CM-P00058860

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Ref.TH.1515-CERN

LIGHT CONE CURRENT ALGEBRA AND REGGE COUPLINGS

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ABSTRACT

Considerations on Regge couplings in hadron-hadron scattering and deep inelastic electroproduction are shown to be suggested by canonical light cone expansion. The particular case in which no canonical scalar fields are present is examined in detail.

Since the Stanford experiment ¹⁾ confirmed the Bjorken scaling conjecture ²⁾, a lot of theoretical work has been performed to gain more insight into the nature of this phenomenon.

A very general mechanism which could account for empirical facts has been identified in the so-called canonical light cone expansion ³⁾⁻⁵⁾ which provides a very natural framework to describe inclusive high mass experiments. The purpose of this letter is to show that by means of the light cone expansion it is possible to get some dynamical information about the structure of Regge pole couplings. In particular, it is possible to justify in some cases the interesting proposal made in Ref. 6) about the role of currents in high energy hadron-hadron scattering. The idea is that, since operator product expansion looks like a crossed channel partial wave expansion, it is possible, by means of a kind of pole dominance, to relate the residue of a Regge trajectory to the matrix elements of the operators which appear in the light cone expansion and so, in some cases, through factorization, we can get information about Regge couplings to hadrons. In other cases, it is possible to outline general features of the deep inelastic Regge limit such as decoupling of trajectories or the presence of fixed singularities.

Let us consider the forward scattering of a vector SU(3) current against an unpolarized target; this process will be described by :

$$\begin{aligned} \int e^{iq \cdot x} \langle P | T^* (J_\mu^{(i)}(x) J_\nu^{(k)}(0)) | P \rangle d^4x = \\ = g_{\mu\nu} T_1^{(i,k)}(q^2, \nu) + p_\mu p_\nu T_2^{(i,k)}(q^2, \nu) + \dots \end{aligned} \quad (1)$$

where i and k denote SU(3) indices, $\nu = 2p \cdot q$ and T^* is a covariant retarded commutator whose definition will be made clearer in a moment.

We suppose that in the so-called Bjorken limit ²⁾ $q^2 \rightarrow -\infty$, $\omega \equiv -\nu/q^2$ fixed the form factors $T_1^{(i,k)}$ and $T_2^{(i,k)}$ scale in the sense :

$$\begin{aligned} T_1^{(i,k)}(q^2, \nu) \xrightarrow{B_j} F_1^{(i,k)}(\omega) \\ \nu T_2^{(i,k)}(q^2, \nu) \xrightarrow{B_j} F_2^{(i,k)}(\omega) \end{aligned} \quad (2)$$

This hypothesis is suggested, although not implied by SLAC experiments.

The behaviour in Eq. (2) is easily accounted for by any canonical light cone expansion. For definiteness, we shall assume the expansion of the form proposed in Ref. 7) which, for example, gives :

$$\begin{aligned}
 4\pi \left[\int_{\mu}^{(i)}(x), \int_{\nu}^{(k)}(0) \right] \xrightarrow{x^2 \rightarrow 0} & i (\partial_{\mu} \partial_{\nu} - g_{\mu\nu} \square) [\epsilon(x_0) \delta(x^2) \times \\
 & \times \left\{ d^{ik\ell} (G_{(x,0)}^{(\ell)} + G_{(0,x)}^{(\ell)}) + i f^{ik\ell} (G_{(x,0)}^{(\ell)} - G_{(0,x)}^{(\ell)}) \right\}] + \\
 & + S_{\mu\nu\rho\sigma} [\partial^{\rho} (\epsilon(x_0) \delta(x^2))] \left\{ d^{ik\ell} (F_{(x,0)}^{(\ell)\sigma} - F_{(0,x)}^{(\ell)\sigma}) + \right. \\
 & \left. + i f^{ik\ell} (F_{(x,0)}^{(\ell)\sigma} + F_{(0,x)}^{(\ell)\sigma}) \right\} + \epsilon_{\mu\nu\rho\sigma} (\dots\dots\dots)
 \end{aligned} \tag{3}$$

and analogous expressions for axial currents. $S_{\mu\nu\rho\sigma}$ is defined as $g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho} - g_{\mu\nu} g_{\rho\sigma}$, the f's and d's are the usual SU(3) objects; in particular $d_{oik} \equiv \sqrt{2/3} \delta_{ik}$. It is to be noted that only singlet and octet bilocal operators appear in (3); this assumption, although not essential, greatly simplifies our considerations.

The bilocal operators $G^{(k)}(x,0)$ which appear in (3) are present only in a theory with canonical scalar fields and their Taylor expansions in x^{μ} begin with a local scalar operator of dimension 2, while the first operator in the expansion of $F^{(k)\sigma}(x,0)$ is a vector operator of dimension 3, proportional to $J^{(k)\sigma}(0)$:

$$F^{(k)\sigma}(0,0) \propto J^{(k)\sigma}(0) \tag{4}$$

It is easily verified that $G^{(k)}(0,0)$ is proportional to the operator Schwinger term in the equal time limit of (3).

