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ABSENCE OF MACDOWELL DOUBLETS, DUALITY SOLUTIONS FOR BARYONS
AND BACKWARD πN SCATTERING AT 6 GeV/c

P. Minkowski

CERN - Geneva

A B S T R A C T

An attempt is made to compare restrictions on baryon exchange Regge residues demanding absence of MacDowell doublets and incorporating an approximate duality solution for the decuplet exchange with experimental data on πN backward cross-sections and polarizations at $p_{\text{lab}} = 6 \text{ GeV/c}$.

Although the gross features of cross-sections and polarizations can be obtained from $\mathcal{N}_\alpha$, Δ_δ , Δ_β exchanges, substantial contributions of another nature are needed to describe all details of the data.

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INTRODUCTION

The aim of this work is to investigate the structure of Regge residue functions for baryon exchange compatible with the absence of MacDowell doublets. The alternative of fixed cuts in the complex j plane proposed by Carlitz and Kislinger ¹⁾ shall not be discussed here.

An approximately dual solution for baryons grouping together the trajectories

$$[8]_{\alpha} \longleftrightarrow [8]_{\gamma}, [1]_{\gamma} ; [10]_{\delta}, [8]_{\delta} \longleftrightarrow [10]_{\beta}, [8]_{\beta}$$

shall serve as a starting point. (The numbers within brackets denote the SU(3) representation, the subscripts the trajectory type.)

The main features of the data to be explained are the following ²⁾.

- a) The pronounced dip in the π^+p cross-section at $|u| \sim 0.12-0.15(\text{GeV})^2$.
- b) The large negative polarization in π^+p scattering for $|u| > 0.3(\text{GeV})^2$. Following the discussion of Barger and Olsson ²⁾ this implies a large negative polarization in the amplitude for $I_u = \frac{1}{2}$ scattering. In the proposed model this has to be achieved through interference of the N_{α} , N_{β} , N_{γ} and N_{δ} exchanges.
- c) The structure of the $I_u = \frac{1}{2}$, $I_u = \frac{3}{2}$ interference term ²⁾

$$D = \text{Re} \left(\underline{f}_{I_u=1/2}^+ \cdot \underline{f}_{I_u=3/2} \right)$$

$$= \frac{3}{4} \left[\left(\frac{d\sigma}{d\Omega} \right)^{\pi^+p} - 2 \left(\frac{d\sigma}{d\Omega} \right)^{\text{EX}} + \frac{1}{3} \left(\frac{d\sigma}{d\Omega} \right)^{\pi^-p} \right]$$

It is found that D has the form $D \sim (u - u_d)^2$ for $u \sim u_d = -0.15 \text{ GeV}^2$. $\underline{f}$ stands for a two-vector $\underline{f} = (f_{-+}, f_{++})$ $f_{\pm\pm}$ denotes the helicity amplitude in a given channel.

- d) The polarization in π^-p scattering changing sign at $|u| \sim 0.5-0.6(\text{GeV})^2$.

The following statements can be made prior to a detailed analysis.

- 1) The splitting of the $\mathcal{N}_\alpha, \mathcal{N}_\gamma$ trajectories reduces the contribution of $\mathcal{N}_\gamma$ relative to $\mathcal{N}_\alpha$ in the exact exchange degenerate situation by a factor

$$R = \left(s_0/s\right)^{\alpha(\mathcal{N}_\alpha) - \alpha(\mathcal{N}_\gamma)} \sim \left(s_0/s\right)^{1/2} \sim \frac{1}{3.5}$$

choosing $s_0 = 1 \text{ GeV}^2$, for $p_{\text{lab}} = 6 \text{ GeV}/c$.

- 2) The $[8]_\alpha \leftrightarrow [8]_\gamma, [1]_\gamma$ duality solution is not realized in the quark oscillator model.
- 3) The double zero in D can be explained if Δ_δ and Δ_β are chosen to have exchange degenerate residue functions at $u = u_d$. This implies

$$|\Delta_\delta| \simeq |\Delta_\beta| \quad \text{for } u = u_d$$

- 4) The duality solution for the (β, δ) system implies

$$|\Delta_\beta| \gg |\mathcal{N}_\beta|, |\mathcal{N}_\delta|$$

- 5) From statements (1) and (4) it follows that $\mathcal{N}_\alpha$ will dominate the $I_u = \frac{1}{2}$ amplitudes.

Thus, the only way to explain point b) above within our model demands either a strong deviation from the (β, δ) duality solution and/or shifts in the trajectory functions. In any case it will be difficult to reconcile points a) and b).

The mathematical analysis of the residue functions is presented in Chapter I. The dual solutions within $SU(3)$ $(3), (4), (5)$ are discussed in Chapter II.

Numerical results from a comparison of the proposed model with cross-section data for $\pi^\pm p$ elastic scattering and $\pi^- p \rightarrow \pi^0 n$ charge exchange scattering and polarization data for $\pi^\pm p \rightarrow \pi^\pm p$ at $p_{\text{lab}} \sim 6 \text{ GeV}/c$ are given in Chapter III.

