), which must still be averaged over the nuclear ground state in the case of elastic scattering. Comparing Eq. (48) with (47) we therefore have
\begin{equation}
-i\Delta(b) = \ln \left\langle \prod_{i=1}^{A} \left[ 1 - \Gamma_i(b - \hat{y}_i) \right] \right\rangle_0 ,
\end{equation}

For reasons that will become clear shortly, we now expand Eq. (49) in powers of $\Gamma$
\begin{equation}
i\Delta(b) = \sum_{i=1}^{A} \langle \Gamma_i \rangle_0 + \frac{1}{2} \sum_{i=1}^{A} \langle \Gamma_i \rangle^2
\end{equation}
\begin{equation}
- \sum_{i>j} \left\{ \langle \Gamma_i \Gamma_j \rangle_0 - \langle \Gamma_i \rangle_0 \langle \Gamma_j \rangle_0 \right\} + O(\Gamma^3)
\end{equation}
Henceforth the shorthand $\Gamma_i \equiv \Gamma_i(b - \hat{y}_i)$ is to be understood.

To simplify matters, assume that the profile functions $\Gamma_i$ are the same for neutrons and protons. Then
\begin{equation}
\langle \Gamma_i \rangle_0 \equiv \langle \Gamma_i \rangle_0 = \frac{1}{A} \int d^2 y \int dz \ n(\hat{y}z) \Gamma_i(b - \hat{y}_i) .
\end{equation}
where $n$ is normalized to $A$.

We shall assume that the nucleus is large compared to the range of the two-body interaction, which means that $n$ varies slowly compared to $\Gamma$. Thus
\begin{equation}
\langle \Gamma_i \rangle_0 \approx \frac{1}{A} \left[ \int d^2 y \ \Gamma_i(y) \right] \int_{-\infty}^{\infty} dz \ n(bz)
\end{equation}
\begin{equation}
= \frac{1}{A} \frac{2\pi}{ik} f(0) \int_{-\infty}^{\infty} dz \ n(bz) ,
\end{equation}
where $f(0)$ is the forward scattering amplitude of the projectile by any nucleon [cf. Eq. (28)].
An order of magnitude estimate of Eq. (52) can be obtained by noting that \( \int dz n \approx 2RA/(4\pi R^3/3) \), while \( f(0) \approx i\sigma_{\text{tot}} k/4\pi \). Hence

\[
\langle i_i^0 \rangle_0 \approx \frac{3}{4} \frac{\sigma_{\text{tot}}}{\pi R^2},
\]

(53)

denotes the ratio of the two-body cross-section to the nuclear area, and therefore small compared to unity. This immediately shows that the second term in Eq. (50) is negligible if \( A \gg 1 \).

The third term in Eq. (50) vanishes if the nucleons are randomly distributed because it depends purely on the fluctuation \( \langle i_i^0 \rangle_0 \langle i_i^0 \rho_i^0 \rangle - \langle i_i^0 \rangle_0 \langle \rho_i^0 \rangle_0 \). It turns out that nuclear correlations are small, and this term is estimated to be about a \( -5\% \) correction to the leading term. Before discussing this correction in detail, let us take stock of where we stand.

With the approximations made so far, Eq. (50) is

\[
i\Delta_i^0(b) = -\frac{2\pi i}{k} f(0) \int_{-\infty}^{\infty} n(bz) \, dz .
\]

(54)

We now want to define the optical potential, but we do not want to use the Schrödinger equation for this purpose because we must have relativistic kinematics for free space propagation. The optical potential \( U \) is therefore defined by requiring that the wave equation

\[
\left\{ \nabla^2 + E^2 - m^2 - U \right\} \psi = 0
\]

(55)
gives the same scattering in the high-energy limit as the Fourier transform (28) of Eq. (48). If we carry out the high-energy approximation on Eq. (55), we can then identify the familiar phase factor with Eq. (54), and so we have the lowest order approximation to the optical potential

\[
U_i^0(r) = -4\pi f(0) n(r).
\]

(56)

This expression, and the resulting amplitude as calculated from Eqs. (54) and (48) via (28), provides an exact solution to the multiple scattering problem in a random system when \( A \gg 1 \). All corrections arise from "1/A" effects and correlations.
Perhaps a brief but important remark is in order concerning the relationship between the optical potential and the multiple scattering series (48). When the scatterer is a large system, the series converges very slowly because repeated scatterings are very important. When one uses the optical model, the wave equation -- or equivalently the integral over z in Eq. (54) and its exponentiation in Eq. (48) -- carries out this sum over repeated scatterings, and the optical potential therefore has a far simpler structure than the original multiple scattering series. Naturally this approach fails when A is not large, in practice for A \leq 10. Thus when interpreting scattering by He the more exact series (47) must be used.

Now we can return to the correlation correction. We define the pair correlation function \( g(r_1 r_2) \) by *

\[
\int |\psi(r_1, r_2, r_3, ..., r_A)|^2 \, d^3r_3 ... d^3r_A = \frac{1}{A^2} \, n(r_1) \, n(r_2) \left[ 1 + g(r_1 r_2) \right].
\]  

(57)

If \( \psi \) is a product wave function (no Pauli principle), \( g = 0 \). Thus \( g \) is a measure of the departure from randomness. \( \psi \)'s normalization requires

\[
0 = \int n(r_1) \, n(r_2) \, d^3r_1 \, d^3r_2 \, g(r_1 r_2) \, d^3r_1 \, d^3r_2.
\]  

(58)

We expect \( g \) to depart from zero significantly for \(|r_1 - r_2|\) of order internucleon distances, and to vanish beyond this (though there can be an asymptotic remainder of order \( A^{-3} \) which depends sensitively on the precise definition of \( g \)). The correlation term in Eq. (50) is therefore

\[
-i \Delta_{\text{corr}}(b) = \int d^3r_1 \, d^3r_2 \, \Gamma(b - y_1) \, \Gamma(b - y_2) \, n(r_1) \, n(r_2) \, g(r_1 r_2)
\]

\[
\approx \int_{-\infty}^{\infty} dz \, [n(b, z)]^2 \int d^2\gamma \, d^2\gamma' \, dz' \, \Gamma(\gamma + \frac{1}{2}y - b) \, \Gamma(\gamma - \frac{1}{2}y - b) \, g(y, z'),
\]

(59)

*) There are many possible definitions of \( g \), and one should be aware of this in reading the literature. From a theoretical standpoint, our definition (57) is inelegant because it does not give a neat formula for the expectation value of a two-body operator. In practice, however, one usually retreats to definition (57) when carrying out numerical calculations. Also note that the second term of Eq. (50), which is of "1/A" character, combines with another 1/A term arising from the third (or correlation) piece of Eq. (50). One must pay close heed to the definition of \( g \) and its normalization if one computes these 1/A terms.
where $n$ is again assumed to be slowly varying compared to $\Gamma$, and $g$ is assumed to depend only on $r_1 - r_2$. We can now identify a correction to the optical potential as

$$U^{(1)}(r) = k [n(b, z)]^2 \sigma_{\text{tot}}^2 \ell_c,$$  \hspace{1cm} (60)

where $\sigma_{\text{tot}}$ is the two-body total cross-section, and $\ell_c$ is a correlation length defined as

$$\ell_c = \frac{2}{\sigma_{\text{tot}}^2} \int d^2Y \int d^2y \int dz \Gamma(Y + \frac{1}{2}y - b) \Gamma(Y - \frac{1}{2}y - b) g(y, z).$$

If $\Gamma$ is a real Gaussian as in Eq. (35)

$$\ell_c = \frac{1}{8\pi B} \int d^2y \int dz \int \frac{e^{-y^2/4B}}{g(y, z)}.$$  \hspace{1cm} (61)

For detailed studies of $\ell_c$ the reader is referred to the literature$^8,^9.$ The conclusion of these authors is that the true value of $\sigma_{\text{tot}}$ is some 10% smaller than what would be obtained if the correlation were neglected.

