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$\pi\pi$ PHENOMENOLOGY BELOW 1100 MeV

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A B S T R A C T

Conclusions are presented that can be drawn from combining all present experimental information on low energy $\pi\pi$ scattering with the requirements of crossing, unitarity and analyticity.

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We present the set of $\pi\pi$ amplitudes consistent with crossing, analyticity and experimental information on the $\pi\pi$ system below 1.1 GeV. In particular, we show how this information reduces considerably the multiplicity of $\pi\pi$ amplitudes containing the rho meson and satisfying analyticity, crossing and unitarity as discussed in the previous paper ¹⁾, hereafter referred to as A. As we shall see, present experimental data do not suffice to determine the $\pi\pi$ scattering lengths completely. However, they must lie in a restricted region which we delineate and we indicate how further experiments can resolve these final ambiguities in their values.

We first enumerate the experimental information, coming from dipion production processes, which we use in our calculations. Note that in the following we use the same notation as in A.

- (i) For the p wave, as in A, we fix the mass and width of the rho meson, $M_\rho = 765$ MeV, and $\Gamma_\rho = 135$ MeV, but allow the scattering length a_1 to be arbitrary ²⁾.
- (ii) The isoscalar s wave phase shift δ_0^0 in the mass range $500 < M_{\pi\pi} < 900$ MeV must lie in the between-down or between-up bands ³⁾⁻⁵⁾ [this up-down ambiguity is resolved by (iv)].
- (iii) The $I = 2$ s wave phase shift δ_0^2 has a rather smooth behaviour with a value at the rho mass in the range $\delta_0^2(M_\rho) = -15^\circ \pm 5^\circ$ ^{3),6)-8)}.
- (iv) Inelasticity due to 4π production below 1 GeV is negligible ^{5),9)-13)}, but there is a strong cusp or S^* effect causing δ_0^0 to accelerate rapidly through 180° and a sharp onset of inelasticity, at the $K\bar{K}$ threshold ^{5),9)-11),14)-16)}.

The behaviour for δ_0^0 allowed by (iv) is shown in Fig. 1. In particular, the "up-down" ambiguity is resolved in favour of the down solution between 750 and 925 MeV ^{5),11)}.

Analyticity and crossing are imposed using the partial wave relations described in A with a cut-off at $N = 110 m_\pi^2$. The s and p wave driving terms $\phi_\ell^I(s)$ are evaluated assuming Regge pole plus Pomeron exchange behaviour above $s = N$. In addition the contributions of the f_0 resonance to the $\phi_\ell^I(s)$ are included, taking a mass $M_{f_0} = 1269$ MeV and elastic partial width $\Gamma_{f_0} = 125$ MeV ²⁾. For the Regge exchange component we take exchange degenerate $\rho + f_0$ poles with the Lovelace-Veneziano residue function ^{17),18)}, evaluated for a universal ρ coupling constant of $f^2/4\pi = 2.4$. The diffractive component is represented as a simple Pomeron exchange pole with a

slope $\alpha_P' = 0.4 \text{ GeV}^{-2}$ and parameters chosen to give an asymptotic cross-section $\sigma_\omega = (1/m_\pi^2) \approx 20 \text{ mb}$ and a logarithmic slope for the differential cross-section of $b = 10 (\text{GeV}/c)^{-2}$ at $s = 10 \text{ GeV}^2$. The driving terms thus obtained can be approximately parametrized between threshold and N by

$$\phi_0^0(s) = (s-4) \left[0.00091 + 0.00098(s-4) \right]$$

$$\phi_1^1(s) = (s-4) \left[0.00003 + 0.000023(s-4) \right]$$

$$\phi_0^2(s) = (s-4) \left[0.00072 + 0.000035(s-4) \right]$$

Here and in the following we use pion mass units, $m_\pi = 1$. The major contribution to $\phi_0^0(s)$ comes from the f_0 resonance, while Pomeron exchange provides the dominant part of $\phi_0^2(s)$ and $\phi_1^1(s)$ is quite small. We estimate the errors due to uncertainties in experimental quantities and neglected contributions to be less than 30% for $\phi_0^0(s)$ and 50% for $\phi_0^2(s)$ and $\phi_1^1(s)$.