I. BARYON EXCHANGE RESIDUE FUNCTIONS

We shall denote the scattering channel by s , the exchange channel by u . Thus considering $\pi^{\pm} p$ scattering the s channel helicity amplitudes for large s ($s \gg m_X^2$) are given by

$$\begin{aligned} f_{++}^{(s)\pm} &\simeq - \frac{\sqrt{-u}}{8\pi m_X} (m_X B)^{(u)\mp} \\ f_{-+}^{(s)\pm} &\simeq \frac{1}{8\pi} (A - m_X B)^{(u)\mp} \end{aligned} \quad (1.1)$$

A, B denote the usual invariant amplitudes for pseudoscalar meson-spin $\frac{1}{2}$ baryon scattering. The isospin decomposition relative to the u channel reads

$$\begin{aligned} A(B)^+ &= A(B)_{I_u=3/2} \quad , \\ A(B)^- &= \frac{1}{3} A(B)_{I_u=3/2} + \frac{2}{3} A(B)_{I_u=1/2} \end{aligned} \quad (1.2)$$

In the limit of $SU(3)$ symmetry and assuming no contributions from exotic channels it follows

$$\begin{aligned} A(B)_{I=3/2, Y=1} &= \frac{1}{2} A(B)^{(u)} [10] \\ A(B)_{I=1/2, Y=1} &= 3 A(B)^{(u)} [8] \end{aligned} \quad (1.3)$$

Differential cross-section and polarization are given by

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= |f_{++}|^2 + |f_{-+}|^2 \\ P \frac{d\sigma}{d\Omega} &= 2 \operatorname{Im} (f_{++} f_{-+}^*) \end{aligned} \quad (1.4)$$

In the following the amplitudes A-mB, mB shall be abbreviated by A', B' respectively. For baryon Regge exchange we shall use a displaced trajectory function

$$\lambda(u) = \alpha(u) - 1/2$$

The contributions of baryon exchanges in a channel with given isospin or SU(3) quantum numbers take characteristic forms according to the trajectory type.

α sequence:

$$\begin{pmatrix} A' \\ B' \end{pmatrix}_{\alpha}^{(u)} = \begin{pmatrix} -\rho_{\alpha}^R \\ \rho_{\alpha}^J \end{pmatrix} \ell(\lambda_{\alpha}) \frac{1}{2} \left[v^{\lambda_{\alpha}} + (-v)^{\lambda_{\alpha}} \right]$$

γ sequence:

$$\begin{pmatrix} A' \\ B' \end{pmatrix}_{\gamma}^{(u)} = \begin{pmatrix} \rho_{\gamma}^R \\ -\rho_{\gamma}^J \end{pmatrix} \ell(\lambda_{\gamma}) \frac{1}{2} \left[v^{\lambda_{\gamma}} - (-v)^{\lambda_{\gamma}} \right]$$

β sequence:

$$\begin{pmatrix} A' \\ B' \end{pmatrix}_{\beta}^{(u)} = \begin{pmatrix} \rho_{\beta}^R \\ \rho_{\beta}^J \end{pmatrix} \ell(\lambda_{\beta}) \frac{1}{2} \left[v^{\lambda_{\beta}} + (-v)^{\lambda_{\beta}} \right]$$

δ sequence:

$$\begin{pmatrix} A' \\ B' \end{pmatrix}_{\delta}^{(u)} = \begin{pmatrix} -\rho_{\delta}^R \\ -\rho_{\delta}^J \end{pmatrix} \ell(\lambda_{\delta}) \frac{1}{2} \left[v^{\lambda_{\delta}} - (-v)^{\lambda_{\delta}} \right]$$

$$\ell(\lambda) = 2\sqrt{\pi} \Gamma(-\lambda) ; \quad v = \frac{t-u}{2} \frac{1}{v_0}$$

v_0 : characteristic square energy

(1.5)

Let u_i , $i=0, 1, 2, \dots$ correspond to the values of u for which for a given trajectory

$$\lambda(u_i) = i$$

The following structure of the residue functions ρ^R, ρ^J assures the absence of MacDowell doublets ^{5), 7)}

$$\rho^R = \frac{1}{2} \left[\tilde{\gamma}(W_n) + \tilde{\gamma}(-W_n) \right]$$

$$\rho^J = \frac{m_N}{2W_n} \left[\tilde{\gamma}(W_n) - \tilde{\gamma}(-W_n) \right]$$

$$W_n = \sqrt{u}$$

$$\tilde{\gamma}(-\sqrt{u_i}) = 0 \quad i = 0, 1, 2, \dots \quad (1.6)$$

Thus, the ratios $r = A'/B'$ interpolate for $u > 0$ the functions $-\sqrt{u}/m_N$ for the (α, γ) sequence and $+\sqrt{u}/m_N$ for the (β, δ) sequence. In the following we specialize to linear trajectories

$$\lambda = \alpha' (u - u_0)$$

and choose units such that $\alpha' = 1$. Furthermore, we set $\gamma_0 = 1/\alpha' = 1$. This is no loss of generality since another choice of γ_0 amounts to a re-definition of ρ^R, ρ^J . A dual ansatz for A' and B' , Refs. 8), 9), would yield ρ^R, ρ^J independent of u . From (1.5) we can deduce the elastic widths of the particles, e.g., on the N_α trajectory

$$\tau_i^{(\alpha)el} = \frac{1}{2\sqrt{\pi}} \frac{E_N(W_i) - m_N}{u_i} \tilde{\gamma}_\alpha(W_i) (4g^2(u_i)) \frac{(i+1)!}{(2i+2)!}$$

$$i = 2, 4, \dots$$

(1.7)

$$W_i = \sqrt{u_i}$$

E_N : nucleon c.m. energy

Similar expressions hold for the other trajectories. For $i \rightarrow \infty$ we obtain the asymptotic behaviour of the elastic widths.

