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MODELS OF WEAK AND ELECTROMAGNETIC INTERACTIONS

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A B S T R A C T

Two models of weak and electromagnetic interactions are described which are based on the "spontaneously broken" gauge group associated with $SU(2) \times U(1)$. In the first model the neutrino field does not appear in the neutral current, in the second it appears only in combination with a heavy neutral lepton. As a consequence, elastic neutrino-proton scattering and elastic mu-neutrino electron scattering are absent in both models. The extension to hadrons is discussed. Finally, a number of observations on unified theories are made.

1. Introduction

Models based on the gauge group associated with $SU(2) \times U(1)$ appear to provide renormalizable unified field theories of weak and electromagnetic interactions. The model most extensively studied [1] is a sort of minimal model based on the known leptons. It implies the existence of weak neutral currents giving rise to processes such as $\nu_\mu + e \rightarrow \nu_\mu + e$ and $\nu + p \rightarrow \nu + p$ with strength comparable to that of the processes $\nu_\mu + \mu \rightarrow \nu_\mu + \mu$ and $\nu + n \rightarrow e + p$ of the usual theory.

In the present note we describe two models based on the possible existence of heavier leptons. In the first model the neutral current does not contain the neutrino field and therefore processes like those mentioned above are absent. In the second model the neutral current contains the neutrino field only in combination with a heavy neutral lepton ν' . It follows that the above mentioned elastic processes are replaced by $\nu_\mu + e \rightarrow \nu'_\mu + e$ and $\nu + p \rightarrow \nu' + p$; these inelastic scattering processes will be appreciable only at sufficiently high energy. Both models satisfy the requirements which are usually said to guarantee the renormalizability of the theory [2]. In particular, the anomalies cancel at the one-loop level [3] (separately for leptons and for quarks) and are therefore expected to be absent to all orders. Also, both our models give the correct value and sign for the amplitude describing the $\pi^0 \rightarrow 2\gamma$ decay. Our aim in discussing them here is mainly to show the possibility of alternatives to the model described by Weinberg. The existence of these alternatives appears especially interesting in view of the fact that the predictions of Weinberg's model are close to the limits provided by present experimental information on neutral currents; more precise experiments may prove incompatible with those predictions [4].

Since both models are based on the group $SU(2) \times U(1)$ (although for the first model, which uses only integral representations, one could say $O(3) \times O(2)$) a number of features will be the same as in the model described by Weinberg. In particular, the vector mesons are the

gauge fields $\vec{A}_\mu$ and B_μ of the group with coupling constants g and g' respectively. The charged intermediate boson is $W_\mu^\pm = \frac{1}{\sqrt{2}} (A_{\mu 1} \mp i A_{\mu 2})$ while the neutral massive intermediate boson and the photon are respectively

$$Z_\mu = (g A_{\mu 3} - g' B_\mu) / \sqrt{g^2 + g'^2}$$

$$A_\mu = (g' A_{\mu 3} + g B_\mu) / \sqrt{g^2 + g'^2}.$$

The electric charge is given by

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}},$$

which implies that both g and g' are larger than e . However, the masses of the vector mesons do not obey the same relations as in Weinberg's model.

2. Neutral current without neutrinos

In our first model we assume the existence of a positive heavy electron e' and a positive heavy muon μ' and attribute the left-handed leptons to the three-dimensional representation of $SU(2)$

$$E_L = \frac{1 + \gamma_5}{2} \begin{pmatrix} e' \\ \nu_e \\ e \end{pmatrix}, \quad M_L = \frac{1 + \gamma_5}{2} \begin{pmatrix} \mu' \\ \nu_\mu \\ \mu \end{pmatrix}$$

The leptons e and μ are negative. The right-handed leptons are taken to be singlets. We write the interaction with the vector mesons, for the

case of the electron multiplet as

$$\begin{aligned}
 & i g \left\{ (-\bar{\nu}_L \gamma_\mu e'_L + \bar{e}_L \gamma_\mu \nu_L) W^-_\mu + \text{h.c.} \right. \\
 & \quad \left. + (\bar{e}'_L \gamma_\mu e'_L - \bar{e}_L \gamma_\mu e_L) A_{\mu 3} \right\} \\
 & + i g' (\bar{e}'_R \gamma_\mu e'_R - \bar{e}_R \gamma_\mu e_R) B_\mu .
 \end{aligned}$$

This attributes definite transformation properties under the $U(1)$ gauge group to the leptons. The muon multiplet is treated in an analogous way.