To summarize, we can now write an easily remembered formula for the optical potential, valid if $\text{Re} f = 0$, and where $n$ is constant. Define the nuclear mean free path $\lambda$ of the projectile as

$$\lambda = \frac{1}{\sigma_{\text{tot}} n}.$$ \hspace{1cm} (62)

Then

$$U = -\frac{i k}{\lambda} \left[ 1 + \frac{\ell_c}{2\lambda} \right].$$ \hspace{1cm} (63)

For a varying $n$, merely replace $\lambda$ by its local value via Eq. (62).

A comparison of an optical model calculation with data on neutron nucleus scattering is shown in Fig. 4.

### 3.5 Coherent production and coupled channel optical models

Elastic scattering is not our main concern here, and so the foregoing must be generalized to the case where there are a number of states that can be scattered amongst themselves while leaving the nucleus in
its ground state. There is now an important complication because these states will have different masses \([\text{e.g. } (\gamma, \omega, \phi, \rho), \text{ or } (\pi, A_1, 5\pi)]\). At each transformation from one mode to another there is therefore a change in wavelength (the energy is always the same, of course, if the nucleus is unexcited), and so the \(z\)-dependence of the wave is no longer trivial. Nevertheless, if \(A \gg 1\), and there are no correlations, a simple matrix generalization of the optical model is possible\(^*)\), just as a matrix generalization of the index of refraction is appropriate to the description of light propagation in a bi-refringent medium.

Let \(\psi_i\) be the wave function of the \(i\)th coherently propagating mode of mass \(m_i\), and \(U_{ij}\) the optical potential for the \((i \leftrightarrow j)\) transition

\[
U_{ij}(r) = -4\pi \int_0^r f_{ij}(0) n(r).
\]  

(64)

Then if \(\psi\) is the column vector made from the \(\psi_i\), \(U\) the matrix with elements (64), and \(M\) the diagonal matrix \(m_i \delta_{ij}\), the wave equation is

\[
(\nabla^2 + E^2 - M^2 - U) \psi = 0.
\]  

(65)

The high-energy approximation is

\[
\psi = e^{ikz} \phi,
\]

with \(k = E - M^2/2E\), and \(\phi\) slowly varying. Then Eq. (65) reduces to

\[
\frac{d\phi}{dz} = \frac{1}{2iE} e^{-ikz} U(bz) e^{ikz} \phi.
\]  

(66)

Note that \([K, U] \neq 0\), and so Eq. (66) cannot be integrated in closed form. Actually we are familiar with a similar problem in quantum electrodynamics, where the time-evolution operator in the interaction picture satisfies an equation like (66), and the solution is the time-ordered series whose pieces are the Feynman graphs. In the same way we can write the \(S\)-matrix that emerges from Eq. (66) (i.e. the asymptotic form of \(\phi\)) as

\[
S(b) = Z \exp \left\{ \frac{1}{2iE} \int_{-\infty}^{\infty} dz \ e^{-ikz} U(bz) e^{ikz} \right\},
\]  

(67)

\(^*)\text{ If there are correlations, the only change is that } U \text{ is no longer given by Eq. (64), but corrections as in Eq. (63) are required.}
where $Z$ is the $z$-ordered product, i.e. the exponential is to be expanded into a power series, and the terms are to be written in the order specified by the magnitude of their $z$-arguments.

3.6 Incoherent scattering

Let $\mathcal{F}$ be the operator that describes the transition of the projectile $a$ into the observed final state $b$ for fixed nucleon positions. The coherent cross-section (if it is not zero) is then

$$\frac{d\sigma^{\text{coh}}}{d\Omega} = |\langle \mathcal{F} \rangle_0 |^2 .$$  \hspace{1cm} (68)

At high energy the cross-section where the nucleus is allowed to go to any and all excited states is $\langle \mathcal{F} \mathcal{F}^+ \rangle_0$ [recall Eq. (4) and its sequel], and the inelastic cross-section is therefore

$$\frac{d\sigma_{\text{in}}}{d\Omega} = \langle \mathcal{F} \mathcal{F}^+ \rangle_0 - \langle \mathcal{F} \rangle_0 \langle \mathcal{F}^+ \rangle_0 .$$  \hspace{1cm} (69)

This is true for any $\mathcal{F}$. If we ignore multiple scattering, and assume that the $a \to b$ amplitude is the same for all nucleons

$$\mathcal{F} = f_{ab} \sum_i e^{i\mathbf{q} \cdot \mathbf{r}_i} ,$$  \hspace{1cm} (70)

just as in our introductory discussion of Section 2. As we now know something about correlations, we can improve a bit on Eq. (7), which assumed a product wave function. If we use the definition (57) of the correlation function we easily find that Eq. (69), with $\mathcal{F}$ given by Eq. (70), is not Eq. (7), but

$$\frac{d\sigma_{\text{in}}}{d\Omega} = A \frac{d\sigma_{ab}^{\theta}}{d\Omega} \left\{ 1 - |F(q)|^2 + G(q) \right\}$$  \hspace{1cm} (71)

$$G(q) = \frac{1}{A} \int d^3r_1 \ d^3r_2 \ n(\mathbf{r}_1) \ n(\mathbf{r}_2) \ g(\mathbf{r}_1, \mathbf{r}_2) \ e^{i\mathbf{q} \cdot (\mathbf{r}_1 - \mathbf{r}_2)} .$$

Here $\frac{d\sigma_{ab}^{\theta}}{d\Omega}$ is the cross-section for $aN \to bN$. In view of Eq. (58),
\( \frac{d\sigma_{\text{in}}}{d\Omega} \) still vanishes *) at \( \tilde{q} = 0 \) (which is not \( \theta = 0 \) unless \( E \to \infty \)).

For a system with many particles F falls off much more rapidly than G, and so \( \frac{d\sigma_{\text{in}}}{d\Omega} \) approaches the incoherent limit A \( \frac{d\sigma_{\text{ab}}}{d\Omega} \) when \( q^2 \gg 1 \).

We must now generalize Eq. (70) to a more realistic form that takes multiple scattering into account. I shall confine myself here to the case \( a = b \), what one might call "elastic" incoherent scattering. Once this is understood it is not too difficult (though somewhat tiring) to go to the more important and relevant case \( a \neq b \).

As always, \( \mathcal{F} \) has the eikonal form

\[
\mathcal{F} = \frac{ik}{2\pi} \int d^2b \ e^{i\mathbf{q} \cdot \mathbf{b}'} \left[ 1 - \prod_i S(y_i - b) \right].
\]

(72)

It is then an easy matter to show that **)\n
\[
\frac{d\sigma_{\text{in}}}{d\Omega} = \left( \frac{k}{2\pi} \right)^2 \int d^2b \ d^2b' \ e^{i\mathbf{q} \cdot (\mathbf{b} - \mathbf{b}')} \left\{ \left\langle \prod_i S_i(b) S_i^*(b') \right\rangle_0 - \left\langle \prod_i S_i(b) \right\rangle_0 \left\langle \prod_i S_i^*(b') \right\rangle_0 \right\}
\]

(73)

A detailed evaluation of this expression including correlations is too involved for the time available ***)). I therefore confine my treatment to the approximation where the joint probability distribution for the A nucleons is taken to be a product of A identical factors \( n(r_i)/A \). For \( A \gg 1 \) one then has

\[
\left\langle \prod_i S_i(b) S_i^*(b') \right\rangle_0 = \left[ 1 - \frac{1}{A} \int d^3x \ n(x) \{ \Gamma_i(b) + \Gamma_i^*(b') - \Gamma_i(b) + \Gamma_i^*(b') \} \right]^{\alpha}
\]

\[
\approx \exp \left\{ - \int d^3x \ n(x) \left[ \Gamma_i(b) + \Gamma_i^*(b') \right] \right\}
\]

(74)

\[
\times \exp \left\{ \int d^3x \ n(x) \Gamma_i(b) \Gamma_i^*(b') \right\}
\]

*) That this must be so can be seen immediately from Eq. (70), which shows that \( \mathcal{F} \) is no longer an operator with respect to the nucleus when \( q = 0 \). Once multiple scattering is included, \( \mathcal{F} \) is an operator even at \( q = 0 \), as one easily verifies from Eq. (72).

