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LIGHT CONE BEHAVIOUR IN THE RELATIVISTIC QUARK
OSCILLATOR MODEL

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ABSTRACT

The light cone expansion of the commutators of bilocal currents is investigated in the relativistic oscillator quark model proposed by Feynman, Kislinger and Ravndal. An infinite component field theory formalism describing the oscillator bound states of quarks is introduced in order to define the current operators. The calculation of the light cone expansions of current commutators shows that the elastic binding force, acting between the quarks, does not affect the light cone behaviour, which remains the same as in the free quark model. The explicit forms of the deep inelastic structure functions W_1 , W_2 are also calculated in the free infinite component field theory model with an arbitrary number N of oscillating quarks ($N = 2$: mesons, $N = 3$: baryons, $N > 3$: exotics). The scaling limits of the structure functions are sums of δ -function terms coming from the individual quarks even if the instability of excited states is taken into account by giving the trajectory an imaginary part. This indicates that non-trivial scaling functions are possible only in a model which accounts better for strong interactions.

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1. INTRODUCTION

The observed scaling behaviour of the structure functions in deep inelastic electron-proton scattering can be understood if the light cone structure of the electromagnetic current commutators is the same as in the free quark model. This led Gell-Mann and Fritzsche to postulate that the free quark light cone algebra of bilocal vector and axial vector currents is of universal validity¹⁾ [see also the work of Cornwall and Jackiw²⁾].

The interaction among the quarks does not affect the current commutators near the light cone, the light cone being a region of very high symmetry. This is in spite of the fact that in perturbation theory, when non-super renormalizable interactions are switched on, anomalous light cone singularities appear, together with logarithmic terms spoiling the scaling behaviour³⁾. The free-quark light-cone structure is preserved, however, in gluon models with the formal use of the canonical equal time commutation relations and field equations of motion^{2,4)}.

In the present paper, we investigate the light cone commutators in another kind of field theory, namely the "relativistic quark model" of Feynman, Kislinger and Ravndal⁵⁾. In this model the quarks are bound by a relativistic harmonic oscillator potential. The field describing the infinity of oscillator bound states is an infinite component field analogous to those considered in dual oscillator models. We define the current operator (in Section 2) in terms of the infinite component field and calculate the leading light cone singularity of the commutators. We find that near the light cone, the $Su(3) \otimes Su(3)$ algebra postulated by Gell-Mann and Fritzsche is preserved. In deriving this we extensively use the ideas and methods developed recently in a paper of Bbi, Gorg and Leutwyler⁶⁾, where the light cone structure of certain infinite component field theories was studied. Actually (as we shall see later on) the infinite component oscillator wave equation, studied in Ref. 6, describes the motion of one of the constituents in an oscillator bound state of two scalar constituents. Thus our results go beyond these of Ref. 6 in that we consider the physically more relevant case of two, three (or more) spin $\frac{1}{2}$ quarks and that our currents are given as the sums of the currents belonging to the individual quarks as defined in Ref. 5.

The explicit form of the structure functions is also calculated (in Section 3) in the "free"*) theory. To be as general as possible this is done in the case of N oscillating quarks, $N = 2$ giving the (ordinary) mesons, $N = 3$ (ordinary) baryons and $N > 3$ giving exotic hadrons**). The scaling limit of the structure functions is consistent with the picture that in deep inelastic collisions the quarks seem to carry a fraction of the hadron's momentum fixed by the ratio of their (bare) masses. The scaling functions are given, therefore, as sums of δ -function terms

*) Free in the sense that no strong interactions of the bound states are considered.

**) In Section 2 only the case of $N = 3$ baryons is considered for simplicity; nevertheless the light cone algebra can also be proved for arbitrary N .

coming from the individual quarks' contributions. It is obvious that in developing the observed scaling functions the role of strong interactions is very important. Our results show that the simple procedure to give a width to the resonances and to average over the resonances contributing in an interval of missing mass squared is not enough for producing in the oscillator model realistic scaling functions^{*)}. The paper contains yet a final section (Section 4), where some conclusions and comments are collected.