Clearly, the photon couples to the electromagnetic current. The coupling of the neutral intermediate boson is

$$\frac{i}{\sqrt{g^2 + g'^2}} \{ g^2 (\bar{e}'_L \gamma_\mu e'_L - \bar{e}_L \gamma_\mu e_L) - g'^2 (\bar{e}'_R \gamma_\mu e'_R - \bar{e}_R \gamma_\mu e_R) \} Z_\mu$$

and, since only charged particles are involved, it would manifest itself as a short-range parity violating correction to electromagnetism, except if $g = \pm g'$, in which case parity is conserved in this interaction.

In order to extend the model to hadrons one must introduce integrally charged quarks (with fractionally charged quarks the model would have anomalies). We take the left-handed triplets

$$\begin{pmatrix} p^+ \\ n_c \\ r^- \end{pmatrix}_L \qquad \begin{pmatrix} p'^+ \\ \lambda_c \\ r'^- \end{pmatrix}_L$$

where $n_c = n \cos \vartheta + \lambda \sin \vartheta$, $\lambda_c = -n \sin \vartheta + \lambda \cos \vartheta$ and ϑ is the Cabibbo angle. The right-handed quarks are taken to be singlets. Finally, in order to construct the baryon octet one must add a singlet (left and right) s which is an $SU(3)$ singlet. Denoting p, n, λ by q , the meson

octet is given as $\bar{q}q$ and the baryon octet as $\bar{q}qs$. As usual in this kind of models, a whole new class of hadrons must be assumed. These hadrons contain e.g. the new quarks, r, p' and r' and are expected to be heavier than the known $SU(3)$ multiplets. Observe that the hadronic neutral current does not contain n or λ . Therefore, no $\Delta Y = 2$ processes (such as $K^0 - \bar{K}^0$ transition) and no $\Delta Y = 1$ couplings to neutral lepton currents (such as $K^0 \rightarrow \mu\bar{\mu}$) occur to lowest order. To second order in the weak interactions these processes vanish if all quarks have the same mass; they can therefore be expected to be small. This result is due to cancellations which involve the new quarks and for this reason one must reject a simpler scheme which would dispense with p' and r' and in which λ_c and s would be treated as singlets.

We now explain in some detail how masses are introduced in the theory. In order to give masses to the spinor fields one must introduce a real scalar triplet

$$\gamma = \begin{pmatrix} \gamma^+ \\ \gamma^0 \\ \gamma^- \end{pmatrix} = \begin{pmatrix} \frac{\gamma_1 - i\gamma_2}{\sqrt{2}} \\ \gamma_3 \\ \frac{\gamma_1 + i\gamma_2}{\sqrt{2}} \end{pmatrix}$$

where the vector $\vec{\gamma}$ has real components $\gamma_1, \gamma_2, \gamma_3$, and a complex scalar triplet

$$\chi = \begin{pmatrix} \chi^{++} \\ \chi^+ \\ \chi^0 \end{pmatrix} = \begin{pmatrix} \frac{\chi_1 - i\chi_2}{\sqrt{2}} \\ \chi_3 \\ \frac{\chi_1 + i\chi_2}{\sqrt{2}} \end{pmatrix}$$

with $\vec{\chi} = \vec{\alpha} + i\vec{\beta}$. One must arrange the self-interaction among these scalar fields in such a way that their vacuum expectation values take the form

$$\langle \gamma \rangle = \begin{pmatrix} 0 \\ u \\ 0 \end{pmatrix}, \quad \langle \chi \rangle = \begin{pmatrix} 0 \\ 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