**) Here we again use the shorthand \( S_i(b) \equiv S(b - y_i); \) also in Eqs. (74) and (75), \( \Gamma_i(b) \equiv \Gamma(b - y) \), etc., and \( d^3x = d^2y \ dz, \ x = (y,z) \).

***) For further details concerning correlations and the \( \theta \to 0 \) limit see Refs. 4b, 4c and 8.
and

\[
\left\langle \prod_{i} S_i(b) \right\rangle_i = \left[ 1 - \frac{1}{\Lambda} \int d^3x \ n(x) \Gamma_i(b) \right]^\Lambda
\]

\[
= \exp \left\{ - \int d^3x \ n(x) \Gamma_1(b) \right\} .
\]  

(75)

Once more we assume that \( n \) is slowly varying compared to \( \Gamma \). It then becomes convenient to introduce a measure of the nuclear thickness at impact parameter \( b \) by means of

\[
\mathcal{T}(b) = \int_{-\infty}^{\infty} n(bz) \, dz .
\]

(76)

It is then easy to simplify Eqs. (74) and (75) considerably; for a purely imaginary \( f(0) \) one finds

\[
\frac{d\sigma_{in}}{d\Omega} = \left( \frac{k}{2\pi} \right)^2 \int d^2b \ e^{-\sigma \mathcal{T}(b)}
\]

\[
\times \int d^2b' \ e^{i\mathbf{q} \cdot \mathbf{b}'} \left\{ e^{\mathcal{T}(b')\Lambda(b')} - 1 \right\}
\]

with

\[
\Lambda(b) = \frac{1}{k^2} \int e^{i\mathbf{q} \cdot \mathbf{b}} |f(q)|^2 \, d^2q .
\]

(78)

As it stands, Eq. (77) is both difficult to understand and evaluate. Fortunately, an expansion of the exponential in the curly bracket of Eq. (77) leads to a series that converges fairly rapidly, and whose terms lend themselves to a straightforward physical interpretation:

\[
\frac{d\sigma_{in}}{d\Omega} = \left( \frac{k}{2\pi} \right)^2 \sum_{s=1}^{s} \frac{1}{s!} N_s \phi_s(q)
\]

(79)

\[
N_s = \int d^2b \ e^{-\sigma \mathcal{T}(b)} [\mathcal{T}(b)]^s
\]

(80)

\[
\phi_s(q) = \int d^2b \ e^{-i\mathbf{q} \cdot \mathbf{b}} [\Lambda(b)]^s .
\]
This is the third type of multiple scattering series that we have encountered: the first was the original, "true", multiple scattering series of Eq. (47); the second was the "irreducible" multiple scattering series for the optical potential of coherent scattering, Eq. (50). It is important not to confuse these different expansions.

In our latest series (79), the collisions have been divided into two categories: \( \theta = 0 \) collisions, which are always summed exactly into the factor \( e^{-\sigma T} \), and \( \theta \neq 0 \) collisions of various orders \( s \), described by \( \phi_s \), which can be cast into the more instructive form

\[
\phi_s(q) = 4\pi^2(k^2)^{-s} \int d^2q_1 \cdots d^2q_s \delta^2 \left( \hat{q} - \sum_{i=1}^{s} \hat{q}_i \right) \prod_{i=1}^{s} |f(q_i)|^2.
\]

In the \( s^{th} \) term each of the \( s \) collisions has, on the average, the momentum transfer \( q/s \). This can be seen even more clearly if \( f \) is taken to be Gaussian, for then

\[
\phi_s(q) = \frac{4\pi B}{s} \left( \frac{\sigma^2}{16\pi B} \right)^s e^{-Bq^2/s}.
\]  

(82)

Thus when \( Bq^2 \ll 1 \) (i.e. inside the two-body diffraction cone) the term \( s = 1 \) dominates, and falls off quickly. As one increases \( q \), higher order terms become important. This can be seen very clearly in the calculations shown in Figs. 5 and 6.

The incoherent multiple scattering formula (77) unfortunately suffers from several defects. One of these is discovered easily by going to the limit of small \( \sigma \), in which case we would expect to recover the Born approximation formula (71) -- naturally with \( G(q) = 0 \). What actually emerges is just \( A \, d\sigma_{ab}^0 / d\Omega; \) the \( |F(q)|^2 \) is missing, and the zero at \( q = 0 \) in Eq. (71) is therefore lost. The reason for this error can be seen if one reconsiders the approximate exponentiations in Eqs. (74) and (75). For large \( A \)

\[
\left(1 + \frac{a}{A}\right)^A \approx e^{a \left(1 - \frac{a^2}{2A} + \ldots\right)},
\]

(83)

where, in our problem, \( a \) has terms of order \( \Gamma \) and \( \Gamma^2 \). Hence the Born approximation, which is purely of order \( \Gamma^2 \), cannot emerge as the small \( \Gamma \)
limit unless we retain the correction terms $a^2/2A$. A detailed analysis of this question shows that such correction terms to Eq. (79) are only important for very small $q$'s, i.e. for scattering angles inside the nuclear diffraction cone $\theta \leq 1/\text{RE}$. In this small-angle region elastic scattering dominates, and these refinements that guarantee the correct $\Gamma \to 0$ limit are therefore only of importance if the inelastic scattering at small angles can be separated experimentally from the elastic, or if one is dealing with a production process which cannot be coherent.

Finally, it should be recalled that once one goes beyond the Born approximation the zero at $\theta = 0$ disappears from the inelastic cross-section.

The discussion following Eq. (73) has ignored all correlations, and from Eq. (75) onwards it was assumed that $n$ varies slowly in comparison to $\Gamma$. Both of these approximations require further comment. It turns out that correlations are numerically more important in incoherent than in elastic scattering. Furthermore, incoherent scattering mainly comes from the nuclear surface, as one easily sees from the structure of the integral (80) that gives $N_s$. For these reasons the numerical precision of the incoherent multiple scattering formula (77) is on a considerably poorer footing than the corresponding formula (56) for elastic scattering: the inelastic scattering originates from the surface where $n$ is not really a slowly-varying function in comparison to $\Gamma$, and where the correlations are not well known from the theory of nuclear structure. Indeed, the whole topic of inelastic scattering merits further experimental and theoretical study.

4. NUCLEAR PHOTOPROCESSES

Vector mesons ($V$) play a fundamental role in hadronic electrodynamics, and for this reason coherent vector meson photoproduction has been studied more extensively than any other type of coherent processes.

*) See footnote following Eq. (71).

**) In this connection it is an amusing exercise to show that formulae (77) and (56) are correctly related by unitarity.
Not only do such experiments provide essential information about $\gamma$-$V$ couplings and VN scattering amplitudes; they also produce a copious supply of $V$'s and therefore allow the study of subtle effects such as $\rho$-$\omega$ mixing and leptonic decays of $V$'s. Another quantity of great theoretical and experimental interest is the total nuclear photo-cross-section. As we shall see, its variation with $A$ and energy provides an incisive test of the hypothesis of vector meson dominance.