2. THE LIGHT CONE STRUCTURE OF THE COMMUTATORS

In order to study the harmonic oscillator bound states of three equal mass quarks, let us denote the position four vectors and the four momenta of the three quarks by x_1, x_2, x_3 and p_1, p_2, p_3 , respectively. The centre-of-mass four vector x and relative position four vectors z_1, z_2 are then defined as

$$\begin{aligned} x &= \frac{1}{3}(x_1 + x_2 + x_3) & x_1 &= x + \frac{1}{2}z_1 + \frac{1}{3}z_2 \\ z_1 &= x_1 - x_2 & x_2 &= x - \frac{1}{2}z_1 + \frac{1}{3}z_2 \\ z_2 &= \frac{1}{2}(x_1 + x_2) - x_3 & x_3 &= x - \frac{2}{3}z_2 \end{aligned} \quad (1)$$

The canonically conjugate momenta are p (the total four momentum) and q_1, q_2 ;

$$\begin{aligned} p &= p_1 + p_2 + p_3 = i \frac{\partial}{\partial x} & p_1 &= \frac{1}{3}p + q_1 + \frac{1}{2}q_2 = i \frac{\partial}{\partial x_1} \\ q_1 &= \frac{1}{2}(p_1 - p_2) = i \frac{\partial}{\partial z_1} & p_2 &= \frac{1}{3}p - q_1 + \frac{1}{2}q_2 = i \frac{\partial}{\partial x_2} \\ q_2 &= \frac{1}{3}(p_1 + p_2) - \frac{2}{3}p_3 = i \frac{\partial}{\partial z_2} & p_3 &= \frac{1}{3}p - q_2 = i \frac{\partial}{\partial x_3} \end{aligned} \quad (2)$$

The mass squared operator according to Ref. 1 is the following

$$\begin{aligned} M^2 &= m_\alpha^2 - 6q_1^2 - \frac{9}{2}q_2^2 - \omega^2 \left(\frac{1}{24}z_1^2 + \frac{1}{18}z_2^2 \right) = \\ &= m_\alpha^2 - 6q_1^2 - \frac{9}{2}q_2^2 - \frac{\omega^2}{36} \left[(x_1 - x_2)^2 + (x_2 - x_3)^2 + (x_3 - x_1)^2 \right]. \end{aligned} \quad (3)$$

The field equation corresponding to this M^2 may be considered as the generalization of the Klein-Gordon equation for the case of relativistic composite systems. It reads as

$$K\psi = (\Box_x^2 + M^2)\psi = \left\{ \Box_x + m_\alpha^2 + 6\Box_{z_1} + \frac{9}{2}\Box_{z_2} - \omega^2 \left(\frac{1}{24}z_1^2 + \frac{1}{18}z_2^2 \right) \right\} \psi(x, z_1, z_2) = 0. \quad (4)$$

*) At least in the case where the original Gaussian form factors are kept. If dipole-like form factors are introduced by hand [see F. Ravndal⁷⁾] then a fit to the scaling functions is expected already on the grounds of the paper by G. Domokos et al.⁷⁾, where dipole form factors and an oscillator-like spectrum were taken phenomenologically and the scaling functions were obtained in good agreement with experiments.

This equation belongs to the general type of bilocal (here "trilocal") field equations introduced 20 years ago by Yukawa⁸⁾ for the description of composite particles. More specifically, in the quark model, equations like Eq. (4) were considered by a number of authors as possible relativistic generalizations of the non-relativistic quark model⁹⁾.