The experimental information (i)-(iv) listed above is now incorporated into our procedure, given in A, for constructing consistent $\pi\pi$ amplitudes. The rho meson parameters and the driving terms $\phi_\ell^I(s)$ are set equal to the values given above. We also investigate the changes introduced by varying these terms by reasonable amounts. The multiplicity of solutions reported in A is reduced, firstly, by fixing the isoscalar s wave amplitude near the $K\bar{K}$ threshold and above to satisfy (iv). Results are given for cases I and II of Fig. 1, which we consider span the experimentally allowed possibilities ^{*}). In each case, we find just one solution for each point in the region of the (a_0^0, a_0^2) plane explored in A. Secondly, we require δ_0^0 to lie (within two standard deviations) in the between-down band of (ii). This reduces the number of possible $\pi\pi$ amplitudes further, leaving only a very narrow band of finite extent in the (a_0^0, a_0^2) plane.

^{*}) We feel I and II represent rather extreme possibilities with some preference for I, however, in view of the data of Ref. 5).

For definiteness, we present the solutions obtained using the s wave phase shifts of Baton et al.³⁾ below 925 MeV (the "no inelasticity" fit) and curve I of Fig. 1 for the behaviour above the $K\bar{K}$ threshold. The effects of varying this input are also analyzed. In Fig. 2, we give three solutions (1-3) for the s wave phase shifts, which illustrate the spread of allowed behaviour. The crucial parameters characterizing these solutions are given in Table I. In addition to the three scattering lengths, we give the value of δ_0^2 at the rho mass, the value of δ_0^0 at 500 MeV and the mass and width m_ϵ , Γ_ϵ of the isoscalar resonance ϵ , as determined by the position of the complex pole. We obtain strong evidence against an ϵ pole with a width less than, say, 400 MeV and favour a super-broad ϵ pole. However, our pole parameters have to be treated with some care since, for such a broad object, they may have little to do with, for example, the position and residue of a real axis pole required to fit $f_0^0(s)$ in the threshold region.

The d and f wave scattering lengths and phase shifts at 1 GeV, evaluated using the partial wave relations for $\ell = 2$ and 3 , are listed in Table II for our three solutions. We note that the isoscalar d wave scattering length a_2^0 has a remarkably stable value within our set of solutions - it is dominated by the ρ and the " ϵ ". The $I = 2$ d wave scattering length a_2^2 is very small while, on the other hand, the f wave scattering length a_3^1 is dominated by the threshold singularities. Even so, however, the d and f wave phase shifts at 1 GeV are remarkably stable. We note that the formula $\delta_\ell^I \approx q^{2\ell+1} a_\ell^I$ is grossly violated for δ_2^2 and δ_3^1 at 1 GeV. The values of δ_2^0 and δ_2^2 at 1 GeV are in good agreement with those of Baton et al.³⁾ and the values of δ_2^0 and δ_3^1 in excellent agreement with those of Protopopescu et al.⁵⁾. The error on δ_2^0 is predominantly due to the error on the elastic partial decay width of the f_0 resonance, the tail of which is responsible for half of the value of δ_2^0 at 1 GeV.

In our solutions, the $I = 2$ s wave phase shift is correlated to the behaviour of δ_0^0 , particularly for the smaller values of a_0^0 , but the experimental information (iii) is not yet precise enough to determinate between our solutions - see Fig. 2b. The allowed region or "universal curve" in the (a_0^0, a_0^2) plane is shown as a full line in Fig. 3a. The surrounding shaded band indicates the uncertainty in our results obtained by varying the rho width by 25 MeV, the driving terms within their estimated errors and the s wave phase shifts from those of Baton et al.³⁾ to those of Protopopescu et al.⁵⁾. Variation in the behaviour of the s wave above the $K\bar{K}$ threshold

causes the major uncertainty in our results ^{*}) and the dot-dashed line in Fig. 3a shows the effect of replacing curve I of Fig. 1 by curve II in our calculations. Our results are quite insensitive to the behaviour of the p wave above 1 GeV and, for example, one could include an inelastic ρ' degenerate with the f_0 meson, having a full width of 240 MeV and partial $\pi\pi$ width of 85 MeV, without altering our results.

