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STATUS OF THE CABIBBO - RADICATI SUM RULE

AND A NEW TEST OF VECTOR MESON DOMINANCE

G.J. Gounaris

CERN - Geneva

and

Nuclear Research Centre "Democritus"

Athens

A B S T R A C T

Using finite energy sum rules the high energy contribution to the Cabibbo-Radicati sum rule is calculated and shown to be of the proper magnitude and sign, so that this sum rule is exactly satisfied. In addition the coupling of the ρ Regge trajectory to two charged photons is calculated and found to be in agreement with the predictions of vector meson dominance and universality for the Reggeized ρ vertex, within the expected accuracy.

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It is well known that the Cabibbo-Radicati ¹⁾ (CR) sum rule states that ²⁾

$$2 \left. \frac{dF_1^V(q^2)}{dq^2} \right|_{q^2=0} + \left(\frac{\mu_V}{2M_N} \right)^2 + \frac{1}{2\pi^2\alpha} \int_{m_\pi^2 + \frac{m_\pi^2}{2M_N}}^{\infty} d\omega \frac{\sigma(\omega)}{\omega} = 0, \quad (1)$$

where

$$\sigma(\omega) \equiv \sigma_T(\gamma^- p) - \sigma_T(\gamma^+ p) = 2 \sigma_T(\gamma^V p \rightarrow I=1/2) - \sigma_T(\gamma^V p \rightarrow I=3/2), \quad (2)$$

and ω is the laboratory energy of the photon. When confronted against the experiment this sum rule seems to be rather well satisfied ^{3),4)}. Stated in more detail the first two terms are respectively $-1720 \mu\text{b}$ and $1520 \mu\text{b}$ while the integral gives about $-50 \mu\text{b}$ if a cut-off at $\omega = 1.1 \text{ GeV}$ is used. We note that this later result comes about after large cancellations between large positive and negative contributions. We conclude therefore that a reasonably small high energy contribution of about $250 \mu\text{b}$ will explain the remaining discrepancy between the left and right-hand side in Eq. (1).

The purpose of the present work is twofold. First we would like to use finite energy sum rules (FESR) to calculate the high energy contribution to CR and see whether it has the sign and magnitude expected to explain the remaining discrepancy mentioned above. Second we would like to calculate the coupling of a Reggeized ρ to two "charged" photons. The reason is the following. We know ^{5),6)} that the couplings of the Pomeron singularity and the A_2 trajectory to a photon and a vector meson obey the vector meson dominance (VMD) predictions to within 22%. We want therefore to see whether the coupling of the Reggeized ρ to two "charged" photons obeys also the VMD predictions to the same accuracy. Below we first give our formalism and then state and discuss our results.

Let us call $e_F^2(\gamma^- p; \omega)$ and $e_F^2(\gamma^+ p; \omega)$ the forward spin non-flip invariant amplitudes for the processes $\gamma^- p \rightarrow \gamma^- p$ and $\gamma^+ p \rightarrow \gamma^+ p$, at a photon laboratory energy ω .

We define

$$F^{(-)}(\omega) = \frac{1}{2} \left\{ F(\gamma^- p; \omega) - F(\gamma^+ p; \omega) \right\}, \quad (3)$$

and demand that $F^{(-)}(\omega)$ is an analytic function in the ω plane with branch points at

$$\pm \omega_0 = \pm \left(m_\pi + \frac{m_\pi^2}{2M_N} \right) .$$

Moreover, we demand that it satisfies the crossing relation

$$F^{(-)}(\omega) = - F^{(-)}(-\omega) ,$$

and the optical theorem

$$\sigma(\omega) = \frac{4\pi\alpha}{\omega M_N} \text{Im} F^{(-)}(\omega) , \quad (4)$$

where $\sigma(\omega)$ is defined by Eq. (2). We expect that for high energies we can always parametrize this amplitude as

$$F^{(-)}(\omega) \simeq F_r^{(-)}(\omega) = \frac{\beta\pi}{2\sin(\pi\alpha_p(0))} \left\{ (\omega)^{\alpha_p(0)} - (-\omega)^{\alpha_p(0)} \right\} , \quad (5)$$