4.1 The amplitude for coherent photoproduction

In general terms we are now dealing with a coupled channel situation having as participating modes $(\gamma, \rho, \phi, \omega)$. There are enormous simplifications, however, because: (a) all amplitudes involving $\gamma$ are down by factors of $e$ or $e^2$ in comparison with hadronic amplitudes; (b) fully coherent $\rho \leftrightarrow \omega$ and $\rho \leftrightarrow \phi$ transitions are impossible; and (c) there does not appear to be any indication that coherent $\omega \leftrightarrow \phi$ transitions have an appreciable magnitude. The equations of Section 3.5 therefore simplify enormously:

\[ (\nabla^2 + E^2 - m_V^2 - U_{VV})\psi_V = U_{V\gamma} \psi_{\gamma} \]  

(84)

To lowest order in $e$, $\psi_{\gamma}$ can be replaced by the incident wave. Thus Eq. (84) can be integrated easily in the high-energy limit to give the profile function for $V$-photoproduction

\[
\Gamma_{VV}(b) = \text{Im} \left[ \int_{\infty}^{\infty} dz \ e^{i z b} \ U_{VV}(b z) \ exp \left( \frac{1}{2 i k_V} \int_{z}^{\infty} U_{VV}(b z') \ dz' \right) \right].
\]

The production amplitude is then obtained via Eq. (28):

\[
T_{VV}(q) = f_{VV}(0) \int dz \ d^2 b \ e^{i q \cdot b} \ e^{i q \cdot z} \ n(b z) \ exp \left[ -\frac{1}{2} \sigma_{VN}(1 - i \alpha_{VN}) \int_{z}^{\infty} n(b z') dz' \right].
\]

(85)

Here the VN forward amplitude has been written as

\[
f_{VV}(0) = \frac{i k \sigma_{VN}}{4\pi} (1 - i \alpha_{VN}).
\]

(86)
Equation (85) reveals an important feature of coherent production that is not present in elastic scattering: sensitivity of the nuclear production amplitude to the real part of the forward amplitude $\alpha_{VN}$ due to interference between the phase factor $e^{i q \cdot z}$ and the oscillating part of the $V$-propagation amplitude. As a consequence, $|T_{VN}|^2$ received a linear contribution from $\alpha_{VN}$ as $\alpha_{VN} \to 0$; this vanishes when $q \to 0$, and one then has the same situation as in elastic scattering, i.e. a leading contribution to the cross-section quadratic in Re $f(0)$. Hence the photoproduction data is both more difficult to analyse and richer in content than elastic scattering.

4.2 Experiments on vector meson photoproduction

A rather recent and quite detailed review of the data and its interpretation can be found in the Proceedings of the Cornell Conference\textsuperscript{10).} I shall therefore confine myself to a brief summary.

In principle, $\phi$ production should be the most straightforward process. It is detected in a two-body ($K\bar{K}$) mode, and the extremely narrow $\phi$ width makes background subtractions rather trivial. Unfortunately, $\phi$ production is intrinsically less copious than $\rho$, and the very much larger value of $q_u$ reduces nuclear coherence considerably at the energies where data now exists ($\lesssim 9$ GeV). The rather extensive measurements at Cornell are summarized in Figs. 7-9. The latter figure shows that the nuclear data, by itself, only determines a relationship between $\sigma_{\phi N}$ and $\alpha_{\phi N}$. If one assumes a value of $\alpha_{\phi N} = -0.3$, which is typical of other diffractive forward amplitudes at these energies, one finds $\sigma_{\phi N} \approx 13$ mb. What is clear, in any case, is that $\sigma_{\phi N}$ is smaller than $\sigma_{\pi N}$ and $\sigma_{\rho N}$ by about a factor of two, and that it is probably also smaller than $\sigma_{K N}$. All this is in very good agreement with the naïve quark model.

Photoproduction of $\omega$ is very difficult to study because of the \pi^+\pi^- decay mode that must be detected. To make matters worse, in the sub-10 GeV region, one-pion exchange contributions to $\omega$-production still compete quite successfully with diffraction dissociation. The existing data is summarized in Ref. 10. Suffice it to say that within the large experimental uncertainties, $\sigma_{\omega N}$ is equal to $\sigma_{\pi N}$ ($\sigma_{\omega N} = 25 \pm 8$ mb).
Photoproduction of $\rho$ is both copious and easy to detect, and has been studied extensively. Unfortunately, the appreciable $\rho$-width inevitably leads to a measure of ambiguity in the definition of the $\rho$. This has been a somewhat controversial and confusing theme throughout the development of the subject, but by now there is fairly widespread agreement as to how the data is to be analysed; as this is a rather specialized topic I merely refer to Yennie's lectures\textsuperscript{10} for an excellent review of these background problems.

There exist three extensive series of measurements on nuclear $\rho$-production (DESY-MIT, Cornell, and Rochester), and they are in excellent agreement with each other. Taken by itself, this data fails to determine $\sigma_{\rho N}$ because of the phase problem just mentioned. An optical model fit to the DESY-MIT data leads to the $\chi^2$-map in the $\sigma_{\rho N}-\alpha_{\rho N}$ plane shown in Fig. 10; observe the long valley of minimum $\chi^2$ that sweeps across this map.

Several experiments have been carried out to eliminate this ambiguity. The most successful is a SLAC experiment\textsuperscript{11} on $\rho$'s photoproduced from deuterium in the double scattering region. As we saw in Section 3.3, the cross-section in this region is $|f_{\gamma\rho}(\frac{1}{2} q)f_{\rho\rho}(\frac{1}{2} q)|^2$. As $f_{\gamma\rho}(q)$ is known from hydrogen experiments, $|f_{\rho\rho}(\frac{1}{2} q)|^2$ is measured by this experiment, and the result therefore depends only on $\alpha_{\rho N}$ quadratically. Another method for acquiring further information is to detect the interference between the amplitudes for $e^+e^-$ production in the nuclear Coulomb field, and the $e^+e^-$ pairs coming from leptonic $\rho$-decay. Figure 10 brings all of this information together, and demonstrates that it is internally consistent. A final and comprehensive analysis of all this data has not yet been carried out, but the answer for $\sigma_{\rho N}$ will be between 27 and 28 mb with an error of about 10%. The $\gamma-\rho$ coupling constant (see definition below in Section 4.4) determined by these experiments is $\gamma^2_\rho/4\pi = 0.62$, with an error of about 10%. This agrees amazingly well with the Orsay storage ring value $0.64 \pm 0.06$.

4.3 $\rho-\omega$ interference\textsuperscript{10}

Conventionally one thinks of the neutral $\rho$ and $\omega$ as being $I = 1$ and $I = 0$ objects, respectively. Naturally such an $I$-spin assignment is only meaningful to the extent that one can ignore the electromagnetic inter-
action, which is an admissible approximation in most hadronic phenomena. But $\rho$ and $\omega$ present a very special situation because they are so nearly degenerate in mass. As a consequence the otherwise exceedingly small electromagnetic mixing of the states is greatly enhanced and reaches measurable (though still small) proportions.

Let $|\rho^0\rangle$ and $|\omega^0\rangle$ be the idealized eigenstates of the Hamiltonian when the electromagnetic interaction $H_{\text{em}}$ is ignored; they are therefore eigenstates of $I$. The perturbation $H_{\text{em}}$ has a non-vanishing off-diagonal element $\delta = \langle \rho^0 | H_{\text{em}} | \omega^0 \rangle$. The influence of $H_{\text{em}}$ can therefore be incorporated by generalizing the diagonal mass matrix that appeared in Eq. (65) to the off-diagonal form

$$
\begin{pmatrix}
    m_{\rho} & \delta \\
    \delta & m_{\omega}
\end{pmatrix}.
$$

In vacuum, where $U = 0$, the eigenfunctions of Eq. (65) are then the physical states $|\rho\rangle$ and $|\omega\rangle$, which are no longer exact eigenstates of $I$, but linear combinations of $|\rho^0\rangle$ and $|\omega^0\rangle$. Consequently the C-violating decay $\omega \to \pi^+ \pi^-$ can occur via the (small) component of $|\rho^0\rangle$ present in $|\omega\rangle$.

The amplitude for $\pi^+ \pi^-$ photoproduction now has two pieces: the familiar $\rho$ contribution having a resonance at $E = m_\rho$, and another term that resonates at $E = m_\omega$:

$$
T(\gamma A \to A \pi^+ \pi^-) = \left\{ \frac{T(\gamma A \to \rho A)}{E^2 - M_\rho^2} + \frac{\delta}{M_\rho - M_\omega} \frac{T(\gamma A \to \omega A)}{E^2 - M_\omega^2} \right\} D(\rho \to \pi^+ \pi^-).
$$

Here the $M'$s are the complex masses of $\rho$ and $\omega$: $M_\rho = m_\rho - \frac{1}{2} i \Gamma_\rho$, etc.; $D(\rho \to \pi^+ \pi^-)$ is the indicated decay amplitude. It is convenient to write this last expression as

$$
T(\gamma A \to A \pi^+ \pi^-) = \frac{T(\gamma A \to \rho A) D(\rho \to \pi^+ \pi^-)}{E^2 - M_\rho^2} F(E).
$$

(87)

The (complex) modulation factor $F(E)$ then shows the interference in the vicinity of $E = m_\omega$. 
Very careful measurements of $\pi^+\pi^-$ photoproduction in the vicinity of $\omega$ have been carried out at Daresbury, Rochester (Cornell) and DESY. A compilation of the modulation factors $F(E)$ is shown in Fig. 11. These measurements provide information about the fundamental electromagnetic matrix element $\delta$; they also set very weak limits on the relative phase of $\omega$ and $\rho$ photoproduction amplitudes.