$$\tau_i^{(2)}(ee) \sim \tilde{\gamma}_\alpha(w_i) \left(\frac{e}{4}\right) \quad i \rightarrow \infty \quad (1.8)$$

(1.7) yields positivity conditions and (1.8) an asymptotic condition on $\tilde{\gamma}^2$ if it is assumed that elastic widths approach zero for $i \rightarrow \infty$:

$$\begin{aligned} \tilde{\gamma}(w_i) &\geq 0 \\ \tilde{\gamma}(w) &= o\left(\left(\frac{4}{e}\right)^{w^2}\right) \quad w \rightarrow +\infty \end{aligned} \quad (1.9)$$

If we demand that in the scattering region the expressions in (1.5) evaluated for fixed angle θ_s should decrease in the same manner as a dual four-point function we obtain further asymptotic conditions on $\tilde{\gamma}$

$$\begin{aligned} |\tilde{\gamma}(\pm i x)| &= o\left[\exp(x^2)\right] \quad x \rightarrow +\infty \\ \limsup_{x \rightarrow +\infty} |\tilde{\gamma}(\pm i x)| \exp[-\delta x^2] &\neq 0 \quad \text{for some } \delta > 0 \end{aligned} \quad (1.10)$$

We are looking for an entire function $\tilde{\gamma}(w)$ satisfying the conditions of equations (1.6), (1.8)-(1.10). The starting point for further investigation is the Weierstrass construction of an entire function having a prescribed set of zeros. Let $\sqrt{u_0}$ be the mass of the lowest lying state on a given trajectory. We consider the function

$$\begin{aligned} \gamma^1(u_0, w) &= \prod_{i=0}^{\infty} \left(1 + \frac{w}{w_i}\right) \exp\left(-\frac{w}{w_i} + \frac{1}{2} \frac{w^2}{w_i^2}\right) \\ w_i &= \sqrt{u_0 + i} \end{aligned} \quad (1.11)$$

γ^1 satisfies the relation

$$\gamma^1(u_0, w) \gamma^1(u_0, -w) = \exp(-w^2 \psi(u_0)) \frac{T(u_0)}{T(u_0 - w^2)}$$

$$\psi(z) = \frac{d}{dz} \log T(z)$$

(1.12)

The asymptotic expansion of γ^1 for large $|w|$ is given by

$$\begin{aligned} \log \gamma^1(u_0, w) &= \\ &= \frac{w^2 - (u_0 - 1/2)}{2} \log(w^2) - \frac{1 + \psi(u_0)}{2} w^2 + \\ &+ \left[2\sqrt{u_0} - \frac{1}{4} \xi\left(\frac{3}{2}, u_0\right) \right] w + \\ &+ \frac{1}{2} \left[\log T(u_0) - \frac{1}{2} \log(2\pi) \right] + \\ &+ O\left(\frac{1}{|w|}\right) \end{aligned}$$

$$|w| \rightarrow \infty \quad |\arg w| < \pi \quad (1.13)$$

$$\zeta(s, a) = \sum_{i=0}^{\infty} \frac{1}{(a+i)^s} \quad : \text{generalized zeta function}$$

γ^1 violates both the condition in (1.9) and the second condition in (1.10). It follows that $\tilde{\gamma}$ must have additional zeros. Without any option on these additional zeros the class of functions satisfying the above conditions is quite large. The properties of

$$\gamma^3 = \prod_{i=0}^{\infty} \left[1 + \left(\frac{w}{w_i} \right)^3 \right]$$

which turns out to be unacceptable too, however, lead us to consider

$$\begin{aligned} \gamma^3(u_0, w) &= \prod_{i=0}^{\infty} \left(1 + \frac{w}{w_i} \right) \left(1 + \frac{w}{w_i} + \frac{w^2}{w_i^2} \right) \exp\left(-2 \frac{w}{w_i}\right) \\ &= \gamma^1(u_0, w) \cdot \gamma^1(u_0, e^{i\varphi} w) \cdot \gamma^1(u_0, e^{-i\varphi} w) \end{aligned}$$

$$\varphi = \frac{\pi}{3} \quad (1.14)$$

From (1.14) and (1.13) one infers the asymptotic expansion of γ^3 .

$$\begin{aligned} \log \gamma^3(u_0, w) &= \\ &= - \frac{\pi}{\sqrt{3}} w^2 + \left[4\sqrt{u_0} - \frac{1}{2} \zeta(3/2, u_0) \right] w - \\ &\quad - 3/2 (u_0 - 1/2) \log w^2 \\ &\quad + 3/2 \left[\log T(u_0) - \frac{1}{2} \log(2\pi) \right] + O\left(\frac{1}{|w|}\right) \end{aligned}$$

$$|w| \rightarrow \infty \quad |\arg w| < \frac{2\pi}{3} \quad (1.15)$$

The Weierstrass construction of an entire function from its zeros satisfying specified asymptotic conditions leaves room in general for a multiplicative function without zeros complying with the asymptotic conditions.