Let us construct the invariants

$$I_1 = \gamma^* \gamma = \vec{\gamma}^2$$

$$I_2 = \chi^* \chi = \vec{\alpha}^2 + \vec{\beta}^2$$

$$I_3 = \gamma^* \chi = \vec{\alpha} \cdot \vec{\gamma} + i\vec{\beta} \cdot \vec{\gamma}$$

$$I_4 = \chi \mathcal{E} \chi = \vec{\alpha}^2 - \vec{\beta}^2 + 2i \vec{\alpha} \cdot \vec{\beta}.$$

I_1 and I_2 have charge zero, I_3 has charge one and I_4 has charge two. Here

$$\mathcal{E} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

The most general self-interaction among the scalar fields acceptable in a renormalizable field theory is a real neutral polynomial in the invariants which is at most quadrilinear in the fields. One can take therefore any linear combination of the expressions I_1 , I_1^2 , I_2 , I_2^2 , $I_3^* I_3$, $I_4^* I_4$ and $I_1 I_2$ with real coefficients. Since the last one gives rise to a mixing term in the scalar mass matrix, we drop it and take

$$\begin{aligned}
 V &= a I_1 + b I_1^2 + c I_2 + d I_2^2 + f I_4^* I_4 + h I_3^* I_3 = \\
 &= a \vec{\gamma}^2 + b (\vec{\gamma}^2)^2 + c (\vec{\alpha}^2 + \vec{\beta}^2) + d (\vec{\alpha}^2 + \vec{\beta}^2)^2 + f ((\vec{\alpha}^2 - \vec{\beta}^2)^2 + 4(\vec{\alpha} \cdot \vec{\beta})^2) \\
 &\quad + h ((\vec{\alpha} \cdot \vec{\gamma})^2 + (\vec{\beta} \cdot \vec{\gamma})^2).
 \end{aligned}$$

In the tree approximation (classical limit) we take a and c negative and b, d, f and h positive (in the Lagrangian $-V$ enters). It is clear that the minimum of V is obtained for $\vec{\alpha} \cdot \vec{\beta} = \vec{\alpha} \cdot \vec{\gamma} = \vec{\beta} \cdot \vec{\gamma} = 0$ and $\vec{\gamma}^2 = -a/2b \equiv u^2$, $\vec{\alpha}^2 = \vec{\beta}^2 = -c/4d \equiv v^2/4$. These relations are then satisfied by the vacuum expectation values of the fields. It is therefore always possible by means of a transformation of the gauge group to achieve the forms

$$\gamma = \begin{pmatrix} 0 \\ u + \gamma^0 \\ 0 \end{pmatrix} \quad \chi = \begin{pmatrix} \chi^{++} \\ \chi^+ \\ \frac{v + \varphi^0}{\sqrt{2}} \end{pmatrix}$$

where the various fields γ^0 , χ and φ^0 have vanishing vacuum expectation value. The field γ^0 is real; φ^0 can also be made real by means of a gauge transformation of the $U(1)$ group. The masses of the various scalar fields are independent and the theory makes no prediction about their values.

If the neutrinos are massless the leptons will interact only with the field χ . The invariants

$$- \sqrt{2} \frac{m_e}{v} (\bar{e}_R \chi^* E_L + \text{h.c.})$$

and

$$- \sqrt{2} \frac{m_{e'}}{v} (\bar{e}_R' \chi E_L + \text{h.c.})$$

give masses to the electron e and to the heavy lepton e' , but not to the electron neutrino ν_e (similarly with the muon triplet). The field γ had to be introduced in order to be able to give a mass to the neutral quarks n and λ . On the other hand, the quark singlet s can be given a mass directly, without making use of the scalar fields. In this manner all quarks can be given different masses.

Once the field γ is present in the theory, it will interact with the vector mesons and both χ and γ contribute to the vector meson mass. The Lagrangian contains the invariants

$$- \left| \partial_\mu \vec{\chi} - ig' B_\mu \vec{\chi} + g \vec{A}_\mu \times \vec{\chi} \right|^2$$

and

$$- \frac{1}{2} (\partial_\mu \vec{\gamma} + g \vec{A}_\mu \times \vec{\gamma})^2.$$

For the masses of the charged and the neutral intermediate vector boson one finds respectively

$$m_W^2 = g^2 \left(\frac{v^2}{2} + u^2 \right) \qquad m_Z^2 = (g^2 + g'^2) \frac{v^2}{2},$$

so that the Fermi constant is given by

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{4m_W^2} = \frac{1}{4 \left(\frac{v^2}{2} + u^2 \right)}.$$

Since $g > e$, $m_W^2 > \frac{e^2 \sqrt{2}}{4G_F} \approx (53 \text{ GeV})^2$.