With these preliminaries, the T^* product needed in Eq. (1) is seen to be obtained, on the light cone, by (3), replacing the $\epsilon(x_0)$ which multiplies $G^{(k)}(x,0)$ by $\theta(x_0)$ and putting a $\theta(x_0)$ function behind the other terms. In fact, one can verify

$$\partial_{\mu} T^* \left(\int_{\mu}^{(i)}(x) \int_{\nu}^{(k)}(0) \right) \xrightarrow{x^2 \rightarrow 0} i \delta^{(4)}(x) f^{ik\ell} \int_{\nu}^{(\ell')}(0) \tag{5}$$

(currents are conserved on the light cone).

Using (3), we can now get an explicit expression for the function $F_1^{(i,k)}(\omega)$ defined in (1) :

$$\begin{aligned} F_1^{(i,k)}(\omega) &= \sum_{n=0}^{\infty} a_n^{(i,k)} \omega^n = d^{i,k,l} \sum_{n=0}^{\infty} S_{2n}^{(l)} \omega^{2n} + \\ &+ f^{i,k,l} \sum_{n=0}^{\infty} S_{2n+1}^{(l)} \omega^{2n+1} + d^{i,k,l} \sum_{n=1}^{\infty} V_{2n}^{(l)} \omega^{2n} + \\ &+ f^{i,k,l} \sum_{n=0}^{\infty} V_{2n+1}^{(l)} \omega^{2n+1} \end{aligned} \quad (6)$$

where

$$F^{(l)\delta}(x,0) = \sum_n x^{\mu_1} \dots x^{\mu_n} F_{\mu_1, \dots, \mu_n}^{(l)\delta}(0) \quad (7)$$

and

$$V_n^{(l)} \propto \frac{\langle P | F_{\mu_1, \dots, \mu_n}^{(l)\delta}(0) | P \rangle}{P^{\delta} p_{\mu_1} \dots p_{\mu_n}}$$

($g_{\mu_1 \mu_2}$ terms are neglected), and the $S_n^{(l)}$ are analogously defined in terms of $G^{(l)}(x,0)$.

Now $T_1^{(i,k)}(q^2, \nu)$ is the t channel non-spin flip amplitude and as such will admit a t channel partial wave expansion of the form

$$T_1^{(i,k)}(q^2, \nu) = \sum_{n=0}^{\infty} b_n^{(i,k)}(q^2) P_n(\cos \Theta_t) \quad (8)$$

In the Bjorken limit ($q^2 \rightarrow -\infty$, ω fixed) we have

$$\cos \Theta_t = \frac{\nu}{2m\sqrt{-q^2}} \rightarrow \infty \quad (9)$$

So if we require scaling as expressed in (2), we must have :

$$b_j^{(i,k)}(q^2) \xrightarrow{q^2 \rightarrow -\infty} \left(\frac{2m}{\sqrt{-q^2}} \right)^J a_J^{(i,k)} \quad (10)$$

so that finally

$$T_1^{(i,k)}(q^2, \nu) \xrightarrow{B_j} F_1^{(i,k)}(\omega) = \sum_{n=0}^{\infty} a_n^{(i,k)} \omega^n \quad (11)$$

Equation (11) tells us that the coefficients which appear in (6) are just the partial wave amplitudes in the t channel for the process (1). As such, when continued at complex n , they have singularities which fix the asymptotic behaviour of $F_1^{(i,k)}(\omega)$ as $\omega \rightarrow \infty$. In particular, if $a_n^{(i,k)}$ has a pole at $n = \alpha$, then $F_1^{(i,k)}(\omega) \xrightarrow{\omega \rightarrow \infty} \omega^\alpha$. Let us make this point more precise : if we choose, for example, the currents which appear in (1) to be $i = 1+i2$ and $k = 1-i2$ so that the ρ trajectory can be exchanged in the t channel, the coefficients in (6) with the ρ exchange quantum numbers are of course $S_{2n+1}^{(3)}$ for the contribution from scalar fields and $V_{2n+1}^{(3)}$ for the spinor field contribution.

At complex n we have :

$$S_n^{(3)} + V_n^{(3)} = \left(\frac{\beta_\chi^{(\rho)} \beta_p^{(\rho)}}{n - \alpha_\rho(0)} + \text{background} \right) \left(\frac{1 - e^{-i\pi n}}{2} \right) \quad (12)$$

where we have assumed factorization of the Regge residue : $\beta_\chi^{(\rho)}$ is the coupling of the ρ trajectory to the current and $\beta_p^{(\rho)}$ is the analogous coupling to the target p . $\beta_p^{(\rho)}$ is a quantity which appears in strong interactions where it is defined by

$$\text{Im } A_{pp' \rightarrow pp'}^{(\rho \text{ exchange})}(t=0, s) \xrightarrow{s \rightarrow \infty} \beta_p^{(\rho)} \beta_{p'}^{(\rho)} s^{\alpha_\rho(0)} \quad (13)$$