In the present case we are left with the following ansatz for $\tilde{\gamma}$:

$$\tilde{\gamma}(w) = D(u) \cdot \exp(-C_2 w^2 + C_1 w) \exp\left(\frac{\pi}{\sqrt{3}} w^2\right) \cdot \gamma^3(u_0, w)$$

$$u = w^2$$

(1.16)

In (1.16) $D(u)$ denotes a function of u which shall be adopted to allow duality solutions for baryons. C_1, C_2 are real constants. They have the status of the Euler constant appearing in the product representation of the inverse gamma function. One easily verifies that the asymptotic conditions are satisfied if

$$D(u_i) \geq 0$$

$$-\log\left(\frac{4}{e}\right) < C_2 < 1$$

(1.17)

Interpreting the factor $\exp(C_2 u)$ as $(\nu_0)^u$, ν_0 being the unit of square energy in the expression $(s/\nu_0)^{\lambda(u)}$ (1.17) implies

$$\frac{e}{4} < \alpha' \nu_0 < e$$

(1.18)

The upper limit on C_2 is obtained considering the Regge expansion for fixed angle which may not be justified.

II. DUALITY SOLUTIONS FOR BARYONS

We work in an $SU(3)$ symmetric situation and denote the amplitudes A' or B' corresponding to given $SU(3)$ quantum numbers by $[\mu]$. Considering only pseudoscalar octet-baryon octet scattering the equations obtained from duality and absence of exotic states have the form ^{3),4)}

$$X_{\mu\mu'}^{tu} J_m [\mu']_{Regge}^{(u)} = 0, \quad t \gg m_N^2, \quad \mu = 10+10, 27$$

$$X_{\mu\mu'}^{su} J_m [\mu']_{Regge}^{(u)} = 0, \quad s \gg m_N^2, \quad \mu = 10, 27$$

(2.1)

X^{tu}, X^{su} denote the $SU(3)$ crossing matrices ¹⁰⁾.

The (N_α, N_δ) trajectories are supposed to be members of $[8]_\alpha, [8]_\delta$ combining with $[1]_\gamma$ to form a dual solution

$$\lambda_\alpha [8] = \lambda_\delta [8] = \lambda_\gamma [1] = \lambda_\alpha$$

$$(F/D)_\alpha = (F/D)_\delta = 1$$

$$D_\delta [8] = \frac{1}{3} D_\alpha [8], \quad D_\delta [1] = \frac{64}{9} D_\alpha [8]$$

$$D_\alpha [8] = D_\alpha$$

(2.2)

The corresponding functions $D(u)$ in (1.16) are constants for the (α, δ) sequence. The pronounced dip at $u \sim -0.12(\text{GeV})^2$ in π^+p backward scattering cross-sections for pion laboratory momenta ranging from 5.9-20 GeV/c seems to indicate that the contribution of N_δ is smaller than (2.2) would imply. Even the reduction factor $(s_0/s)^{\alpha(N_\alpha) - \alpha(N_\delta)}$ mentioned in the Introduction does not seem to be sufficient. Also, there is no evidence for the above solution from oscillator quark models for baryons ¹¹⁾. The most economic solution comprising $\Delta 8$ would combine exchange degenerate $[10]_8, [8]_8$ and $[8]_8$ trajectories. Since a nucleon isobar with $J^P = \frac{1}{2}^-$, $m \sim 750$ MeV and a $J^P = \frac{3}{2}^+$ octet degenerate with the decuplet are not observed, it has been proposed by Rosner ⁵⁾ to consider the system $[10]_6, [8]_6 \leftrightarrow [10]_3, [8]_3$ and to construct a solution with lowest lying states

$$J^P = 5/2^-, 7/2^+, 9/2^- \quad \text{for } [8]_3, [8]_6, [10]_3$$

respectively. Candidates for these resonances are

$$N^*(\frac{5}{2}^-, 1670) \text{ and } N^*(\frac{7}{2}^+, 1990)$$

Equation (2.1) implies

$$\lambda_s [8] = \lambda_\beta [8] = \lambda_s [10] = \lambda_\beta [10] = \lambda_s$$

$$(\mathcal{F}/\mathcal{D})_\beta = (\mathcal{F}/\mathcal{D})_s = -\frac{1}{3}$$

$$\frac{1}{4} \mathcal{D}_\beta [10] = 9 \mathcal{D}_s [8] - 3 \mathcal{D}_\beta [8]$$

$$\frac{1}{4} \mathcal{D}_s [10] = -3 \mathcal{D}_s [8] + 9 \mathcal{D}_\beta [8]$$

$$\lambda_s = 0 : \quad \mathcal{D}_{\beta, s} [8, 10] = 0$$

$$\lambda_s = 1 : \quad \mathcal{D}_s [8] = 0$$

$$\lambda_s = 2 \quad \mathcal{D}_\beta [10] = 0 \quad (2.3)$$

There is a large class of solutions to (2.3). If we demand $\mathcal{D}_{\beta, s} [8, 10]$ to be polynomials in λ_s of minimal, i.e., second degree the residue functions depend, apart from a common multiplicative constant, on one parameter

$$\mathcal{D}_s [10] = \mathcal{D}_s' \cdot \lambda_s \cdot (\rho \lambda_s + 1 - \rho)$$

$$\mathcal{D}_\beta [10] = \mathcal{D}_s' \cdot \lambda_s \cdot (\lambda_s - 2) \cdot \frac{1}{3}$$

$$\mathcal{D}_s [8] = \mathcal{D}_s' \cdot \lambda_s \cdot (\lambda_s - 1) \cdot \frac{1 + \rho}{96}$$

$$\mathcal{D}_\beta [8] = \mathcal{D}_s' \cdot \lambda_s \left[(\rho + \frac{1}{9}) \lambda_s + \frac{7}{9} - \rho \right] \frac{1}{32} \quad (2.4)$$

$$\rho \geq 0$$

The constructive interference between the $I_u = \frac{1}{2}$ and $I_u = \frac{3}{2}$ helicity flip amplitudes near $u=0$ will force us to choose D_δ' such that it has different signs in the resonance region ($u > (m_N + m_\pi)^2$) and in the scattering region ($s \rightarrow \infty$) respectively.