where β is a constant and the crossing symmetry of the amplitude has been taken into account. Here $\alpha_p(0)$ is the intercept of an effective trajectory which describes the contribution of the ρ trajectory as well as the contributions of associated cuts and lower lying trajectories. Inspired by the data on charge exchange πN scattering ⁷⁾, we will assume $\alpha_p(0) \simeq 0.5$. Using Eqs. (3)-(5) and the lowest moment FERS

$$\int_{\omega_0}^{\Omega} \text{Im} (F^{(-)}(\omega) - F_r^{(-)}(\omega)) d\omega = 0 , \quad (6)$$

we find

$$\frac{\beta}{M_N} = \frac{\alpha_p(0)+1}{2\pi^2\alpha} \frac{\int_{\omega_0}^{\Omega} \omega \sigma(\omega) d\omega}{(\Omega)^{\alpha_p(0)+1}} , \quad (7)$$

where Ω is the cut-off in the FESR. We now express the high energy contribution to the integral appearing in CR in terms of β/M_N . To do this we assume that there exists an energy Ω' such that for $\omega > \Omega'$ we are allowed to use in the afore-mentioned high energy contribution the Regge expression for $\sigma(\omega)$. From (4) and (5) we find this Regge expression to be

$$\sigma_r(\omega) = 2\pi^2\alpha \frac{\beta}{M_N} \omega^{\alpha_f(\omega)-1} \quad (8)$$

and then

$$\begin{aligned} \frac{1}{2\pi^2\alpha} \int_{\omega_0}^{\infty} \frac{\sigma(\omega)}{\omega} d\omega &= \frac{1}{2\pi^2\alpha} \int_{\omega_0}^{\Omega'} \frac{\sigma(\omega)}{\omega} d\omega + \frac{1}{2\pi^2\alpha} \int_{\Omega'}^{\infty} \frac{\sigma_r(\omega)}{\omega} d\omega = \\ &= \frac{1}{2\pi^2\alpha} \int_{\omega_0}^{\Omega'} \frac{\sigma(\omega)}{\omega} d\omega - \frac{\beta}{M_N} \frac{(\Omega')^{\alpha_f(\omega)-1}}{\alpha_f(\omega)-1} \end{aligned} \quad (9)$$

To proceed further we need to know $\sigma(\omega)$. In order to obtain it we use the following procedure. We split $\sigma(\omega)$ into two parts

$$\begin{aligned} \sigma(\omega) &= \left\{ 2\sigma_T(\gamma^V p \rightarrow (\pi N)_{I=1/2}) - \sigma_T(\gamma^V p \rightarrow (\pi N)_{I=3/2}) \right\} + \sigma'(\omega) = \\ &= \sigma_{\pi N}(\omega) + \sigma'(\omega) \end{aligned} \quad (10)$$

where $\sigma_{\pi N}(\omega)$ describes the contribution of the πN channel while $\sigma'(\omega)$ comes from all the other channels. For $\sigma_{\pi N}(\omega)$ we note that Walker's ⁸⁾ phenomenological analysis of the pion photoproduction in terms of the Born term, a few resonances and a background, gives a reasonable description of the data up to energies ^{8),9)} of 1.1 GeV. We assume that by including into his amplitudes ¹⁰⁾ $P_{11}(1750)$ and $F_{37}(1940)$ we get a good description for $\sigma_{\pi N}(\omega)$ for $\omega \leq 1.73$ GeV. We remark here that Walker's background is presumably coming mainly from weak s channel resonances which he ignores, as well as from the tails of the u channel resonances. At least this is what is suggested by applying the two component duality hypothesis to the process $\gamma N \rightarrow \pi N$. We are then led to the assumption that the true background in the process $\gamma N \rightarrow \pi N$ is coming mainly from the pion pole in the Born term ¹¹⁾. Now we turn to $\sigma'(\omega)$ for which we know how to calculate the contributions of the various resonances but we do not know how to estimate the background. Ignoring therefore hereafter the background for $\sigma'(\omega)$, we have that the resonance contribution is

$$\sigma'(\omega) = \sum_{R_1} 2\sigma_T(\gamma^V p \rightarrow R_1 \rightarrow \pi N) \left(\frac{\Gamma_t(R_1)}{\Gamma(R_1 \rightarrow \pi N)} - 1 \right) - \sum_{R_3} \sigma_T(\gamma^V p \rightarrow R_3 \rightarrow \pi N) \left(\frac{\Gamma_t(R_3)}{\Gamma(R_3 \rightarrow \pi N)} - 1 \right) , \quad (11)$$