Similar experiments showing $\rho-\omega$ interference in the $e^+e^-$ mode have also been carried out at DESY and Daresbury$^{10}$.

4.4 Qualitative tests of Vector Meson Dominance

Let us recall the hypothesis of Vector Meson Dominance (VMD). In any process involving one or more real photons, say $\gamma a \to b$, or $\gamma a \to \gamma b$, where $a$ and $b$ are hadrons, the amplitudes are related to vector meson amplitudes by the linear relations

$$T(\gamma a \to b) = \sum \frac{e}{2\gamma V} T(Va \to b)$$

(88)

$$T(\gamma a \to \gamma b) = \sum_{\gamma V'} \left( \frac{e}{2\gamma V'} \right) \left( \frac{e}{2\gamma V} \right) T(Va \to V'b)$$

(89)

where the $\gamma V'$'s are universal constants. For example, in the case of the $\rho$, $\gamma_\rho$ can be defined by the process $e^+e^- \to \gamma \to \pi^+\pi^-$. [If the photons are virtual, a number of modifications of Eqs. (88) and (89) are required.]

The propagation of light through nuclear matter shows some unexpected features if VMD is valid. This section is devoted to these aspects of nuclear optics.

The qualitative tests of VMD stem from the following paradox$^*$.  

**Argument I**

The $\gamma N$ total cross-section is of order 120 $\mu b$, hence the nuclear mean-free-path $l_\gamma$ of photons is $\sim 500$ fm. Thus every nucleon is illuminated by the full incident beam, and $\sigma_{tot}(\gamma A) = Z\sigma_{tot}(\gamma p) + (A-Z)\sigma_{tot}(\gamma n) \approx A\sigma_{tot}(\gamma N)$.

* The same paradox occurs in neutrino reactions$^{1,2}$ because pions play a role in weak interactions analogous to the role played by $\rho$'s in electrodynamics.
Argument II

According to VMD, the forward amplitudes for the processes $\gamma A \rightarrow \gamma A$ and $VA \rightarrow VA$ are proportional to each other. But V's are hadrons, and their short nuclear mean-free-paths $\ell_V$ imply that $\sigma_{\text{tot}}(VA) = 2\pi R^2$ if $R \gg \ell_V$.

These arguments therefore say, in turn, that $\sigma_{\text{tot}}(\gamma A)$ is proportional to the nuclear volume and to its surface. The reconciliation can be put in several (equivalent) ways.

Consider the physical photon state $|\gamma\rangle$ of energy $E$ expanded to lowest order in $e$ in terms of the eigenstates $|n\rangle$ of the Hamiltonian without the electromagnetic interaction $H'$:

$$ |\gamma\rangle = |\tilde{\gamma}\rangle + \sum_n |n\rangle \frac{\langle n|H'|\tilde{\gamma}\rangle}{\Delta E_n}. \quad (90) $$

For large $E$

$$ \Delta E_n = \frac{m_n^2}{2E}; \quad (91) $$

the time during which the hadronic vacuum fluctuation $|n\rangle$ lives is $1/\Delta E_n$, and the distance it traverses is then also $1/\Delta E_n$. Whether $|n\rangle$ is shadowed or not in nuclear matter then depends on whether $1/\Delta E_n$ is large or small compared to $\ell_n$, the nuclear mean free path of $|n\rangle$. Thus if $(1/\Delta E_n) \ll \ell_n$, the hadronic vacuum fluctuation $|n\rangle$ is unlikely to undergo a nuclear collision, and so the beam will not be appreciably attenuated, and $\sigma(\gamma A)$ will be proportional to $A$. On the other hand, if $(1/\Delta E_n) \gg \ell_n$, the vacuum fluctuation will be absorbed long before it would normally (i.e. in vacuo) reconvert to the photon state, and the beam will be attenuated; if, moreover, $\ell_n \ll R$ for all $n$, $\sigma(\gamma A)$ will be proportional to the nuclear area. Hence if the sum over $n$ in Eq. (90) receives its major contribution from low masses (and this is what we mean by VMD), we expect that the total nuclear photon cross-section should display a transition for a behaviour proportional to $A$ to a behaviour proportional to $A^{2/3}$ as the photon energy rises. The characteristic energy where this transition should occur is seen to be

$$ E_n \approx \ell_n \frac{m_n^2}{2}; \quad (92) $$
For the $V$'s we find $E_\rho \sim 6$ GeV, $E_\omega \sim 7$ GeV, $E_\phi \sim 23$ GeV. As the $\phi$ contribution (and also that of $\omega$) is not very important in Eqs. (88) and (89), we expect the volume-to-surface transition to occur in the 5-10 GeV region.

One can readily generalize the above conclusions in two ways. Firstly, if the photon is virtual, as in inelastic electron scattering by nuclei, $\Delta E_n$ is increased to $(m_n^2 + Q^2)/2E$, where $Q^2 > 0$ for space-like momentum transfer. The characteristic transition energy $E_n$ is therefore increased:

$$ E_n(Q^2) \approx \tilde{E}_n \left( m_n^2 + Q^2 \right); \quad (93) $$

that is, at fixed $E$ the shadowing of the photon decreases as $Q^2$ increases, and this is closely related to the expectation that photons should appear to become "smaller" as $Q^2$ increases. The second extension concerns incoherent processes, for example $\gamma A \rightarrow \pi^0 A^*$; it is based on the obvious point that if there is a volume-to-surface transition in the total cross-section, most, if not all, the partial cross-sections must also undergo a related transition. Crudely speaking, for $E \ll E_n$ incoherent processes should appear as if they emanate from the whole downstream semi-surface, whereas for $E \gg E_n$ only the nuclear rim should contribute.

A detailed theoretical description of the $A$- and $E$-dependence of $\sigma(\gamma A)$ can be given in terms of the coupled channel optical model, the coupled modes being $(\gamma, \omega, \rho, \phi)$. These equations are

$$ (\mathbf{V}^2 + E^2 - U_{VV})\psi_\gamma = \sum_V U_{\gamma V} \psi_V \quad (94) $$

$$ (\mathbf{V}^2 + E^2 - m_V^2 - U_{VV})\psi_V = U_{VV} \psi_\gamma. \quad (95) $$

These can be solved quite easily if we bear in mind that

$U_{\gamma\gamma} = 0(e^2)$, $U_{\gamma V} = 0(e)$. Hence to $O(e^2)$ the photon wave function $\psi_\gamma$ only has three contributions:

i) the incident wave $e^{i k z}$;

ii) a wave that has been Compton scattered once by $U_{\gamma \gamma}$;

iii) a wave that has converted to $V$ and reconverted to $\gamma$, and is therefore proportional to $U_{\gamma V} U_{VV}$. 
No other powers or combinations of $U_{YY}$ and $U_{VV}$ can occur.) For simplicity, assume a nucleus of constant density, in which case the differential equations can be integrated easily. The photon wave function $\psi_Y(z)$ at a depth $z$ can then be cast into the following form:

$$
e^{-ikz} \psi_Y(z) - 1 = \frac{iz}{2k} U_{YY} + \frac{iz}{2k} \sum_V \frac{U_{YV} U_{VV}}{\varepsilon_V} - \sum_V \frac{U_{YV} U_{VV}}{\varepsilon_V^2} \left[ 1 - e^{-i\varepsilon_V z/2k} \right]$$

(96)

where

$$\varepsilon_V = m_V^2 + U_{VV} = m_V^2 - ik/\lambda_V .$$

(97)

The profile for nuclear Compton scattering is simply Eq. (96) evaluated at $z = 2\sqrt{R^2 - b^2}$, and the total cross-section is then determined from Eq. (31'):

$$\sigma(\gamma A) = \pi \int_0^{2R} z \, dz \, \text{Re} \left[ e^{-ikz} \psi_Y(z) - 1 \right] .$$

(98)

We now see that terms proportional to $z$ in the amplitude $\left[ e^{-ikz} \psi_Y(z) - 1 \right]$ produce a contribution to the cross-section proportional to the target volume, whereas constant terms give contributions proportional to the target's cross-sectional area. If $R \gg \lambda_V$, terms of both types are present in Eq. (96); for smaller nuclei, where $R$ is comparable to $\lambda_V$, the last term of Eq. (96) produces a contribution to $\sigma(\gamma A)$ that is rather smaller than the cross-sectional area.

