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NEW TESTS OF MULTIPARTICLE PRODUCTION MODELS

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A B S T R A C T

The two quantities known in statistical theory as the median and modal multiplicities form a better basis than the multiplicity moments for tests of multiparticle production models, and distinguish clearly between multiperipheral and diffractive models. The data indicate multiperipheral dominance.

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Most discussions of the multiplicity distributions in high energy collisions have taken one of two general approaches to the experimental data. In the first kind of approach, the distributions are compared directly with a theoretical expression - for example, a Poisson-like ¹⁾, normal-like ²⁾, or more complicated formula ³⁾. In the second approach, some moments or combinations of moments of the distribution are compared with theoretical predictions ^{4),5)}.

Both of these forms of analysis suffer from the defect that they are unpleasantly sensitive to the cross-sections for abnormally high or low multiplicities, which most models do not profess to treat correctly. This may be seen even in the case of a prediction of the mean multiplicity,

$$\langle n \rangle = \sum n \sigma_n / \sum \sigma_n , \quad (1)$$

where σ_n is the cross-section for the inelastic production of n particles of the type detected. The derivatives of Eq. (1)

$$\frac{\partial \langle n \rangle}{\partial \sigma_n} = \frac{(n - \langle n \rangle)}{\sum \sigma_n} , \quad (2)$$

reveal that the prediction is most sensitive to the cross-sections for multiplicities that are farthest from the mean.

This problem becomes more severe for the higher moments, and leads one to expect in general that model predictions of progressively higher moments will not become valid until higher and higher energies. This gives rise to an unfortunate situation in which, if the moments predicted by a model do not agree with experiment, then the proponents of the model can claim that present energies are not yet sufficiently high.

The purpose of this paper is to point out that there are two well-known characteristics of a multiplicity distribution that are of lower order than even the first moment, in the sense that they are less sensitive to the cross-sections for abnormally high or low multiplicities. These quantities are the median multiplicity, M_d , which is given by the equation

$$\sum_{n < M_d} \sigma_n = \sum_{n > M_d} \sigma_n , \quad (3)$$

and the modal multiplicity, M_0 , which expresses the position of the peak (or mode) of the distribution. Any model that claims to account for the majority of the inelastic cross-section and to give the correct mean multiplicity should also predict the correct values of M_d and M_0 , because these quantities are more intimately connected than the mean multiplicity with the most typical inelastic events.

Together with the mean, the median and mode constitute the three commonly used estimates of the centre of a probability distribution ⁶⁾. However, in some multiparticle production models the median and modal charged multiplicities are expected to have an energy dependence that is quite different from that of the mean. This surprising situation occurs in a class of models that I shall call diffractive ^{5),7)}. According to these models, particles are produced in one or two clusters which have mass spectra that are independent of the collision energy, apart from a kinematic cut-off at large masses. As the collision energy is increased, the multiplicity distribution becomes constant, except that higher multiplicities become kinematically accessible as the cut-off moves out to higher cluster masses. Thus, for suitably chosen mass spectra, the mean multiplicity, which emphasizes these higher multiplicities, can be arranged to increase logarithmically with energy, but the median and modal multiplicities, which avoid such an emphasis, approach constant values in much the same way as the total cross-section.

In sharp contrast to this diffractive picture of multiparticle production is a class of models that may be loosely described as multiperipheral ⁸⁾. In the multiperipheral picture, the increase in the mean multiplicity is associated with the production of more and more particle clusters of approximately constant mass, and the entire multiplicity distribution moves out to higher multiplicities as the collision energy increases. The median and mode are carried along with the distribution, so that these quantities have the same kind of energy dependence as the mean multiplicity.

The question of which of these quite different mechanisms, or what proportion of each, correctly describes the dynamics of multiparticle production is probably the matter of greatest urgency in the study of these processes, and we see that a study of the median and mode of the charged multiplicity distribution has a good chance of shedding some light on this problem. In the remainder of this paper, I shall first define these quantities more precisely, then summarize the model predictions more quantitatively and finally make a comparison with the recent data on high energy pp collisions.