In the calculation, we approximated the analytic function D_δ' by a multiple of the step function

$$D_\delta' \sim D_\delta \cdot \varepsilon(\lambda_\delta - \lambda_0)$$

$$0 < \lambda_0 < 1$$

$$\varepsilon(x) = \text{sign}(x)$$

(2.5)

III. COMPARISON WITH DATA AT 6 GeV/c

Gathering the results from Chapters I and II and taking into account the fact that $SU(3)$ duality is not exact, the contributions of $N_\alpha, N_\gamma, N_\beta, N_\delta, \Delta_\delta, \Delta_\beta$ depend on the parameters

$$\alpha', D_\alpha, C_1^\alpha, C_2^\alpha, R_\gamma, C_1^\gamma, D_\delta, \rho, C_1^\delta, C_2^\delta, R_\beta^\Delta, \\ C_1^\beta, \varepsilon, R_\beta^N, R_\delta^N$$

in the following way

$$N_\alpha: \quad \lambda_\alpha = \alpha' (u - m_N^2) \\ \tilde{\gamma}_\alpha[8] = D_\alpha \cdot \exp(-C_2^\alpha u + C_1^\alpha w) \cdot \gamma^3(\alpha' m_N^2, w)$$

$$N_\gamma: \quad \lambda_\gamma = \alpha' (u - m_\gamma^2) + 1 \\ \tilde{\gamma}_\gamma[8] = \frac{1}{3} R_\gamma \cdot D_\alpha \cdot \exp(-C_2^\alpha u + C_1^\gamma w) \times \\ \times \gamma^3(\alpha' m_\gamma^2 - 1, w)$$

$$\begin{aligned}
 \Delta_\delta : \quad & \lambda_\delta = \alpha' (u - m_\Delta^2) + 1 \\
 & \tilde{\gamma}_\delta [10] = \mathcal{D}_\delta [10] (\lambda_\delta, \rho) \cdot \exp(-c_2^\delta u + c_1^\delta w) \times \\
 & \quad \times \gamma^3(\alpha' m_\Delta^2, w) \\
 \Delta_\beta : \quad & \lambda_\beta [10] = \lambda_\delta - \varepsilon \\
 & \tilde{\gamma}_\beta [10] = R_\beta^\Delta \cdot \mathcal{D}_\beta [10] (\lambda_\beta) \cdot \exp(-c_2^\delta u + c_1^\beta w) \times \\
 & \quad \times \gamma^3(\alpha' m_\Delta^2, w) \\
 \mathcal{N}_\beta : \quad & \lambda_\beta [8] = \lambda_\delta \\
 & \tilde{\gamma}_\beta [8] = R_\beta^\mathcal{N} \cdot \mathcal{D}_\beta [8] (\lambda_\delta, \rho) \cdot \exp(-c_2^\delta u + c_1^\delta w) \cdot \gamma^3(\alpha' m_\Delta^2, w) \\
 \mathcal{N}_\delta : \quad & \lambda_\delta [8] = \lambda_\delta \\
 & \tilde{\gamma}_\delta [8] = R_\delta^\mathcal{N} \cdot \mathcal{D}_\delta [8] (\lambda_\delta, \rho) \cdot \exp(-c_2^\delta u + c_1^\delta w) \times \\
 & \quad \times \gamma^3(\alpha' m_\Delta^2, w)
 \end{aligned}$$

(3.1)

R_δ , $R_\beta^\mathcal{N}$, $R_\delta^\mathcal{N}$, R_β^Δ are non-negative constants modifying the strength of $[8]_\gamma / [8]_\alpha$, $[8]_\beta$, $[10]_\delta$ and $[10]_\beta / [10]_\delta$ relative to the dual solutions for which $R_\delta = R_\beta^\mathcal{N} = R_\delta^\mathcal{N} = R_\beta^\Delta = 1$ and, furthermore, $\varepsilon = 0$, $c_1^\alpha = c_1^\gamma$, $c_1^\delta = c_1^\beta$. Parametrizing A' , B' by a combination of Euler B functions would imply

$$c_2^\alpha = c_2^\delta = 0$$

D_α , R_γ , D_δ can be calculated from the remaining parameters and the $\pi \mathcal{N}$ coupling constant, the widths of $\mathcal{N}(\frac{3}{2}^-, 1520)$ and $\Delta(\frac{3}{2}^+, 1236)$ respectively. We remark that in a situation where a single Regge exchange dominates, the ratio of spin non-flip to spin-flip helicity amplitudes have the following asymptotic behaviour for small and large momentum transfers in our model

$$f_{++}^{(s)} / f_{-+}^{(s)} \sim \begin{cases} \sigma C_1 \sqrt{-u\alpha'}, (-u) \rightarrow 0 & s \gg u_N^2 \\ \sigma t g \left[(\zeta_1 + \bar{\zeta}_1) \sqrt{-u\alpha'} - d \frac{\pi}{2} \right], & s \gg -u \gg u_0 \end{cases}$$

$$\bar{C}_1 = 4 \sqrt{u_0 \alpha'} - \frac{1}{2} \xi \left(\frac{3}{2}, u_0 \alpha' \right)$$

$$d = 3 \left(u_0 \alpha' - \frac{1}{2} \right)$$

$$\sigma = \begin{cases} 1 & \text{for the } (\alpha, \delta) \text{ sequence} \\ -1 & \text{for the } (\beta, \delta) \text{ sequence} \end{cases}$$