There is also an infinite component field equation equivalent to Eq. (4). It is obtained if the operator $\psi(x, z_1, z_2)$ is considered as the representative of an element of the (infinite dimensional) Hilbert space H_z . The elements of H_z characterize the internal motion of the bound system and they will be denoted by kets like $|\psi\rangle$ ^{*}). This settled, the field operator $\psi(x, z_1, z_2)$ is given by

$$\psi(x, z_1, z_2) = (z_1, z_2 | \psi(x)) \quad (5)$$

and the field equation Eq. (4) goes over into the infinite component field equation

$$(\square_x + m_0^2 - \omega a_1^+ \cdot a_1 - \omega a_2^+ \cdot a_2) |\psi(x)\rangle = 0 \quad (6)$$

Here $m_0^2 = m_a^2 + 4\omega$, and the creation and annihilation operators of the internal oscillations a_i^+ , a_i ($i = 1, 2$) are defined by

$$a_1 = z_1 \sqrt{\frac{\omega}{24}} + i q_1 \sqrt{\frac{6}{\omega}} \quad a_2 = z_2 \sqrt{\frac{\omega}{18}} + i q_2 \sqrt{\frac{9}{2\omega}} \quad (7)$$

The internal $Su(3)$ quantum numbers A_1, A_2, A_3 of the three quarks and their Dirac indices $\alpha_1, \alpha_2, \alpha_3$ are understood in Eq. (6), that is it should read in detail as

$$(\square_x + m_0^2 - \omega a_1^+ \cdot a_1 - \omega a_2^+ \cdot a_2) |\psi(x)^{A_1 \alpha_1, A_2 \alpha_2, A_3 \alpha_3}\rangle = 0 \quad (8)$$

In what follows the indices A_i, α_i will be omitted in general but they will always be understood.

Our aim is to write down the current operators in terms of the field $|\psi(x)\rangle$. As the charges associated with currents are carried by the individual quarks the current operators are expressed as the sum of three terms each coming from a single quark. In order to obtain the currents of the quarks it is useful to introduce the fields describing the motion of individual quarks. The field $|\psi(x)\rangle$ depends on the centre-of-mass coordinate; therefore it is associated with the centre-of-mass motion

^{*}) To avoid confusion the states of the whole system (including the centre-of-mass motion) will be denoted by kets like $|\psi\rangle$.

of the whole system. In order to have a field $|\psi_1(x_1)\rangle$ depending, say, on x_1 (the position of the first quark) let us define the trilocal field depending on x_1, z_1, z_2 instead of x, z_1, z_2 :

$$\psi_1(x_1, z_1, z_2) \equiv \psi(x_1 - \frac{1}{2}z_1 - \frac{1}{3}z_2, z_1, z_2) . \quad (9)$$

By virtue of Eqs. (1) and (4) the field $\psi_1(x_1, z_1, z_2)$ obeys the field equation

$$\left\{ 3\Box_{x_1} + m_a^2 - 2i \frac{\partial}{\partial x_1} \cdot \left(3i \frac{\partial}{\partial z_1} + \frac{3}{2} i \frac{\partial}{\partial z_2} \right) + 6\Box_{z_1} + \frac{9}{2}\Box_{z_2} - \omega^2 \left(\frac{1}{24}z_1^2 + \frac{1}{18}z_2^2 \right) \right\} \psi_1(x_1, z_1, z_2) = 0. \quad (10)$$

The infinite component field $|\psi_1(x_1)\rangle$ corresponding to $\psi_1(x_1, z_1, z_2)$ is defined by

$$\psi_1(x_1, z_1, z_2) = (z_1, z_2 | \psi_1(x_1)) . \quad (11)$$

The field equation for $|\psi_1(x_1)\rangle$ can be obtained from Eq. (10):

$$\left(3\Box_{x_1} + M^2 - 2i \frac{\partial}{\partial x_1} \cdot G_{(1)} \right) |\psi_1(x_1)\rangle = 0. \quad (12)$$