Consider now the coefficient of the volume-proportional term,

$$K \equiv U_{YY} - \sum_V \frac{U_{YV} U_{VV}}{m_V^2 + U_{VV}} .$$

At high energy $\varepsilon_V \to U_{VV}$, because $U_{VV}$ is proportional to $k$; for a purely imaginary VN forward amplitude, $\varepsilon_V \to -ik/\lambda_V$, as one sees from Eq. (97). Thus when the inequality

$$k \gg \lambda_V m_V^2$$

(99)

*) In Eqs. (94) and (95), et seq., we have ignored coherent mixing of $V$'s, which is believed to be very small. The final results are not sensitive to this approximation.
is satisfied, \( K \) reduces to

\[
K = U_{\gamma\gamma} - \sum_{\nu} \frac{U_{\nu\nu} U_{\nu\gamma}}{U_{\nu\nu}}. \tag{100}
\]

If, in addition, VMD is correct, Eqs. (88) and (89) imply

\[
U_{\nu\nu} = \frac{e}{2\gamma} U_{\nu\nu} = \frac{2\gamma}{e} U_{\nu\nu},
\]

and \( K \) vanishes. Thus if VMD is valid, and the energy is high enough, the leading volume-proportional term of \( \sigma(\gamma A) \) disappears, leaving only the surface term. We have therefore recovered our earlier result, Eq. (92), but now we have a detailed description via Eq. (98) of the transition from the low- to high-energy regimes\(^*\).

Experiments to test these ideas have been carried out at SLAC, DESY, and Daresbury. Figure 12 shows that the total cross-sections for real photons definitely show shadowing, though not quite as much as VMD would require. Unfortunately the precision of these very difficult experiments is such that they do not tell us whether the \( E \)-dependence is correctly given by the theory. On the whole, though, the present agreement of theory with experiment for real photons is not discouraging.

There are two further experiments which, however, cast great doubt on the validity of the theory. Firstly, there is inelastic electron scattering, which determines \( \sigma(\gamma A) \) for virtual photons. Here the data shows\(^10\) no shadowing — see Fig. 13. Finally, there is a new and extensive Cornell experiment on \( \pi^0 \) photoproduction\(^14\). It does not show any trace of the behaviour expected on the basis of the argument given just after Eq. (93) above.

At this time there is no convincing explanation of why VMD seems to be so remarkably successful in relating \( e^+ e^- \rightarrow \pi^+ \pi^- \) to \( \pi^+ \pi^- \)-photoproduction, why it works reasonably well in predicting total cross-sections for real photons, while failing utterly for virtual photons and in the incoherent process of \( \pi^0 \)-photoproduction.

\(^*\) Although we have used purely absorptive optical potentials without correlations in this derivation, the result is completely general, even if hadronic mixing between different \( V \)’s is important\(^13\).
5. PROCESSES INDUCED BY HADRONS

5.1 The amplitude for a coherent hadronic process

The general problem of coherent hadronic production is completely encompassed by the multichannel equations of Section 3.5. If, however, the production amplitudes — i.e., the off-diagonal elements of $U$ in Eq. (65) — are much smaller than those on the diagonal, great simplifications occur. At first sight one would tend to believe that this is the situation found in nature, because production amplitudes of well-defined states are certainly small in comparison to elastic amplitudes. Assuming then that this is the case one finds an amplitude which is a straightforward generalization of the photoproduction amplitude (85), viz:

$$T_{ab}(q) = f_{ab}(0) \int dz \, d^2b \exp \left[ -\frac{1}{2} \sigma_{aN}(1-\alpha_{aN}) \int n(bz') \, dz' \right]$$

$$\times e^{iq \cdot b} e^{iq \cdot z} n(bz) \exp \left[ -\frac{1}{2} \sigma_{bN}(1-\alpha_{bN}) \int n(bz') \, dz' \right].$$  \hspace{1cm} (101)

In contrast to photoproduction, there is now an amplitude for the propagation of the incident particle $a$ from $-\infty$ to $z$ which allows for its attenuation.

5.2 Data on coherent hadronic processes induced by $\pi$ and K beams

Unfortunately the existent reviews of the data, though excellent, are somewhat inaccessible\(^3,^{15}\). I shall summarize the results of the most extensively studied processes:

$$\pi^- A \rightarrow \pi^- \pi^+ \pi^- A$$  \hspace{1cm} (102)

$$\rightarrow \pi^- \pi^+ \pi^- \pi^- A$$  \hspace{1cm} (103)

and

$$K^- \rightarrow K^- \pi^+ \pi^- A .$$  \hspace{1cm} (104)

Reactions (102) and (103) have been studied by counter techniques at CERN at 15.1 GeV/c\(^{16}\), while reaction (104) was examined at BNL and
CERN with a variety of heavy liquid mixtures at K-momenta of 5.5 to 12.7 GeV/c\(^+\)). The final state in reaction (102) is in the \(A_1\) region, which is a very prominent lump in the mass region of 1 to 1.8 GeV. This object is dominantly a \(\pi\rho\) system of \(I = 1, J^P = 1^+\). The \(K^-\pi^+\pi^-\) is a closely related structure called Q, i.e. mainly \(K^*\pi, J^P = 1^+, I = \frac{1}{2}\).

The analysis of reactions (102) and (103) has been done with the amplitude (101), assuming that each and every mass bin of the \(n\pi\) system can be treated as if it were a particle. Angular distributions and the \(A\)-dependence of the cross-section are shown in Figs. 14 and 15. Assuming \(\alpha_{bN} = 0\), the results are shown in Table 1.

<table>
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<th>(m_{3\pi})</th>
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<th>(m_{5\pi})</th>
<th>(\sigma_{5\pi,N}) (mb)</th>
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<td>1.5–1.7</td>
<td>10 ± 7</td>
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<td></td>
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<tr>
<td>1.7–1.9</td>
<td>37 ± 8</td>
<td></td>
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If \(\alpha_{bN}\) is allowed to become appreciable and negative (recall that \(\alpha \approx -0.3\) is typical at these energies), the cross-sections tabulated above decrease quite appreciably.

There is considerably less information on Q-production. The result of the analysis\(^{17}\) is \(\sigma_{QN} = 21 \pm 8\) mb in the 10–13 GeV/c region.

These results are very striking and unexpected. Naïvely one would have supposed that in the case of such ill-defined structures as \(A_1\) and Q the nuclear cross-section should be roughly the sum of the constituent cross-sections, perhaps depleted appreciably -- but not drastically -- by mutual shadowing effects\(^{18}\). This is clearly not the case -- after all, \(5\sigma_{^5\pi N} \approx 150\) mb! One might cast a suspicious eye on the experiments, for it is true that they have not yet yielded as detailed and precise
data as those on $\rho$-photoproduction $^\star$). But it would take quite a stretch of the imagination to assert that the strange results for $\sigma_{\text{QN}}$ and $\sigma_{\text{R\pi,N}}$ are merely due to faulty experiments. Surely it is our naive theoretical understanding of these processes which most needs improvement.