The universal curve of Fig. 3a is similar to that of Morgan and Shaw ¹⁹⁾. However, we find that it is not possible to obtain such a universal curve from the existence of the rho meson and the behaviour of the s wave in the ρ region alone. In fact, if we use only the experimental information (i)-(iii) and allow arbitrary behaviour above 900 MeV, we obtain a broad band of solutions - e.g., for fixed a_0^2 , values of a_0^0 are allowed in a range of width $\Delta a_0 = 0.15$. It is necessary to fix the behaviour near the $K\bar{K}$ threshold before we obtain an essentially one parameter set of solutions of finite extent. We note here that the $I = 2$ information (iii) is not used as an input but is checked a posteriori. In Ref. 19), a unique solution below 850 MeV is chosen, by taking a value for a_0^0/a_0^2 from low energy data then available, and a value $a_0^0 = 0.16 \pm 0.04$ obtained. This corresponds to our central solution (2) in Fig. 2 and agrees with the Weinberg ²⁰⁾ - Lovelace ¹⁷⁾ models. However, the ratio a_0^0/a_0^2 is not reliably determined and, at present, there are three conflicting sources of information on the value of a_0^0 , which essentially cover the region obtained from our analysis. In addition to the evidence ²¹⁾ supporting the Weinberg value, the K_{e4} results ²²⁾ and $\pi^0\pi^0$ data ²³⁾ favour a higher value for a_0^0 similar to our solution 1, while a recent low energy πN data analysis ²⁴⁾ favours a lower value of the type corresponding to our solution 3. These three solutions (subject to our discussion of the errors due to uncertainties in the input) span the differences allowed by our present knowledge of the $\pi\pi$ system.

We now consider whether it is possible to restrict our set of $\pi\pi$ amplitudes further, by requiring a "reasonable" high energy behaviour. For this purpose, it is useful to discuss the asymptotic contribution L_ω (defined in A) to the Olsson sum rule ²⁵⁾ for s wave scattering lengths. Including Regge ρ exchange with the Lovelace-Veneziano residue function

^{*}) However, we note that the precise value of $\delta_0^0(2M_K)$ is of minor importance.

above the cut-off N and the f_0 meson below N , we obtain for the expected phenomenological value $L_\omega = 0.23 \pm 0.08$. The error is estimated by allowing a 50% variation in the magnitude of the $\mathcal{P}$ residue function at $t = 0$. In Fig. 3b we show the values of L_ω obtained for our set of solutions, with the same notation for errors estimated from uncertainties in the input as in Fig. 3a. Our estimated range of phenomenologically allowed values for L_ω is marked by dotted lines. We see that all of our solutions are compatible with this sum rule *).

Finally, we discuss how future experimental information will select between our various solutions. We note that the errors on $\delta_0^0(500)$, the isoscalar phase shift at 500 MeV, for the three solutions of Table I are predominantly due to the experimental uncertainty on δ_0^0 in the rho region - see Fig. 2a. A knowledge of δ_0^0 from 500 to 900 MeV with a true uncertainty of $\pm 3^\circ$ would allow us to select a_0^0 with an uncertainty of 0.05 for the lower values of a_0^0 and of 0.15 for the higher values of a_0^0 . An important check on such a result would come from measuring δ_0^2 with a similar accuracy and using our predicted correlations between δ_0^0 and δ_0^2 . The final confirmation would come from an accurate K_{e4} experiment with known (small) systematic errors. All these data should gradually become available within the next few years.

A more complete account of this work will be given elsewhere.

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*) There is a tendency for the sum rule to discriminate in favour of solutions with the behaviour I of Fig. 1 rather than II above the $K\bar{K}$ threshold.