Although the distribution of multiplicity is discrete, its median and modal values can be defined as continuous quantities by interpolation of the data. In the case of the median, suppose M is the smallest integer such that

$$\sum_{n=0}^M \sigma_n > \sum_{n=M+1}^{\infty} \sigma_n . \quad (4)$$

With the comparison with the pp data in mind, from now on σ_n will be taken to be the cross-section for the production of n negatively charged particles in an inelastic pp collision. Then the number of charged particles in the final state is given by

$$n_{ch} = 2n + 2 , \quad (5)$$

and a suitable interpolation formula for the median charged multiplicity, M_d , is

$$M_d = 2M + 2 + \left(\sum_{n=M+1}^{\infty} \sigma_n - \sum_{n=0}^{M-1} \sigma_n \right) / \sigma_M . \quad (6)$$

The derivatives of Eq. (6),

$$\frac{\partial M_d}{\partial \sigma_n} = \pm \frac{1}{\sigma_M} \quad (n \neq M) , \quad (7)$$

compared with Eq. (2), show that the median is less sensitive than the mean to the cross-sections for multiplicities that are far from the mean.

In order to locate the modal value M_o of the charged multiplicity, I have simply made a quadratic interpolation in the logarithms of the three largest topological cross-sections. If these occur at the values $n = N, N \pm 1$, this procedure gives

$$M_o = 2N + 2 + \ln \left(\frac{\sigma_{N+1}}{\sigma_{N-1}} \right) / \ln \left(\frac{\sigma_N^2}{\sigma_{N+1} \sigma_{N-1}} \right) . \quad (8)$$

It is clear that the modal multiplicity defined by Eq. (8) does not depend at all on the cross-sections for abnormally high or low multiplicities, but rather summarizes the relative behaviour of the most important cross-sections.

The interpolation rules (6) and (8) are not unique, but any alternative definitions will differ by at most a small constant, which is irrelevant for our purposes. Figure 1 illustrates the results of their application when the production of negatively charged particles is described by a Poisson distribution,

$$\sigma_n \propto e^{-\langle n \rangle} \frac{\langle n \rangle^n}{\Gamma(n+1)} \quad (9)$$

In this case, Eq. (9) can be taken to define a smooth interpolating distribution function for non-integral n , and one can show that for large $\langle n \rangle$ the median and mode of this function have the properties

$$M_d \approx \langle n_{ch} \rangle - \frac{1}{3} \quad , \quad (10)$$

$$M_o \approx \langle n_{ch} \rangle - 1 \quad . \quad (11)$$

It may be seen from Fig. 1 that the interpolation rules (6) and (8) give good agreement with the analytical results (10) and (11), even for small values of $\langle n \rangle$.

The properties (10) and (11) of the Poisson-like distribution are a particular case of an important asymptotic relation between the mean, median and mode which holds for a much wider class of distribution functions ; it was first derived by Haldane ⁹⁾. In language appropriate to the present discussion, we may summarize Haldane's results as follows : if the correlations ^{10),11)} that occur in inclusive cross-sections have short (i.e., finite) range ^{*)}, then

$$M_d = \langle n_{ch} \rangle - \frac{1}{3} d + O[(\ln s)^{-1}] \quad , \quad (12)$$

$$M_o = \langle n_{ch} \rangle - d + O[(\ln s)^{-1}] \quad , \quad (13)$$

^{*)} The range referred to here is in longitudinal rapidity : see Refs. 10)-12). Haldane's discussion is in terms of the cumulants of the distribution [see, for example, Ref. 6), pp. 67-75, p. 179] ; the hypothesis of short-range correlations implies that the cumulants are of order $\ln s$.

where d is a constant, which happens to be 1 for the Poisson-like distribution. It is convenient to eliminate d from Eqs. (12) and (13) and to express them as a prediction of a quantity H , which I shall call Haldane's ratio,

$$H \equiv \frac{\langle n_{ch} \rangle - M_0}{\langle n_{ch} \rangle - M_d} \quad (14)$$

$$= 3 + O[(\ln s)^{-1}] \quad (15)$$

The hypothesis of short-range correlations that leads to the results (12), (13) and (15) is for practical purposes equivalent to the multiperipheral mechanism discussed above¹²⁾. Thus the multiperipheral models^{8),12)} predict that

$$\langle n_{ch} \rangle, M_d, M_0 \sim c \ln s \quad (16)$$

and that the relation between the mean, median and mode is asymptotically given by (15). One should notice, however, that (15) is more of an asymptotic statement than (16), and is therefore a less reliable test of multiperipheral models at present energies. Nevertheless, it would be embarrassing for these models if, say, H was equal to 10 and not decreasing at a beam momentum of 200 GeV/c ($\ln s \approx 6$).

