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SEPARATED HIGH-ENERGY ELECTRON BEAMS USING SYNCHROTRON RADIATION

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It has already been remarked that the new 300-500 GeV accelerators can provide high-energy electron or photon beams not otherwise available, thus opening up a new range of possibilities for the study of deep inelastic scattering, of the electro- and photo-production of bosons, and so on¹⁾. For such experiments it is important that the beam does not contain pions and other hadronic particles. A simple way of isolating the electrons is to pass the momentum-selected beam through a region of high magnetic field. The electrons then lose energy by synchrotron radiation, and may be separated from the other particles by means of a conventional spectrometer.

The purpose of this note is to emphasize the feasibility of such a device, and to summarize the relevant design formulae. A specific 100-150 GeV beam design for a 300 GeV machine is discussed by way of example.

1. THEORY

When an electron of high energy $E = \gamma m_0 c^2$ has orbit radius ρ in a magnetic field B , the energy loss per radian^{2,3)} is

$$-\frac{dE}{d\theta} = \frac{2}{3} \frac{e^2}{\rho} \gamma^4. \quad (1)$$

Hence the fractional energy loss per unit length is

$$\kappa = -\frac{1}{E} \frac{dE}{dx} = \frac{2}{3} \frac{e^4}{(m_0 c^2)^3} \gamma B^2. \quad (2)$$

Therefore the total energy loss in length L (metres) is

$$\frac{\Delta E}{E} = 6.44 \times 10^{-12} \gamma B^2 L \quad (3)$$

where B is in kilogauss.

The fluctuation in the energy loss is discussed in the Appendix, where we show that for a mono-energetic initial beam the final energy spectrum is approximately Gaussian with standard deviation

$$\sigma_E = n_{\text{eff}}^{-1/2} \Delta E. \quad (4)$$

In this expression the "effective number of photons" of synchrotron radiation emitted is

$$\begin{aligned} n_{\text{eff}} &= 0.336 \alpha \gamma \quad \text{per radian} \\ &= 0.144 BL, \end{aligned} \tag{5}$$

where the units for B and L are as above, and α is the fine-structure constant.

In the practical cases examined below, n_{eff} is reasonably large and the fluctuation is small compared to the intrinsic momentum spread of the beam. As the photons are emitted close to the forward direction, the fluctuations of beam direction due to recoil effects are negligible.

2. TYPICAL DESIGN

Equations (3), (4) and (5) suffice for a preliminary beam design. As an example we find that 100 GeV electrons in a field of 100 kG lose 1.28% of their energy per metre, and emit 14 photons per metre. A typical design might involve a 4 m long superconducting 100 kG magnet, giving a 5.1% energy loss with a standard deviation of 0.7%. (Note that a smaller fluctuation is obtained if the same energy loss is produced in lower fields.)

A convenient arrangement is to divide the magnet into three parts of lengths $\frac{1}{4} L$, $\frac{1}{2} L$, and $\frac{1}{4} L$ with field directions +, -, +, respectively. We call this a *chicane* of length L. Hadrons of any momentum are neither displaced laterally nor deflected by a *chicane*. Electrons which have radius of curvature ρ in the field and fractional energy loss κ per unit length also suffer virtually no net deflection: they are, however, displaced laterally by the distance $s = \kappa L^3 / 16\rho$. For a *chicane* 4 metres long with a 100 kG field, $s \sim 0.25$ mm and is negligible.

Thus a chicane may be introduced anywhere into a beam transport system to make the electrons lose energy, without altering the optical properties of the beam. Clearly several chicanes may be placed in series, the total effect being calculated by adding the individual contributions. We then have the configuration $\frac{1}{4} L$, $\frac{1}{2} L$, $\frac{1}{2} L \dots \frac{1}{2} L$, $\frac{1}{4} L$ with alternating fields directions.

The effect of energy fluctuations on the final electron direction is being investigated, but does not appear to be serious.

3. ELECTRON INTENSITY

The electron intensity has been estimated assuming that the π^0 production is the average of π^+ and π^- , that 0.5 of the incoming protons interact, and that 0.5 of the π^0 γ -rays produce pairs in the target (for example, a beryllium target, 50 cm long). The $\pi^\pm$ spectra at zero angle are taken from Grote, Hagedorn and Ranft⁴⁾ for 300 GeV incident protons. The percentage of electrons in a typical $\pi^\pm$ beam are then 60%, 18%, 4% and 0.7% at 50, 100, 150, and 200 GeV, respectively. Assuming that the electrons are collected over an emission solid angle of 10 μ sr, we then find for $\pm 1\%$ momentum spread, with 10^{13} protons per pulse, 5×10^8 electrons at 100 GeV, and 1×10^8 electrons at 150 GeV. With 500 GeV primary protons, electron yields at 170 and 250 GeV should be approximately four times higher for the same solid angle and the same dp/p.

4. SYNCHROTRON RADIATION IN NORMAL BEAM TRANSPORT

The effect of synchrotron radiation by electrons in the normal quadrupoles and bending magnets used to form and analyse the beam can be obtained from Eq. (3). In a typical 100 GeV beam we expect to have about 20 m of bending magnet with a field of the order of 15 kG, and about 25 m of quadrupole with a field of about 10 kG maximum. So for a particle near the edge of the beam, which sees the largest quadrupole fields,

$$\int B^2 dL \sim 4500 \text{ (bending)} + 1500 \text{ (quads)} \sim 6000 \text{ kG}^2 \cdot \text{m}.$$

The corresponding energy loss for a 100 GeV electron is 0.8%. As this is occurring uniformly along the beam, it will have very little effect on the fundamental beam optics, but clearly some magnet currents will have to be readjusted.