5.3 Towards a more sophisticated theory of coherent production

Where could the flaw in the theory be? The general spirit of the Glauber approach, whereby the process is treated separately for each impact parameter, is surely a very good approximation at these energies. What is more likely to be at fault is the notion that the transformation from the incident particle to the finally observed state takes place at a well localized point $(\underline{b},z)$. We already saw that in Compton scattering the photon does not scatter at a point, but it converts into other states ($\rho,\omega,\phi$ in the naive theory) which can exist for ever longer distances along $z$ as the energy grows. Presumably this is just a special case of a general phenomenon: as the energy increases all characteristic times governing the evolution of a state as observed in the lab. frame grow, and as a consequence the transformations become more and more non-localized. The object that is propagating through the nucleus is therefore not at all the asymptotic state, but is something that we do not observe in conventional scattering experiments.

These ideas have recently been explored by Van Hove$^{19}$, who has constructed a simple model embodying them. In his model the internal state of the evolving system is described by the simplest conceivable number of variables, one, and this is taken to be its mass, which ranges over an appreciable interval $^{**}$. When the system scatters from a single nucleon its mass changes are described by a scattering amplitude that is a continuous function of the incident and produced mass. Thus Van Hove

*) Thus in Ref. 16 it was necessary to integrate over the coherent peak, and this therefore includes an angular range where incoherent subtractions are much more important than near $\theta = 0$, where the $V$-photoproduction analysis is carried out. In Ref. 17 one deals with mixtures of nuclei, not with data on a well-defined set of nuclei spanning the periodic table.

**) Another variable that might be important is the number of pions, or perhaps, in a totally different framework, the parton distribution.
deals with a coupled-channel optical model where the matrices, such as $U$, are kernels in integral equations. He has solved his integral equation under the following assumptions:

i) the coupling from the discrete incident state (e.g. $\pi$) to the continuum (e.g. $3\pi$, $5\pi$) is small;

ii) the energy is sufficiently high so that all minimum momentum transfer effects are ignorable;

iii) the one-nucleon scattering amplitude for $m \leftrightarrow m'$ transitions is a smooth function of $m - m'$.

Of these (i) appears to be reasonable, but (ii) may not be valid at present energies; this is a shortcoming that can be repaired without changing anything essential. But (iii) is still controversial, because a simple argument\textsuperscript{20}) indicates that a term $\delta(m - m')$ should be included in the amplitude $f(m,m')$. If this is the case, Van Hove's results would be affected considerably.

Having listed the assumptions and objections, I can now summarize Van Hove's results, which are very interesting, and raise a whole new set of questions. Van Hove shows that his model gives a nuclear amplitude that can only be reproduced by the naïve amplitude (101) if, in the latter, $\sigma_{bN}$ is considerably smaller than is the quantity one might call the total one-nucleon cross-section in the continuum model. He therefore shows that his picture produces an effect of the correct sign. His calculations also contain several predictions: the $A$-dependence of the mass spectrum, which has diffraction-like patterns that depend on nuclear dimensions, and a mass spectrum for, say, $A_1$ production from deuterium in the double-scattering region that should differ markedly from that for production from hydrogen.

This type of approach to coherent processes may become of great importance at NAL energies if the school of thought\textsuperscript{21}) that believes that high-energy multiple production is mainly a manifestation of diffraction dissociation is vindicated. For if that is the case, an enormously extensive range of masses would be produced coherently in a nucleus, and the produced states could rescatter coherently amongst themselves. A possible practical consequence would be that secondary pion beams from high $A$ converters would be considerably more copious than one would expect in incoherent production.
5.4 Incoherent processes and processes that
go to a specified nuclear state

These are two types of phenomena that have received rather little
attention. In view of the rather unsophisticated state of multiple
scattering theory for incoherent processes, one might wonder whether
processes of this type can yield useful information. But this could be
rectified if one had precise experimental information on known processes
(K charge exchange is an ideal example) so that the theory could be
calibrated. It is certain that a rather precise theory could be formu-
lated given such a stimulus \(^*\)), and then one could measure interesting
quantities such as \(\sigma_{\pi N}, \sigma_{K^*N}\), which are not accessible to coherent pro-
cesses.

The excitation of specific nuclear levels\(^{22}\) can, in principle, be
detected by the subsequent \(\gamma\)-decay, and information about the orientation
of the excited state can be obtained by measuring the angular correlation
between the \(\gamma\)-ray and the produced particle. In this way the spin-parity
of the production mechanism could be studied, the nucleus acting some-
what like a polarized target.

5.5 Coulomb production

A particle \(a\) can "collide" with a Coulomb photon to produce
another system \(b\). A measurement of this cross-section determines the
rate for the electromagnetic process \(b + a \rightarrow \gamma\). If \(a = \gamma, b = \pi^0\), we
have the original Primakoff effect from which the \(\pi^0\)-lifetime is deter-
mined. Other examples of interest are \(\Xi^0 \rightarrow \Lambda^0 \gamma\), and \(K^* \rightarrow KY\); the latter
is presently being measured at CERN. For the sake of concreteness I
consider this last example here\(^{23}\).

The effective \(K\gamma K^*\) coupling can be described by the gauge-invariant
Lagrangian density

\[
L = \lambda \, F_{\mu \nu} \epsilon_{\nu \lambda \sigma} \, K^{* \lambda} \partial^\sigma K. \quad (105)
\]

For a static electric field this provides the interaction Hamiltonian

\[
H' = \lambda \int d^3x \, \mathbf{E} \cdot (\mathbf{K}^* \times \mathbf{V} K). \quad (106)
\]

\(^*\) Also contemplate the remarkable success shown in Fig. 6.
If \( K \) and \( K^* \) are treated in the plane wave approximation (momenta \( \vec{k} \) and \( \vec{p} \), respectively), the amplitude is immediately found to be

\[
\langle p|H'|k \rangle = e^{i\lambda Z} \frac{F(q^2)}{q} [\vec{e}^* \times \vec{k}] \cdot \vec{q}_+ ,
\]

(107)

where \( \vec{e}^* \) is the polarization vector of \( K^* \), and \( F(q^2) \) the electric form factor of the nucleus. Note that the matrix element (107) vanishes if the \( K^* \) helicity is zero, in conformity with our result of Section 2.4. Equation (107) displays the characteristic features of the Primakoff effect: the Rutherford divergence \( 1/q^2 \), which is \( 1/q_+^2 \) at \( \theta = 0 \), and the zero at \( \theta = 0 \) from the presence of \( \vec{q}_+ \). The angular distribution near \( \theta = 0 \) therefore has the dramatic form

\[
\frac{(k\theta)^2}{(m_a^2 - m_b^2)^2 + (k\theta)^2} .
\]

(108)

The plane wave approximation is certainly not valid because
(a) nuclear absorption of \( K \) and \( K^* \) is important, and (b) ordinary Coulomb scattering of \( K \) and \( K^* \) is significant for large \( Z \). As one must do these experiments in heavy materials a more refined theory is mandatory.

Both of the effects just mentioned require fairly obvious changes in the wave functions of \( K \) and \( K^* \). Consider first the nuclear interactions. The \( K^* \)'s wave function, which was \( e^{ikz} \) in Eq. (107) must now be

\[ e^{ikz} \exp \left\{ -\frac{1}{2} \sigma_{KN}(1 - i\alpha_{KN}) \int_{-\infty}^{z} n(b,z') \, dz' \right\} , \]

and a similar wave function, involving however \( \int_{-\infty}^{z} n(b,z') \, dz' \), must be used for \( K^* \). If \( \sigma_{KN} = \sigma_{K^*N} \), \( \alpha_{KN} = \alpha_{K^*N} \), the \( z \)-integrals combine to give the familiar factor

\[ e^{-\frac{1}{2} \sigma(1-i\alpha)\Pi(b)} \]

in the amplitude:

\[
\langle p|H'|k \rangle = i\lambda \int d^2b \, dz \, e^{-\frac{1}{2} \sigma(1-i\alpha)\Pi(b)} e^{i\lambda b} e^{iq^2} (\vec{e}^* \times \vec{k}) \cdot \vec{\nabla} \phi(bz) .
\]