where the sums over R_1 and R_3 run over all contributing resonances of isospin $\frac{1}{2}$ and $\frac{3}{2}$ respectively ¹²⁾. Here $\Gamma_t(R)$ and $\Gamma(R \rightarrow \pi N)$ stand for the total and the partial widths to the πN channel, of resonance R . Adding $\sigma_{\pi N}(\omega)$ and $\sigma'(\omega)$ we have our result for $\sigma(\omega)$. This result is presumably valid for $\omega \leq 1.73$ GeV, and it is shown by the continuous line in Fig. 1.

Assuming now that FESR is satisfied when ¹³⁾ when $\Omega = 1.73$ [i.e., right after the $F_{37}(1940)$ resonance] we find from (7)

$$\frac{\beta}{M_N} = + 0.423 (\text{GeV})^{-1-\alpha_p(0)} \quad (12)$$

Substituting (12) in (8), we obtain the Regge expression $\sigma_r(\omega)$ for $\sigma(\omega)$ which is the broken line in Fig. 1. As it is shown there, $\sigma_r(\omega)$ "averages" $\sigma(\omega)$, for low ω , as expected from the duality hypothesis ¹⁴⁾. Using now in (9) $\Omega' = \Omega$ we calculate the integral appearing in CR. The result is presented in the first column of the Table, where the contributions of the other terms are also given. As we see from this Table, the high energy contribution to CR is exactly what is needed to make this sum rule satisfied. We emphasize that our FESR method determines both the sign and magnitude of the high energy contribution to CR. We also remark that, since the integral in (1) emphasizes the low energy region, it should not depend critically on the detailed form of $\sigma(\omega)$ for high ω . This makes us expect that CR should still be satisfied even if Ω' is considerably smaller than Ω . The Table shows that this is indeed the case. This means that so far as the Cabibbo-Radicati sum rule is concerned $\sigma_r(\omega)$ "averages" $\sigma(\omega)$ even for laboratory energies of the photon as low as 0.85 GeV.

We now come to the second aim of the present work, namely to provide still another check of VMD. As we see from Fig. 2 and Eq. (12), VMD tells us that

$$\left(\frac{\beta}{M_N}\right)_{\gamma^* p \rightarrow \gamma^* p} = \frac{1}{f_p^2} \left(\frac{\beta}{M_N}\right)_{\rho^* p \rightarrow \rho^* p}$$

Assuming that at $t = 0$ the main contribution comes from the ρ trajectory ¹⁵⁾, and assuming Reggeized ρ universality we have that

$$\frac{\beta}{M_N} = \frac{1}{f_p^2} \left(\frac{\beta}{M_N}\right)_{\pi N \rightarrow \pi N} = - \frac{1}{f_p^2} \frac{\pi}{4 \cos(\pi \alpha_\rho(0)/2)} \gamma_\rho(t=0)$$

where $\gamma_\rho(t=0)$ is the coupling of ρ trajectory in πN scattering defined by Barger and Phillips ¹⁶⁾. From Eq. (12) and their result for $\gamma_\rho(t=0)$ we find ¹⁷⁾

$$f_p^2/4\pi = 1.8. \quad (13)$$

The new Orsay data suggest ¹⁸⁾

$$f_p^2/4\pi = 2.54 \pm 0.23 ,$$

while the old Orsay data ¹⁹⁾ were giving ¹⁸⁾

$$f_p^2/4\pi = 2.10 \pm 0.11 .$$

We see therefore that, at worse, we have a 30% violation of the VMD plus universality assumption for the coupling of a Reggeized ρ to two charged photons. Actually, part of this violation may well be due to our assumption that a particular Regge trajectory dominates. Nevertheless we remark that a similar phenomenon is also observed for the couplings of the Pomeron and A_2 singularities to a photon and a vector meson. Indeed the sum rule ⁶⁾

$$\sum_{\rho, \omega, \eta} \frac{1}{g_p} \sqrt{16\pi (e/f_v)^2 \frac{d\sigma^0(\gamma p \rightarrow v p)}{dt} / (1 + \eta_v^2)} = 1 \quad (14)$$

is violated by 22% when the new Orsay data are used ⁵⁾. Above $d\sigma^0/dt(\gamma p \rightarrow Vp)$ refers to the vector meson photoproduction cross-section when extrapolated to $t = 0$ and η_V stands for the ratio of the real to the imaginary part of the forward vector meson production amplitude.