As we have seen, the behaviour of the median and modal multiplicities in diffractive models is entirely different from that in the multiperipheral case. The cross-sections σ_n approach constant values at high energy, and so also do the positions of the median and mode,

$$M_d \sim \text{constant} \quad , \quad (17)$$

$$M_0 \sim \text{constant} \quad . \quad (18)$$

The mean, on the other hand, can still be arranged to increase like $\ln s$, if we suppose that

$$\sigma_n \propto n^{-2} \quad (19)$$

at large n ; the existence of a kinematic cut-off in the vicinity of $n \propto \sqrt{s}$ then provides the necessary energy dependence of $\langle n \rangle$. A simple illustration of these properties is provided by the example

$$\sigma_n(s) = n_0 \sigma \Theta(c\sqrt{s} - n) \frac{\exp(-n_0/n)}{n^2} , \quad (20)$$

which gives

$$\begin{aligned} \sigma_T &\approx \sigma - O(s^{-1/2}) , \quad M_d \approx \frac{2}{\ln 2} n_0 + 2 - O(s^{-1/2}) , \\ M_0 &\approx n_0 + 2 , \quad \langle n_d \rangle \sim n_0 \ln s , \end{aligned} \quad (21)$$

where σ_T is the total inelastic cross-section.

The quantitative predictions of a more realistic diffractive model, the nova model of Jacob, Slansky and Berger^{5),7)}, will be discussed after we have looked at the experimental data.

Diffractive models involve very long-range correlations, which invalidate Haldane's results. Instead of Eq. (15), we see from Eqs. (17) and (18) that in the diffractive case

$$H = 1 + O[(\ln s)^{-1}] . \quad (22)$$

In addition to the purely multiperipheral or diffractive models, we should consider the possibility that significant amounts of both processes occur, leading to a class of models that I shall call hybrid¹³⁾. In these models, one usually supposes that the two mechanisms make their main contributions in different parts of phase space, so that their interference may be neglected. In this approximation, every cross-section may be written as the sum of a diffractive and a multiperipheral part,

$$\sigma_n = \sigma_n^D + \sigma_n^M , \quad (23)$$

$$\sigma_T = \sigma_T^D + \sigma_T^M . \quad (24)$$

If $\langle n_D \rangle$ and $\langle n_M \rangle$ are the mean charged multiplicities associated with each mechanism separately, we have

$$\langle n_{ch} \rangle = (\sigma_T^D \langle n_D \rangle + \sigma_T^M \langle n_M \rangle) / \sigma_T \quad . \quad (25)$$

At extremely high energies, the hybrid multiplicity distribution would be expected to be bimodal, with one static diffractive mode and one moving mode in the vicinity of $\langle n_M \rangle \sim c_M \ln s$. At lower energies, only one mode may be apparent, and, depending on whether $\sigma_T^D > \sigma_T^M$ or $\sigma_T^M > \sigma_T^D$, it will be either static or moving. Similarly, the median multiplicity may be either asymptotically constant or of the order of $c_M \ln s$, depending on which mechanism is dominant : it will always lie in the vicinity of the more prominent mode ^{*}). Thus the median and modal multiplicities have the useful property that they retain the kind of energy dependence associated with the dominant mechanism, even in the presence of a secondary contribution of a different type. However, in a hybrid model dominated by multiperipheral processes we can no longer expect Eq. (16) to be true with a common constant of proportionality c , because in general $M_d, M_o \sim \langle n_M \rangle \neq \langle n_{ch} \rangle$. Furthermore, the median and mode may increase with $\ln s$ either faster or slower than $\langle n_{ch} \rangle$, depending on the behaviour of the diffractive multiplicity $\langle n_D \rangle$.

Hybrid models necessarily involve long-range correlations, so Haldane's result (15) is invalid and one finds that in most situations $H \rightarrow 1$ as $s \rightarrow \infty$. However, the behaviour of H is not such a good test of hybrid models because the limit may be approached extremely slowly. In the case of multiperipheral dominance, for example,

$$H = 1 + O[(\ln s)^{-1/2}] \quad , \quad (26)$$

provided $\langle n_D \rangle \neq \langle n_M \rangle$.

^{*}) By the more prominent or principal mode I mean the peak containing most of the cross-section, not necessarily the highest peak.

Experimental data on the charged multiplicity distribution in pp collisions are now available at a variety of beam momenta up to 205 GeV/c. Using Eqs. (6) and (8), I have computed the median and modal charged multiplicities above 10 GeV/c. The sources of the data used are listed in Ref. 14), and the results are compared with the mean multiplicity in Figs. 2 and 3. It may be seen that the median and modal multiplicities increase with energy in approximately the same way as the mean, and do not appear to be approaching constant asymptotic values. Furthermore, the value of Haldane's ratio H , which is shown in Fig. 4 ^{*)}, approaches 3 in a way that is in good agreement with the predictions of models of the multiperipheral type.