5. CONCLUSION

The energy loss due to synchrotron radiation can be used to select pure beams of high-energy electrons from multi-GeV proton accelerators. Yields of more than 10^8 electrons per pulse in the range 100-250 GeV appear feasible.

We wish to thank Dr. D. Newton for several stimulating discussions.

APPENDIX

FLUCTUATIONS IN ENERGY-LOSS

The energy spectrum of an initially mono-energetic beam after it has passed through a magnet has been obtained in two different ways which lead to the same result: i) by considering the variance in the number of photons of each energy emitted; and ii) by setting up a diffusion equation for the electron energy.

1. STATISTICS OF PHOTON EMISSION

The number of photons⁵⁾ of energy ε to $(\varepsilon + d\varepsilon)$ emitted while the electron is deflected through an angle $d\theta$ in the magnetic field is

$$n(\varepsilon) d\varepsilon d\theta = \frac{\sqrt{3}}{2\pi} \alpha \gamma d\theta \frac{1}{\varepsilon_{cr}} \int_{\varepsilon/\varepsilon_{cr}}^{\infty} K_{5/3}(\eta) d\eta d\varepsilon, \quad (A.1)$$

where

$$\varepsilon_{cr} = \frac{3}{2} \gamma^3 \hbar c / \rho. \quad (A.2)$$

Putting $y = \varepsilon/\varepsilon_{cr}$, this becomes

$$n(\varepsilon) d\varepsilon d\theta = \frac{\sqrt{3}}{2\pi} \alpha \gamma d\theta \int_y^{\infty} K_{5/3}(\eta) d\eta dy. \quad (A.3)$$

$K_{5/3}(\eta)$ is a Bessel function of the second kind.

We assume that the statistical fluctuation in this number of photons is $[n(\varepsilon) d\varepsilon d\theta]^{1/2}$. Therefore the energy loss is

$$\Delta E = d\theta \int_0^{\infty} \varepsilon n(\varepsilon) d\varepsilon \quad (A.4)$$

and the variance of ΔE is

$$V_{\Delta E} = d\theta \int_0^{\infty} \varepsilon^2 n(\varepsilon) d\varepsilon. \quad (A.5)$$

Using Eq. (A.3) and the integrals⁶⁾

$$\int_0^\infty y \int_y^\infty K_{5/3}(\eta) d\eta dy = \frac{8\pi}{9\sqrt{3}} \quad (A.6)$$

and

$$\int_0^\infty y^2 \int_y^\infty K_{5/3}(\eta) d\eta dy = \frac{55\pi}{81} \quad (A.7)$$

gives

$$\Delta E = \frac{4}{9} \alpha \gamma \epsilon_{cr} d\theta \quad (A.8)$$

[in agreement with Eq. (1)] and

$$V_{\Delta E} = \frac{55}{18\sqrt{3}} \alpha \gamma \epsilon_{cr}^2 d\theta. \quad (A.9)$$

It is convenient to define an effective number of photons from which $V_{\Delta E}$ can be readily calculated:

$$n_{eff} \equiv (\Delta E)^2 / V_{\Delta E} = \frac{32}{55\sqrt{3}} \alpha \gamma d\theta = 0.336 \alpha \gamma d\theta. \quad (A.10)$$

Note that this is much smaller than the total number of photons obtained by integrating Eq. (A.3); as

$$\int dy \int_y^\infty K_{5/3}(\eta) d\eta = \frac{5\pi}{3}, \quad (A.11)$$

$$\text{the total number of photons}^{7)} = \frac{5}{2\sqrt{3}} \alpha \gamma d\theta. \quad (A.12)$$

Equation (A.12) would give an incorrect estimate of the energy fluctuations as most of these photons carry away very little energy.

2. DIFFUSION EQUATIONS

The above result may also be obtained by writing a diffusion equation as indicated by Rossi⁸⁾. When a beam of initial energy E_0 is bent through an angle θ in the magnetic field let $N(E, \theta)$ be the number of electrons of energy E , then we have approximately, neglecting the slow dependence of

the photon spectrum on electron energy,

$$\begin{aligned} \frac{\partial N}{\partial \theta} = & - \int N(E, \theta) n(\epsilon) d\epsilon \\ & + \int N(E + \epsilon, \theta) n(\epsilon) d\epsilon . \end{aligned} \quad (\text{A.13})$$

Expanding $N(E + \epsilon)$ as far as terms in ϵ^2 , we get

$$\begin{aligned} \frac{\partial N}{\partial \theta} = & \frac{\partial N}{\partial E} \int \epsilon n(\epsilon) d\epsilon + \frac{1}{2} \frac{\partial^2 N}{\partial E^2} \int \epsilon^2 n(\epsilon) d\epsilon \\ = & X \frac{\partial N}{\partial E} + Y \frac{\partial^2 N}{\partial E^2} , \end{aligned} \quad (\text{A.14})$$

where

$$\left. \begin{aligned} X = & \int \epsilon n(\epsilon) d\epsilon \\ Y = & \frac{1}{2} \int \epsilon^2 n(\epsilon) d\epsilon \end{aligned} \right\} \quad (\text{A.15})$$

The solution, obtained for example using a Fourier transform, is

$$N(E, \theta) = N_0 (4\pi Y \theta)^{1/2} \exp \left\{ \frac{-(E - E_0 - X \theta)^2}{4Y \theta} \right\} . \quad (\text{A.16})$$

The distribution is a Gaussian function of energy, centred at

$$\{E_0 - \iint \epsilon n(\epsilon) d\epsilon d\theta\} ,$$

as expected, and the standard deviation is

$$2Y = \iint \epsilon^2 n(\epsilon) d\epsilon d\theta$$

agreeing with our previous result, Eq. (A.5).

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