The charged fields χ^{++} and χ^+ cannot be eliminated by a gauge transformation. In particular, χ^+ has couplings to the leptons and hadrons analogous to those of the W_μ^+ field, but proportional to the masses of the fields to which it couples. Order of magnitude estimates

show that these interactions are negligible when electrons are involved. In processes involving muons these effects can also be neglected if the mass of the χ^+ is at least a couple of GeV's. On the other hand, if the mass of the χ^+ is only about 0.6 to 0.8 GeV there would be an effect on $K_{\mu 3}$ and in particular the values of F_π and F_K , as well as the form of the Callan - Treiman relation would be modified. The effect of such a small χ^+ mass on other processes is perhaps not unacceptable, in view of the present theoretical and experimental uncertainties. This point is worth investigating in more detail [5].

3. Neutral current inducing only inelastic neutrino processes

In our second model we assume the existence of a positive heavy electron e' and also of a neutral heavy neutrino ν_e' and treat the left-handed leptons

$$E_{1L} = \frac{1 + \gamma_5}{2} \begin{pmatrix} e' \\ \frac{\nu_e + \nu_e'}{\sqrt{2}} \end{pmatrix}, \quad E_{2L} = \frac{1 + \gamma_5}{2} \begin{pmatrix} \frac{\nu_e - \nu_e'}{\sqrt{2}} \\ e \end{pmatrix}$$

as SU(2) doublets. Similarly, for the muon,

$$M_{1L} = \frac{1 + \gamma_5}{2} \begin{pmatrix} \mu' \\ \frac{\nu_\mu + \nu_\mu'}{\sqrt{2}} \end{pmatrix}, \quad M_{2L} = \frac{1 + \gamma_5}{2} \begin{pmatrix} \frac{\nu_\mu - \nu_\mu'}{\sqrt{2}} \\ \mu \end{pmatrix}.$$

All right-handed leptons are taken as singlets. Again we give the vector interaction for the electron multiplet only, the muon multiplet being perfectly analogous. It is

$$\begin{aligned} & \frac{ig}{2} (\bar{E}_{1L} \gamma_\mu \vec{\tau} E_{1L} + \bar{E}_{2L} \gamma_\mu \vec{\tau} E_{2L}) \cdot \vec{A}_\mu \\ & + ig' \left(\frac{1}{2} \bar{E}_{1L} \gamma_\mu E_{1L} - \frac{1}{2} \bar{E}_{2L} \gamma_\mu E_{2L} + \bar{e}'_R \gamma_\mu e_R - \bar{e}_R \gamma_\mu e_R \right) B_\mu \end{aligned}$$

This formula attributes well defined gauge properties under the U(1) group to the various lepton fields. The above interaction can be written as

$$\begin{aligned} & \frac{ig}{2} \{ [\bar{e}'_L \gamma_\mu (v_L + v'_L) + (\bar{v}_L - \bar{v}'_L) \gamma_\mu e_L] W^-_\mu + \text{h.c.} \\ & + [\bar{e}'_L \gamma_\mu e'_L - \bar{v}'_L \gamma_\mu v_L - \bar{v}_L \gamma_\mu v'_L - \bar{e}_L \gamma_\mu e_L] A_{\mu 3} \} \\ & + ig' \{ \frac{1}{2} \bar{e}'_L \gamma_\mu e'_L + \frac{1}{2} \bar{v}_L \gamma_\mu v'_L + \frac{1}{2} \bar{v}'_L \gamma_\mu v_L \\ & - \frac{1}{2} \bar{e}_L \gamma_\mu e_L + \bar{e}'_R \gamma_\mu e'_R - \bar{e}_R \gamma_\mu e_R \} B_\mu \end{aligned}$$