The simplest purely diffractive models, such as that represented by Eq. (20), are in serious disagreement with the data on the median and modal multiplicities. In more realistic diffractive models, there remains some suspicion that perhaps the asymptotic predictions could be postponed to energies greater than 200 GeV, so that a fit to the data could be obtained at present energies. I have investigated this possibility in the case of the nova model of Jacob, Slansky and Berger ^{5),7)}.

According to the nova model, one or both protons may be excited into resonant (nova) states with a mass spectrum

$$\rho(M) \propto (M-M_p)^{-2} \exp[-2M^*/(M-M_p)] \quad , \quad (27)$$

where M^* , the most probable nova mass, is 1.9 GeV. A nova of mass M decays into a mean number of pions N which is proportional to its excitation energy,

$$N = c(M-M_p) \quad , \quad (28)$$

and the corresponding number of negatively charged pions produced is taken to be binomially distributed with mean

$$\langle n(\pi^-) \rangle = \frac{1}{3}(N-1) \quad . \quad (29)$$

*) The errors in H are smaller than one might expect from an examination of Figs. 2 and 3. This is because the mean, median and mode are strongly correlated.

By choosing the ratio of double to single nova excitation to be $3/7$, and the constant c to be 1.9 pions per GeV (both values being close to those suggested by Berger et al. ⁵⁾), one can obtain good agreement with the mean charged multiplicity ^{*}), as shown by the solid curve in Figs. 2 and 3. The median multiplicity, indicated by the dashed curve in Fig. 2, does indeed approach its asymptotic value (about 6.4) rather slowly, but it is already in disagreement with the data at 50 GeV/c. Furthermore, the rise in the predicted median multiplicity between 10 and 200 GeV/c is associated in the nova model with a comparable increase in the predicted total cross-section, which is not observed experimentally.

The predicted modal multiplicity (dot-dashed curve in Fig. 3) is always in the vicinity of its asymptotic value, and although this value can be altered by slight adjustments of parameters, it is difficult to see how the energy dependence of the mode could be brought into agreement with the data. Haldane's ratio (solid curve in Fig. 4) is predicted to be falling through the value 3 at 200 GeV/c, but the agreement with the data at this energy is an accident resulting from the unacceptably low modal multiplicity, and it is spoiled if the model parameters are adjusted to shift the position of the mode.

The hybrid models that are in best agreement with the data in Figs. 2-4 are those in which the multiperipheral type of component is dominant, so that the median and modal multiplicities increase logarithmically with energy. The existence of a smaller diffractive component with the property $\langle n_D \rangle > \langle n_M \rangle$ would imply that these quantities increase slightly less rapidly than the mean multiplicity, a possibility that is consistent with the data.

In conclusion, the present experimental data on median and modal multiplicities are in good agreement with the hypothesis that pp collisions at high energies are dominated by processes of a multiperipheral type. The same data are difficult to understand on the basis of the diffractive models that we have considered. These results therefore tend to confirm the multiperipheral picture that the increase in the mean multiplicity is due to the

^{*}) This is true if one introduces a sharp cut-off in the nova production amplitude when the sum of the nova masses is equal to $\sqrt{s}$. If one includes factors of the form $e^{Bt_{\min}}$, where $|t_{\min}|$ is the minimum momentum transfer, a fit to the mean multiplicity cannot be obtained without substantial changes in the parameter values.

production of more clusters of particles, rather than an increase in the mean mass of a fixed number of clusters, as is the case in the diffractive picture. However, the data are also consistent with a hybrid model in which there is a significant secondary contribution of the diffractive type. Any difference in the coefficients of $\ln s$ in the energy dependences of the mean, median and modal multiplicities, or a deviation of Haldane's ratio from the value 3 at high energies, would support the existence of such a contribution. It will therefore be interesting to refine our knowledge of these quantities as more precise data become available at high energies.

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FIGURE CAPTIONS

- Figure_1 Results of applying Eqs. (6) and (8) to a Poisson-like distribution. The mean charged multiplicity is $\langle n_{ch} \rangle = 2 \langle n \rangle + 2$.
- Figure_2 Data on the mean and median charged multiplicities in pp collisions, and nova model predictions of the mean (solid curve) and median (dashed curve).
- Figure_3 Data on the modal charged multiplicity in pp collisions, and the nova model prediction (dot-dashed curve). For comparison, the data and prediction for the mean multiplicity, already given in Fig. 2, are also shown here.
- Figure_4 Data on Haldane's ratio H , defined by Eq. (14), and the nova model prediction (solid curve).

FIG.1

FIG.2

FIG.3

FIG.4