Solution	a_0^0	a_0^2	a_1^1	$\delta_0^2(M_\rho)$	$\delta_0^0(500)$	m_ϵ (MeV)	Γ_ϵ (MeV)
1	-0.05	-0.115 ± 0.005	0.034 ± 0.002	$-22^\circ \pm 3^\circ$	$24^\circ \pm 5^\circ$	550 ± 60	820 ± 150
2	0.16 ± 0.02	-0.048 ± 0.003	0.035 ± 0.002	$-16^\circ \pm 3^\circ$	$30^\circ \pm 5^\circ$	480 ± 120	950 ± 300
3	0.60	0.043 ± 0.004	0.041 ± 0.002	$-12^\circ \pm 3^\circ$	$42^\circ \pm 3^\circ$	350 ± 300	$900 < \Gamma_\epsilon < \infty$

TABLE I : Values of the scattering lengths, δ_0^2 at the rho mass, δ_0^0 at 500 MeV and the mass m_ϵ and width Γ_ϵ corresponding to the isoscalar resonance pole position for the three solutions of Fig. 2. Errors shown have the same origin as the bands in Fig. 3. The effect of using the S^* curve II rather than curve I of Fig. 1 is as follows : to increase a_0^2 by 0.01 (for fixed a_0^0 - see Fig. 3a), to decrease a_1^1 by 0.003 and to increase $\delta_0^2(M_\rho)$ by 3° . There is no clear effect for $\delta_0^0(500)$, m_ϵ and Γ_ϵ .

Solution	a_2^0	a_2^2	a_3^1	$\delta_2^0(1 \text{ GeV})$	$\delta_2^2(1 \text{ GeV})$	$\delta_3^1(1 \text{ GeV})$
1	0.0014 ± 0.0001	$(-2.5 \pm 1.0) \times 10^{-4}$	$(1.5 \pm 0.2) \times 10^{-5}$	$11^\circ \pm 2^\circ$	$-2.2^\circ \pm 0.2^\circ$	$0.54^\circ \pm 0.05^\circ$
2	0.0014 ± 0.0001	$(-1.5 \pm 1.0) \times 10^{-4}$	$(4.0 \pm 1.0) \times 10^{-5}$	$11^\circ \pm 2^\circ$	$-2.2^\circ \pm 0.2^\circ$	$0.57^\circ \pm 0.05^\circ$
3	0.0018 ± 0.0002	$(5.3 \pm 2.0) \times 10^{-4}$	$(12 \pm 3) \times 10^{-5}$	$11^\circ \pm 2^\circ$	$-2.2^\circ \pm 0.2^\circ$	$0.64^\circ \pm 0.05^\circ$

TABLE II : d and f wave scattering lengths and phase shifts from the partial wave relations for $\ell = 2$ and 3.

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FIGURE CAPTIONS

Figure 1 Behaviour of $f_0^0(s)$ above the $K\bar{K}$ threshold as given by :

- (a) Protopopescu et al. ⁵⁾ (data points) corresponding to $M_{S^*} = 980 \pm 5$ MeV, $\Gamma_{S^*} = 74 \pm 16$ MeV,
- (b) a qualitative one pole fit (curve I) to those data using our own parametrization giving $M_{S^*} = 996$ MeV, $\Gamma_{S^*} = 131$ MeV, and
- (c) a form (curve II) corresponding to total absorption at 1033 MeV as described by one of the solutions of Beusch ¹⁶⁾ and thus having $M_{S^*} = 1033$ MeV, $\Gamma_{S^*} = 0$. In all three cases the S^* pole is situated on the second sheet reached by going through the elastic 2π cut below the $K\bar{K}$ threshold.

Figure 2 Typical solutions for :

- (a) δ_0^0 (together with the input η_0^0 from I of Fig. 1), and
- (b) δ_0^2 .

Data points for :

- (a) Baton et al. ³⁾ ; Protopopescu et al. ⁵⁾ ;
Baillon et al. ⁴⁾ ; Sonderegger et al. ²³⁾ ;
Bensinger et al. ²³⁾, and Zylbersztejn et al. ²²⁾ ;
- (b) Baton et al. ³⁾ ; Colton et al. ⁸⁾ ;
Katz et al. ⁷⁾, and Walker et al. ⁶⁾.

Figure 3 (a) Allowed region in the (a_0^0, a_0^2) plane. The solid and dot-dashed curves correspond to models I and II respectively of Fig. 1 for the S^* effect. The shaded band indicates our estimated errors due to other uncertainties in the input (see text). We also indicate the Weinberg predictions.

(b) Asymptotic contributions L_ω to the Olsson sum rule with the same notation as in (a). The dotted lines indicate the range expected phenomenologically.

FIG.1

FIG. 2a

FIG.2b

FIG. 3a