(3.2)

The asymptotic expansions in (3.2) are reminiscent of the geometric impact parameter representation of f_{++}, f_{-+}

$$J_m f_{++}^{(s)} / J_m f_{-+}^{(s)} \sim \frac{J_1(b\sqrt{-u})}{J_0(b\sqrt{-u})}$$

(3.3)

Before numerical calculations can start the question of the interference between $I_u = \frac{1}{2}$ and $I_u = \frac{3}{2}$ amplitudes demands attention. We follow the discussion of Barger and Olsson ²⁾ and define

$$f^{\pm} = \left(f_{-+}^{\pi^{\pm}p}, f_{++}^{\pi^{\pm}p} \right), \quad \left(\frac{d\sigma}{d\Omega} \right)^{\pi^{\pm}p} = \sigma^{\pm}$$

$$f^0 = \left(f_{-+}^{CEX}, f_{++}^{CEX} \right), \quad \left(\frac{d\sigma}{d\Omega} \right)^{CEX} = \sigma^0$$

$$N = \left(f_{-+}^{I_n=1/2}, f_{++}^{I_n=1/2} \right)$$

$$\Delta = \left(f_{-+}^{I_n=3/2}, f_{++}^{I_n=3/2} \right)$$

(3.4)

CEX denotes the charge exchange reaction $\pi^-p \rightarrow \pi^0n$. It follows

$$f^+ = \frac{2}{3} N + \frac{1}{3} \Delta, \quad f^- = \Delta, \quad f^0 = \frac{1}{\sqrt{2}} (f^+ - f^-) = \frac{\sqrt{2}}{3} (N - \Delta)$$

$$\sigma^+ = \frac{4}{9} |N|^2 + \frac{4}{9} D + \frac{1}{9} |\Delta|^2$$

$$\sigma^- = |\Delta|^2$$

$$\sigma^0 = \frac{2}{9} |N|^2 - \frac{4}{9} D + \frac{2}{9} |\Delta|^2$$

$$D = \operatorname{Re}(N^* \Delta)$$

(3.5)

The interference term D is positive in the very backward direction and shows a quadratic zero in the vicinity of the dip at $u = u_d \approx -0.15(\text{GeV})^2$ observed in σ^+ . Neglecting as a first approximation the helicity non-flip amplitudes in the region $-0.2 \leq u \leq u_b$ [$u_b = (m_N^2 - m_\pi^2)^2/s$] it follows

$$(Re N_{-+})(Re \Delta_{-+}) + (Im N_{-+})(Im \Delta_{-+}) \sim (u - u_d)^2 \text{ for } u \simeq u_d \quad (3.6)$$

If we assume that N_{-+} is dominated in the above region by N_α exchange

$$N_{-+} \sim -\rho_\alpha^R e^{-i\frac{\pi}{2}\lambda_\alpha} \cos\left(\frac{\pi}{2}\lambda_\alpha\right) \quad (3.7)$$

Equation (3.6) implies

$$Im \Delta_{-+} \simeq 0 \text{ for } u = u_d \quad (3.8)$$

If, furthermore, Δ_δ and Δ_β exchanges dominate Δ_{-+}

$$\Delta_{-+} \sim e^{-i\frac{\pi}{2}\lambda_\delta} \left[\rho_\beta^R \cos\left(\frac{\pi}{2}\lambda_\delta\right) + i \rho_\delta^R \sin\left(\frac{\pi}{2}\lambda_\delta\right) \right] \quad (3.9)$$

(3.9) implies that $\Delta_\delta, \Delta_\beta$ are nearly exchange degenerate in the region of the dip:

$$\left. \begin{aligned} \rho_\beta^R &\simeq \rho_\delta^R, \quad \Delta_{-+} \sim \rho_\delta^R \\ D &\sim -\rho_\delta^R \rho_\alpha^R \cos^2\left(\frac{\pi}{2}\lambda_\alpha\right) \end{aligned} \right\} \text{ for } u \simeq u_d \quad (3.10)$$

In order to give D the correct sign near $u = u_d$ and remembering that $\rho_\alpha^R > 0$ near the backward direction in our model we obtain

$$\rho_\beta^R \simeq \rho_\delta^R < 0 \text{ for } u = u_d \quad (3.11)$$

Comparing (3.11) with (2.14) it follows

$$D_s' \lambda_s (\rho \lambda_s + 1 - \rho) \simeq R_\beta^4 D_s' \lambda_s (\lambda_s - 2) \cdot \frac{1}{3}$$

for $u = u_\alpha$ or $\lambda_s \simeq -0.65$

(3.12)

From Eq. (3.12) the sign of D_s' discussed in Chapter II is inferred

$$D_s' < 0 \quad \text{for } u = u_\alpha$$

$$\rho \simeq \frac{R_\beta^4 (2 - \lambda_s) + 3}{3 (1 - \lambda_s)} = \frac{3 + 2.65 R_\beta^4}{4.95} \quad \text{for } \lambda_s = -0.65$$