(109)
Here $\phi$ is the electrostatic potential of the nucleus. Coulomb production is most interesting in large nuclei because, as we saw in Eq. (107), the amplitude is proportional to $Z$. Such nuclei are very opaque to hadrons, and a rough-and-ready approximation can therefore be obtained by setting $\sigma \to \infty$ in Eq. (109). Once this is done Eq. (109) only receives contribution from $b$'s outside the charge distribution, where $\phi$ is just the Coulomb field of a point charge. Hence Eq. (109) simplifies to:

$$\langle p|\phi'|k \rangle = i\lambda eZ \int_{b>R} d^2b \, dz \, e^{iq_{b} \cdot b} \, e^{iq_{z} \cdot \hat{k}} \frac{\hat{\hat{e}} \times \hat{k} \cdot \hat{b} \cdot \hat{e}}{4\pi (b^2 + z^2)^2}$$

$$\equiv \int d^2b \, e^{iq_{b} \cdot b} \, \Gamma(b) ,$$

where the last expression defines the profile function for the process:

$$\Gamma(b) = \frac{i\lambda eZ}{2\pi} \frac{\hat{\hat{e}} \times \hat{k} \cdot \hat{b}}{b} \frac{q_{b}}{b} K_1(q_{b}b) . \tag{110}$$

if $b > R$, and zero otherwise; here $K_1$ is the modified Hankel function.

The nuclear Coulomb field seen by the incoming and outgoing charged particles produces a further phase factor

$$\exp \left( \pm 2i \alpha \ln kb \right) \tag{111}$$

which must be appended to Eq. (110), where the signs in the factor (111) go with the indicated charge of the particles. This is the eikonal phase for a point Coulomb field, which is all that we need to concern ourselves with, because Eq. (110) already forces the particles to remain outside the charge distribution.

Because of the remarkable angular distribution (108) that is characteristic of the Primakoff effect, Coulomb production will dominate strong production at sufficiently high energy and small angle. In practice, however, hadronic production, such as $\omega$-exchange in the $KA \to K^*A$ case, can provide a very important background. The interference of the Primakoff and hadronic amplitudes is therefore of importance, and it follows from a closer study of Eqs. (110) and (111) that this interference has a complicated structure as a function of angle and energy.
because of the Coulomb phase factors. One may therefore hope to di-
sentangle the Coulomb and hadronic amplitudes from a detailed study of
the angular distribution\textsuperscript{23}).

* * *

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Figure captions

Fig. 1 : Various components of the pd elastic cross-section at 10 GeV, taken from Ref. 24. Single and double scatterings are shown separately and combined. $S_0$ means without the D-state, $S_Q$ with the D-state. The curve does not show a complete interference because the real parts of the nucleon-nucleon amplitudes are not set equal to zero, i.e. $\xi \neq 1$ in our notation.

Fig. 2 : Analysis of Alberi and Bertocchi\textsuperscript{25} of $\pi^-$d elastic scattering at 895 MeV/c; the data are from Bradamante et al.\textsuperscript{26}). The dashed curve ignores the D-state, and the full curve includes it.

Fig. 3 : The pd elastic cross-section\textsuperscript{24}). Agreement between theory and experiment is excellent in the single scattering and interference regions, but there are indications of disagreement for larger values of $q^2$.

Fig. 4 : Total neutron cross-sections at 5.7 GeV/c. The density $\tilde{n}$ in the optical potential has a Woods-Saxon shape, whose parameters are computed as follows\textsuperscript{27}). The true density $n$ in Pb is extracted from electron scattering and the resulting central density is held fixed for all $A > 20$, as is the surface thickness; $\tilde{n}$ is then determined from the known slope of NN elastic scattering. An estimate of the correlation correction has been included. The shaded region represents the uncertainties which are almost entirely due to the errors in the two-body cross-sections. For C a shell-model density (which also fits electron scattering) is used.

Fig. 5 : Multiple scattering calculation of elastic $\alpha$-$\alpha$ collisions by Kofod-Hansen\textsuperscript{28}). The curves marked 2, 5, 8 keep terms up to two-, five-, and eightfold-scattering. The curve marked (11, 16) is the full multiple-scattering series.
Fig. 6: Angular distribution of protons scattered from Pb compared to a multiple scattering calculation by Koford-Hansen$^{29}$). The density is a Woods-Saxon form taken from electron scattering. Coulomb scattering is included, but correlations are not.

Fig. 7: Angular distribution of photoproduced $\phi$'s in carbon$^{30}$).

Fig. 8: A dependence of the $\theta = 0^\circ$ $\phi$ production cross-section$^{30}$).

Fig. 9: $\sigma_{\phi N}$ and $\gamma^2/4\pi$ as functions of $\alpha_{\phi N}$. The storage ring value is indicated by $e^+e^- \to \phi$, and the leptonic decay result by $\phi \to e^+e^-$. For details see Ref. 10.

Fig. 10: $\chi^2$ map in the $\sigma_{\rho N} - \alpha_{\rho N}$ plane comparing the DESY-MIT $\rho$ data to an optical model analysis. The nuclear parameters used are the same as in Fig. 4. Also shown are limits on $\sigma_{\rho N}$ from $\rho$ production in deuterium$^{11}$), and on $\alpha_{\rho N}$ from $\rho \to e^+e^-$ and the Compton dispersion relation.

Fig. 11: Modulation factor for $\rho - \omega$ interference $F(\Omega)$ as defined by Eq. (87).

Fig. 12: Comparison of measured total photon cross-sections $\sigma(\gamma A)$ to theory. The quantity $A_{\text{eff}}$ is defined by $A_{\text{eff}} = \sigma(\gamma A)/\sigma(\gamma p)$. The full curve is the vector meson dominance prediction, whereas in the dashed curve measured values of $f_{\gamma p}$ and $f_{\rho p}$ are used, instead of the VMD expectation (these only amount to a $\sim 10\%$ change). For further details, see Ref. 10.

Fig. 13: The quantity $S = A_{\text{eff}}/A$ for the total cross-sections of virtual photons from Au, as measured by inelastic electron scattering at SLAC.

Fig. 14: Angular distribution of $A_1$-production on various nuclei$^{16}$).
Fig. 15 : Fits of optical potential cross-sections for $3\pi$-production in various mass intervals$^{16}$). The quantity $\sigma_2$ is the $(3\pi,N)$ total cross-section.

Fig. 16 : Angular distribution of $5\pi$-production from Ta$^{16}$).

Fig. 17 : Fits of optical potential cross-sections for $5\pi$-production in various mass intervals$^{16}$).
pd→pd at 10 GeV/c

Fig. 1
Fig. 2
\( p + Pb \rightarrow Pb + p' \)

- Bellettini et al. 19.3 GeV/c
- Allaby et al. 19.1 GeV/c (Preliminary)

- Elastic scattering
- Inelastic single scattering
- Inelastic double scattering
- Inelastic triple scattering
- Inelastic 1+2+3+4 fold
- Elastic + inelastic

Fig. 6
6.4 GeV
Carbon $t_\perp$ - Dependence

$\frac{d\sigma}{dt}$ ($\mu b / GeV^2$)

Kölbig & Margolis
Incoherent Calculation

$t_\perp (GeV^2)$

Fig. 7
\[ \frac{\gamma_{\phi}}{4\pi} \]

- 8.3 GeV Data
- 6.4 GeV Data
- All Data Combined

\[ \frac{\gamma_{\phi}}{4\pi} \text{ from } e^+e^- \rightarrow \phi \]

\[ \alpha_{\phi N} \text{ from } \phi - e^+e^- \]

\[ \sigma_{\phi N} \text{ (mb)} \]

\[ \alpha_{\phi N}(0) = \frac{\text{Re} f_{\phi N}(0)}{\text{Im} f_{\phi N}(0)} \]

Fig. 9
\[(\rho, \omega) \rightarrow \pi^+ \pi^-\]

Fig. 11
$A_{\text{eff}}/A$ from Total Photon Cross Sections

![Graphs showing $A_{\text{eff}}/A$ vs. $\nu$ (GeV) for C, Cu, and Pb with VMD predictions.](image)

Fig. 12
Fig. 15
Fig. 17