The photon field couples to the electromagnetic current. The coupling of the neutral intermediate boson is

$$\begin{aligned} & \frac{i}{\sqrt{g^2 + g'^2}} \left\{ \frac{g^2 - g'^2}{2} (\bar{e}'_L \gamma_\mu e'_L - \bar{e}_L \gamma_\mu e_L) - \frac{g^2 + g'^2}{2} (\bar{v}_L \gamma_\mu v'_L + \bar{v}'_L \gamma_\mu v_L) \right. \\ & \left. + g'^2 (\bar{e}'_R \gamma_\mu e'_R - \bar{e}_R \gamma_\mu e_R) \right\} Z_\mu. \end{aligned}$$

Observe that the neutrino field v enters only in combination with the field v' of the heavy neutrino.

The present model can be extended to hadrons by introducing integrally charged quarks (again in order to avoid the occurrence of anomalies) attributed to SU(2) doublets as

$$\frac{1 + \gamma_5}{2} \begin{pmatrix} p^+ \\ \frac{n' + n_c}{\sqrt{2}} \end{pmatrix} \qquad \frac{1 + \gamma_5}{2} \begin{pmatrix} \frac{n' - n_c}{\sqrt{2}} \\ r^- \end{pmatrix}$$

$$\frac{1 + \gamma_5}{2} \begin{pmatrix} p'^+ \\ \frac{n'' + \lambda_c}{\sqrt{2}} \end{pmatrix} \qquad \frac{1 + \gamma_5}{2} \begin{pmatrix} \frac{n'' - \lambda_c}{\sqrt{2}} \\ r'^- \end{pmatrix}$$

where n_c and λ_c are defined as in section 2. All right-handed quarks are singlets. This assignment is perfectly analogous to that of the leptons. The current coupled to the neutral intermediate vector boson Z_μ does not contain terms bilinear in n_c or in λ_c . The only terms involving neutral quarks are proportional to

$$\bar{n}_L' \gamma_\mu n_c + \text{h.c.} \quad \text{and to} \quad \bar{n}_L'' \gamma_\mu \lambda_c + \text{h.c.}$$

Therefore, to lowest order, processes like $K^0 \rightarrow \mu\bar{\mu}$ and $K^0 \rightarrow \bar{K}^0$ are absent. In second order they are also absent, if all quarks have the same mass, due again to cancellations among diagrams in which different quarks are exchanged. Denoting the SU(3) triplet p, n, λ by q and treating for instance n' as an SU(3) singlet, the meson octet can be constructed as $\bar{q}q$ and the baryon octet as $\bar{q}qn'$.

The particular properties of the neutral current in this model follow from the particular mixing chosen above for the fields v_L and v_L' or n_L' and n_c etc. One could, in principle, consider more general mixing angles. However, if one requires universality for the weak interactions involving charged currents, the mixing angle must be the same for the electron multiplet, for the muon multiplet and for the quarks. On

the other hand, the requirement that the hadronic neutral current should not contain terms bilinear in n_c or in λ_c determines the mixing angle as chosen above. Therefore, even though universality and absence of the mentioned terms in the hadronic neutral current are put in by hand, these physical requirements determine the form of the leptonic neutral current and thus the absence in it of terms bilinear in ν_L .

One can give masses to the leptons and to the quarks by using, just as in Weinberg's model, a scalar doublet

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$$

which, due to its self-interaction, acquires a vacuum expectation value. By means of a gauge transformation one can always obtain the form

$$\varphi = \begin{pmatrix} 0 \\ \frac{v + \varphi^0}{\sqrt{2}} \end{pmatrix}$$

where the field φ^0 has no vacuum expectation value. The interaction of the field φ with the vectors is just as in Weinberg's model and the mass relations are the same

$$m_W^2 = \frac{g^2 v^2}{4}, \quad m_Z^2 = (g^2 + g'^2) \frac{v^2}{4},$$

so that $m_Z > m_W$. However, the constant g enters differently in the expression for the charged current and the Fermi constant is given by

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{16 m_W^2} = \frac{1}{4v^2}$$

which differs from the formula valid in Weinberg's model. Since $g > e$, $m_W^2 > \frac{e^2 \sqrt{2}}{16 G_F} \approx (26.5 \text{ GeV})^2$.