(3.13)

Equation (3.13) rules out the most economic set of zeros in the duality solution proposed by Rosner⁵⁾ corresponding to $\rho = 0$. The 12 parameters defining the amplitudes according to (3.1) were varied to fit the experimental cross-sections of backward $\pi^\pm_p$ elastic scattering at 5.91 GeV/c from Owen et al.¹²⁾, the charge exchange reaction $\pi^-p \rightarrow \pi^0n$ from Boright et al.¹³⁾ and the polarization measurements for the elastic $\pi^\pm_p$ reactions at 6 GeV/c from Aoi et al.¹⁴⁾. The parameters fall into three groups

- i) α', ε
- ii) $c_1^\alpha, c_2^\alpha, c_1^\delta, c_2^\delta, \rho, c_1^\beta = c_2^\delta, c_1^\beta, R_\beta^4$
- iii) $R_\beta^N, R_\delta^N, c_1^r, c_2^r = c_2^\alpha$

The main computation time goes into calculating the functions

$$y^3(u_\alpha, w\sqrt{\alpha'})$$

which have to be redetermined every time α' or ε change.

The product representation (1.14) for γ^3 converges very slowly. The following procedure was used to render convergence faster by at least one order of magnitude. Let

$$\gamma^3 = \prod_{i=0}^{\infty} \frac{1+x_i}{1-x_i} (1-x_i^3) \exp(-2x_i) \quad , \quad x_0 = \frac{W}{W_2}$$

Using the fact that $\sum x_i^3$ converges one obtains an equivalent representation of γ^3

$$\begin{aligned} \gamma^3 &= \exp \left[-\frac{1}{3} \cdot \xi\left(\frac{3}{2}, \alpha' u_0\right) \cdot (W \sqrt{\alpha'})^3 \right] \times \\ &\times \prod_{i=0}^{\infty} \frac{1+x_i}{1-x_i} (1-x_i^3) \exp \left[-2x_i + \frac{1}{3} x_i^3 \right] \\ &= \exp \left[-\frac{1}{3} \xi\left(\frac{3}{2}, \alpha' u_0\right) \cdot (W \sqrt{\alpha'})^3 \right] \cdot \prod_{i=0}^{\infty} (1 + P(x_i)) \end{aligned} \quad (3.14)$$

The extra factors $\exp(\frac{1}{3} x_i^3)$ assure that the remainder term

$$R_N = \prod_{i=N}^{\infty} (1 + P(x_i))$$

decreases like $N^{-\frac{3}{2}}$ as compared with $N^{-\frac{1}{2}}$ without the above correction.

Nevertheless, a full calculation of the amplitudes in (3.1) demands about $10^5 - 10^6$ algebraic operations for a set of 32 momentum transfers. This number is reduced by one order of magnitude if α' , ε remain fixed and the functions γ^3 are calculated only once in the minimization process. It turned out that no substantial improvement on the fit was obtained choosing other values for α' , ε than $1(\text{GeV})^{-2}$ and 0 respectively. Polarizations and cross-sections are mainly determined by the seven parameters in the second group. Thus, the amplitudes are dominated in our model by three Regge poles $\mathcal{N}_\alpha$, Δ_δ and Δ_β .

$\mathcal{N}_\gamma$, $\mathcal{N}_\beta$, $\mathcal{N}_\delta$ constitute small corrections and can very well be neglected, the three parameters in group three are, therefore, redundant. Changing the $\mathcal{N}_\beta$ trajectory function to coincide with λ_α as indicated by the masses of the $(\frac{5}{2}^+, 1688)(\frac{5}{2}^-, 1670)$ resonances does not alter this situation. The best fit was found for the following values of the parameters

$$\alpha' = 1 (\text{GeV})^{-2}, \quad \varepsilon = 0$$

$$C_1^\alpha = 1.22, \quad C_2^\alpha = 0.99, \quad C_1^\delta = 0.16, \quad C_2^\delta = 0.3$$

$$\rho = 0.71, \quad C_1^\beta = -0.43, \quad R_\beta^\delta = 0.27$$

$$C_1^\gamma = 2.91 \quad (\Rightarrow R_\gamma = 0.08), \quad R_X^\beta = 0.37, \quad R_X^\delta = 0.29 \quad (3.15)$$

The above value for ρ implies a vanishing contribution from $\Delta\delta$ at $u = 0.1(\text{GeV})^2$ very near the backward direction $u \approx 0.66(\text{GeV})^2$. The resulting cross-sections and polarizations together with the experimental ones are shown in Figs. 1 and 2. The π^+p cross-section (Fig. 1a) and polarization (Fig. 3a) indicate that there is a contribution to the $I_u = \frac{1}{2}$ amplitudes which is missing in our model where the polarization in the $I_u = \frac{1}{2}$ amplitude P_N is essentially zero. Isospin inequalities ²⁾ yield the relation

$$P_N \simeq P(\pi^+p) \quad \text{for } (-u) \gtrsim 0.48 (\text{GeV})^2$$

Argand diagrams of the amplitudes $(f_{\pm+})^{\pi^\pm p}$ are shown in Figs. 4-6. We note that near $u = u_d$ $\text{Im}(f_{-+})^{\pi^-p}$ (Fig. 6) is small as required in Eq. (3.8). $(f_{-+})^{\text{CEX}}$ indeed would move the position of the dip in the charge exchange cross-section to the position indicated by experiment. However, $(f_{++})^{\text{CEX}}$ gives a non-negligible contribution for $u \sim u_d$ which moves the dip position back to $u \approx u_d$.