In conclusion we summarize our results. We have shown in the present note that :

- 1) FESR for the forward amplitude of charged photon scattering indicates that the high energy contribution to the Cabibbo-Radicati sum rule is of correct sign and magnitude to make the sum rule exactly satisfied ;
- 2) so far as this sum rule is concerned, the high energy expression for $\sigma(\omega)$ averages the low energy data up to an energy as low as 0.85 GeV ;
- 3) comparing our FESR result for charge photon Compton scattering with the one for πN scattering, we conclude that the VMD assumption for the coupling of a Reggeized ρ to two isovector photons is violated by almost 30% and this is in agreement with what we would have expected in analogy to the situation in photoproduction of vector mesons ^{5),6)}.

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TABLE

Contributions to the Cabibbo-Radicati sum rule ^{*)}

$$2 \frac{dF_1^V(q^2)}{dq^2} \Big|_{q^2=0} + \left(\frac{\mu_V}{2M_N} \right)^2 + \frac{1}{2\pi^2\alpha} \int_{\omega_0}^{\Omega'} \frac{d\omega}{\omega} \left[2\sigma_T(\gamma^V p \rightarrow I=1/2) - \sigma_T(\gamma^V p \rightarrow I=3/2) \right] +$$

$$+ \frac{1}{2\pi^2\alpha} \int_{\Omega'}^{\infty} \frac{d\omega}{\omega} \left[2\sigma_T(\gamma^V p \rightarrow I=1/2) - \sigma_T(\gamma^V p \rightarrow I=3/2) \right] = 0$$

Ω' (GeV)	$2 \frac{dF_1^V(q^2)}{dq^2} \Big _{q^2=0}$ (μ_b)	$\left(\frac{\mu_V}{2M} \right)^2$ (μ_b)	$\int_{\omega_0}^{\Omega'} \frac{d\omega}{\omega}$ (μ_b)	$\int_{\Omega'}^{\infty} \frac{d\omega}{\omega}$ (μ_b)	Sum of terms in left-hand side of the sum rule (μ_b)
1.73	-1720	1520	- 48	250	2
1.50	-1720	1520	- 40	270	30
1.30	-1720	1520	- 37	290	53
1.10	-1720	1520	- 54	310	56
0.85	-1720	1520	-112	350	38
^{*)} For the computation of the integral $\int_{\Omega'}^{\infty} \dots$, a Regge parametrization is used (see the text).					

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- 11) This is essentially assumed in the analysis of Ref. 10).

- 12) We use here the resonances $P_{11}(1470)$, $S_{11}(1520)$, $D_{13}(1520)$, $D_{15}(1670)$, $F_{15}(1690)$, $P_{11}(1750)$, $S_{31}(1630)$, $F_{37}(1940)$. The characteristics of these resonances are taken from Rankin's article mentioned in Ref. 10). We use Rankin's values instead of Walker's to make sure that we also take into account the contributions of the weaker resonances which Walker absorbs into his background. Notice also that our results do not depend on the presence or not of an iso-tensor coupling in $P_{33}(1236)$.

- 13) As it is seen from Eq. (6), the lowest moment FESR stresses equally the low and the high energy contributions. This suggests that we should keep Ω as high as possible to make sure that this sum rule is satisfied.

- 14) Actually the average would have been more convincing if $\omega\sigma(\omega)$ had been plotted.

- 15) This is reasonable since cuts are usually assumed to be important at higher t values. Moreover at least in the parametrization of Barger and Phillips [Ref. 16] for πN scattering, the s' coupling vanishes at $t = 0$.

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FIGURE CAPTIONS

Figure 1 Input values for $\sigma(\omega) = 2\sigma_T(\gamma^{\nu} p \rightarrow I=\frac{1}{2}) - \sigma_T(\gamma^{\nu} p \rightarrow I=\frac{3}{2})$ as a function of ω (continuous line). Broken line gives the Regge asymptotic expression for $\sigma(\omega)$ when extrapolated to low energies.

Figure 2 Diagrams for the coupling of $\mathfrak{J}$ trajectory to "charged" photons according to VMD plus universality.

FIG. 1

FIG.2