The neutral heavy lepton ν'_μ will contribute to the $g-2$ of the muon as an internal line in a diagram in which a W meson is emitted, interacts with the electromagnetic field, and is reabsorbed by the muon. Unless the ν'_μ mass is extremely large this contribution is expected to be of the same order of magnitude as that in which ν'_μ is replaced by the ordinary neutrino ν_μ and is therefore acceptable, within present experimental errors on the $g-2$ of the muon [6].

4. Discussion

The gauge vector fields of an abelian group like $U(1)$ can be coupled with arbitrary coupling constants to different fields. In the case of a simple non-abelian gauge group like $SU(2)$, on the contrary, the coupling constants of the gauge vectors to different fields are determined by the transformation properties of these fields, as long as they belong to irreducible representations. This observation leads to the hope that a unified gauge theory of weak and electromagnetic interactions will account for the universality properties of both interactions. In practice this hope has not been fully realized [7]. Models based on $SU(2) \times U(1)$ may have automatic universality of the weak interactions involving charged currents (as in Weinberg's model and in the first model described in the present note) but will not have automatic universality for the electromagnetic interactions. If mixing occurs (as in the second model described here) even the universality of the charged currents must be introduced as an ad hoc assumption. In view of these remarks it is interesting to observe that the condition of cancellation of the anomalies can act as an additional constraint on the charges of the various particles, even though it does not determine them uniquely.

It would appear that the simplest way of introducing the strong interactions in both our models is to couple all quarks with equal strength to a singlet vector gluon in a manner which would be invariant under

$SU(7) \times SU(7)$ for the first model and under $SU(8) \times SU(8)$ for the second. However, this additional vector interaction would introduce anomalies and spoil the renormalizability of the models (the same happens in Weinberg's model with $SU(4)$ quarks). The only feasible alternative seems to be the introduction of scalar and pseudoscalar fields belonging to the $(7_L, \bar{7}_R) + (7_R, \bar{7}_L)$ and $(8_L, \bar{8}_R) + (8_R, \bar{8}_L)$ representations respectively.

In sections 2 and 3 we have remarked that all quarks can be given different masses, for an arbitrary value of the Cabibbo angle ϑ . (This destroys the hope of determining ϑ at the present level of sophistication). The breaking of the $SU(3) \times SU(3)$ group of the strong interactions appears naturally as a linear combination of u_o , u_8 , and u_3 , the well known scalar fields of the $(3_L, \bar{3}_R) + (3_R, \bar{3}_L)$ representation. This is a property of all known models of the kind considered here. The presence of u_3 (non electromagnetic breaking of isospin) is to be noticed. Indeed, the theory gives no indication about why it should be small, smaller or about the same as the effect of electromagnetism.

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For models based on the group $SU(2) \times U(1)$, the condition for cancellation of the one-loop anomalies can be written as

$$\text{Tr} C_3^2 Q = 0 ,$$

where the trace is taken over all spinor fields, Q is the (diagonal) charge matrix, and C_3 is the matrix (also diagonal) which represents the corresponding generator of the group $SU(2)$ (the representation will in general be reducible). The correct $\pi^0 \rightarrow 2\gamma$ amplitude will be obtained if $2\text{Tr} T_3 Q^2 = 1$, where T_3 is the third component of the hadronic isospin matrix.

- [4] While we were writing this note a paper by H. Georgi and S.L. Glashow appeared in Phys.Rev.Letters 28, 1494 (1972). It describes a model based on the group $O(3)$ in which neutral currents other than the electromagnetic current are totally absent. In footnote 9 of their paper Georgi and Glashow mention that models with neutral currents without neutrinos can be constructed.

More recently we received a preprint by B.W. Lee, to be published in Phys.Rev., where a model essentially identical to our first model is described.

- [5] We are grateful to C. Bouchiat and to M.K. Gaillard for discussing some of these points with us.
- [6] We are grateful to R. Gastmans and B. Lautrup for estimating this contribution.
- [7] As the authors themselves observe, the $SU(2)$ (or $O(3)$) model of Georgi and Glashow does not have automatic universality of the charged currents. It does, however, have electromagnetic universality, although one may ask why particles of charge $e/2$ do not exist.