It is to be noted in (12) that the signature factor accounts for the fact that the light cone expansion which corresponds to ρ quantum numbers exchange has only odd n coefficients. Now $\alpha_\rho(0) \approx \frac{1}{2}$ so that if we believe in some sort of pole dominance, we can neglect the background contribution when n is near $\frac{1}{2}$. In particular for $n=1$ we have :

$$\frac{\beta_\gamma^{(e)} \beta_P^{(p)}}{1 - \alpha_p(0)} \approx S_1^{(3)} + V_1^{(3)} \quad (14)$$

If Eq. (14) is valid, we have of course

$$\beta_P^{(e)} \approx C_p \frac{\langle P | J_\mu^{(3)} + S_\mu^{(3)} | P \rangle}{P_\mu} \quad (15)$$

where $S_\mu^{(3)}$ is the vector operator in the Taylor expansion of $G^{(3)}(x,0)$, C_p is a constant which does not depend on the target so that Eq. (15) can be seen as a sort of universality principle for the couplings of vector meson Regge trajectories.

In particular, in a theory in which no canonical scalar fields are present, which could be of physical relevance due to the approximate transversality observed at SLAC, $S_\mu^{(3)}(0)=0$ and (15) tells us that the ρ trajectory couples through the vector current which has its quantum numbers. The same argument applies also for ω and φ trajectories, and hadron experiments seem to show approximate agreement with this pattern of couplings⁹⁾.

For strangeness changing vector trajectories, like K^* , we can no longer identify the light cone expansion with the crossed channel partial wave expansion because the matrix element in (1) must be kept between particles of different masses and so an uncertainty of the order of $SU(3)$ breaking is to be added to the pole dominance approximation.

Let us now see what happens for positive signature trajectories. Consider the general case :

$$S_n^{(1)} + V_n^{(1)} = \left(\frac{\beta_\gamma^{(1)} \beta_P^{(1)}}{n - \alpha(0)} + \text{background} \right) \left(\frac{1 + e^{i\pi n}}{2} \right) \quad (16)$$

where, for the moment, the quantum numbers of the crossed channel are unspecified. If a trajectory happens to have an intercept near 0 (now at $n=1$ the signature factor will be zero), we can invoke pole dominance to get :

$$S_0^{(')} \approx \frac{\beta_x^{(')} \beta_p^{(')}}{-\alpha'(0)} \quad (17)$$

[It is to be noted, by Eq. (6), that $v_0^{(')} = 0$.]

Therefore, in this case :

$$\beta_p^{(')} \approx C \langle p | h^{(')} (0) | p \rangle \quad (18)$$

where $h^{(')} (0)$ is the scalar operator in the Taylor expansion of $G^{(')} (x, 0)$. Thus, in a theory with no elementary scalar fields, (17) reads

$$\beta_x^{(')} \beta_p^{(')} \approx 0 \quad (19)$$

unless some other phenomena happen. We have several possibilities. The first one is the decoupling of the trajectory from the process described in (1) as suggested by Eq. (19), or the cancellation between trajectories if several of them are present in the same channel; the second one is the presence of a Kronecker δ term at $n=0$, the only type of fixed singularity at sense points tolerated by linear unitarity; finally the third one is the very unlikely requirement that the background exactly compensates at $n=0$ the contribution of the Regge singularity.

Let us now see what happens in the deep inelastic electroproduction. In the channel with vacuum quantum numbers, the Pomeron and the f_0 trajectories seem to be present in the proton inelastic form factor measured at SLAC. So, unless a cancellation between P and f_0 takes place, a Kronecker δ singularity must be present at $n=0$ in T_1 , and this should imply, through gauge invariance, a fixed $J=0$ pole with vacuum quantum numbers in T_2 ¹⁰⁾. If we call Δ the coefficient of the Kronecker δ singularity, we have (factorization is not required any more) :

$$\Delta + \frac{\beta^{(P)}}{-\alpha_p(0)} + \frac{\beta^{(f_0)}}{-\alpha_{f_0}(0)} \approx \Delta - \beta^{(P)} - 2\beta^{(f_0)} \approx 0 \quad (20)$$

and Δ is also the residue of the fixed $J=0$ pole in $F_2(\omega)$. Such a fixed pole seems to be consistent with fits of inelastic form factors ¹¹⁾.

In the $I_t=1$ channel we have the A_2 trajectory. Since no other trajectories with these quantum numbers are known, cancellation seems to be very unlikely. So, in a theory in which only fermion fields are present, A_2 can give rise to a fixed $J=0$ singularity or else decouple.

The analysis of experiments performed at SLAC on the difference of the inelastic form factors of the proton and of the neutron ¹⁾ could be compatible with the decoupling of the A_2 trajectory, but more accurate data are needed to reach such a conclusion.

It is a pleasure to thank Professor D. Amati for innumerable discussions without which this work could not have been written.

R E F E R E N C E S

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