In the fit to $\pi^\pm p$, $(\pi p)^{\text{CEX}}$ cross-sections by Halzen et al. ¹⁵⁾ using kinematic cuts ¹⁾ which affect mainly the helicity non-flip amplitude, the position of the dip in the charge exchange cross-section is well reproduced. We take this as an indication that our model is more reliable for the helicity flip than for the helicity non-flip amplitudes.

The loop in $(f_{-+})^{\pi^+p}$ (Fig. 4) is the clearest manifestation of our special choice of residue functions (γ^3).

The predictions of the proposed model for $p_{\text{lab}} = 13.8 \text{ GeV}/c$ agree fairly well with experimental data for $\pi^\pm p$ at $13.73 \text{ GeV}/c$ from Owen et al. ¹²⁾, for $(\pi p)^{\text{CEX}}$ from Boright et al. ¹³⁾. The situation relative to the dip at $u \approx -0.24(\text{GeV})^2$ remains the same as for $6 \text{ GeV}/c$. The polarization for both $\pi^\pm p$ at the higher energy does not deviate much from the results at $6 \text{ GeV}/c$.

CONCLUSIONS

Implementing the statements 1)-5) in the Introduction, we conclude.

- 1) We find a large deviation of R_γ from the value implied by the duality solution $[8]_\alpha \approx [8]_\gamma, [1]_\gamma$. $R_\gamma = 0.08$ as compared with

$$\left(s_0/s\right)^{\alpha(N_\alpha) - \alpha(N_\gamma)} \approx \frac{1}{3.5}$$

- 2) The $(\beta\delta)$ duality solution is although distorted not completely lost, remembering that we consider a very special realization of this solution

$$R_\beta^\Delta = 0.27 \longleftrightarrow 1$$

$$R_\beta^\gamma = 0.37 \longleftrightarrow \left(s_0/s\right)^{\alpha(\Delta_S) - \alpha(N_\beta)} \sim \frac{1}{3}$$

$$\text{for } p_{\text{lab}} = 6 \text{ GeV}/c \text{ and } s_0 = 1 (\text{GeV})^2$$

$$R_\delta^\gamma = 0.29 \longleftrightarrow 1$$

In the region $|u| \geq 0.4(\text{GeV})^2$ we find the hierarchy of contributions

$$|N_\alpha| > |\Delta_S| > |\Delta_\beta| \gg |N_\beta|, |N_\delta|, |N_\gamma|$$

- 3) The structure of D is reproduced fairly well since we find

$$J_m f_{-+}^{I_n=3/2} \approx 0 \quad \text{for } u \approx u_\alpha$$

The helicity non-flip amplitude then dominates near $u \approx u_d$ and pushes the dip in the charge exchange cross-section back to $u = u_d$.

- 4) The hierarchy of contributions described in 2) does not produce the required large negative polarization in the $I_u = \frac{1}{2}$ amplitudes for $|u| > 0.4(\text{GeV})^2$. Shifting the trajectory function of $\mathcal{N}_3$ to be exchange degenerate with $\mathcal{N}_2$ does not alter this situation. Within a pure pole model a refined manipulation of trajectories and residue functions within the $I_u = \frac{1}{2}$ exchanges as opposed to the logic of the present work would be needed to reproduce the polarization correctly. We rather take the standpoint that the pure pole model gives a good description of the helicity flip amplitudes whereas significant contributions to the helicity non-flip amplitudes are missing. We do not speculate whether these contributions are of non-asymptotic nature.
- 5) The π^-p polarization can be well reproduced. Thus, we feel that some evidence for the need of a Δ_3 trajectory, in accordance with quark oscillator models is definitely obtained.
- 6) The consequences of the residue functions determined from absence of MacDowell doublets can only be marginally tested for $P_{\text{lab}} = 6 \text{ GeV}/c$. The prominent feature due to the proposed mechanism is the loop in the Argand diagram of $f_{-+}^{\pi^+p}$ (Fig.4) setting in at $u \simeq -0.25(\text{GeV})^2$.

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FIGURE CAPTIONS

- Figure 1 : Differential cross-sections for $\pi^\pm p$ scattering ($p_{\text{lab}} = 6 \text{ GeV/c}$) versus data from Owen et al. ¹²⁾.
- Figure 2 : Differential cross-section for $\pi^- p \rightarrow \pi^0 n$ scattering ($p_{\text{lab}} = 6 \text{ GeV/c}$) versus data from Boright et al. ¹³⁾.
- Figure 3 : Polarization for $\pi^\pm p$ scattering ($p_{\text{lab}} = 6 \text{ GeV/c}$) versus data from Aoi et al. ¹⁴⁾.
- Figure 4 : Argand diagram of $f_{-+}^{\pi^+ p}$ ($p_{\text{lab}} = 6 \text{ GeV/c}$).
- Figure 5 : Argand diagram of $f_{++}^{\pi^+ p}$ ($p_{\text{lab}} = 6 \text{ GeV/c}$).
- Figure 6 : Argand diagram of $f_{-+}^{\pi^- p}$ ($p_{\text{lab}} = 6 \text{ GeV/c}$).

FIG. 1a

FIG. 1b

FIG. 2

FIG. 3a

FIG.3b

FIG. 4

FIG. 5

FIG.