For brevity, here the following notations were introduced

$$M^2 = m_0^2 - \omega a_1^+ \cdot a_1 - \omega a_2^+ \cdot a_2 , \quad (13)$$

$$G_{(1)} = \frac{\sqrt{\omega}}{2} \left[i(a_1^+ - a_1) \sqrt{\frac{3}{2}} + i(a_2^+ - a_2) \frac{1}{\sqrt{2}} \right] = 3q_1 + \frac{3}{2}q_2 .$$

The equations describing the motion of the other quarks can be derived analogously. They have the same form as Eq. (12), namely

$$\left(3\Box_{x_k} + M^2 - 2i \frac{\partial}{\partial x_k} \cdot G_{(k)} \right) |\psi_k(x_k)\rangle = 0 \quad (k=1, 2, 3). \quad (14)$$

The quantities $G_{(k)}$ ($k = 2, 3$) are given by

$$G_{(2)} = \frac{\sqrt{\omega}}{2} \left[-i(a_1^+ - a_1) \sqrt{\frac{3}{2}} + i(a_2^+ - a_2) \frac{1}{\sqrt{2}} \right] = -3q_1 + \frac{3}{2}q_2$$

$$G_{(3)} = \frac{\sqrt{\omega}}{2} \left[-i(a_2^+ - a_2) \sqrt{2} \right] = -3q_2 . \quad (15)$$

Equation (14) is of the same type of oscillator equation as that studied by Leutwyler¹⁰⁾. Here we see that the Leutwyler equations describe the motions of individual constituents in the oscillator bound state. It is easy to connect the solutions of Eqs. (6) and (14). This is what is called by Leutwyler the "tilting operation"¹⁰⁾.

$$\begin{aligned}
 |\psi_k(x_k)\rangle &= \exp\{-2D_{(k)} \cdot \vec{\partial}_{x_k}\} |\psi(x_k)\rangle \\
 D_{(1)} &= \frac{1}{2\sqrt{\omega}} \left[(a_1^+ + a_1) \sqrt{\frac{3}{2}} + (a_2^+ + a_2) \frac{1}{\sqrt{2}} \right] = \frac{1}{4} z_1 + \frac{1}{6} z_2 \\
 D_{(2)} &= \frac{1}{2\sqrt{\omega}} \left[-(a_1^+ + a_1) \sqrt{\frac{3}{2}} + (a_2^+ + a_2) \frac{1}{\sqrt{2}} \right] = -\frac{1}{4} z_1 + \frac{1}{6} z_2 \\
 D_{(3)} &= \frac{1}{2\sqrt{\omega}} \left[-(a_2^+ + a_2) \sqrt{2} \right] = -\frac{1}{3} z_2 .
 \end{aligned} \tag{16}$$

The equivalence of Eqs. (6) and (14) can be seen from Eq. (16) on the grounds of the commutation relations following from Eqs. (1) and (2):

$$[G_{(k)\mu}, D_{(k)}] = ig_{\mu\nu} \left\{ \delta_{k\nu} - \frac{1}{2} (1 - \delta_{k\nu}) \right\} . \tag{17}$$

The quarks have spin $\frac{1}{2}$ and Eq. (14) does not involve spin. In order to derive the current operator we take into account the spin $\frac{1}{2}$ character of quarks in the same way as was done in Ref. 5, namely we interpret $\square_{x_k}$ as $\gamma \cdot \partial_{x_k} \gamma \cdot \partial_{x_k}$. This changes the equation in no way, but does change the perturbational effects if an electromagnetic potential $A_\mu(x)$ is switched on. Namely, the Lagrangian of the "free" field equation (14) is then

$$\begin{aligned}
 I_k &= (\tilde{\psi}_k(x_k) | 3 \vec{\partial}_{x_k} \cdot \gamma^{(k)} \vec{\partial}_{x_k} \cdot \gamma^{(k)} - M^2 + i(\vec{\partial}_{x_k} - \vec{\partial}_{x_k}) \cdot G_{(k)} | \psi_k(x_k)) \\
 (\tilde{\psi}_k(x_k)_{A_1 \alpha_1, A_2 \alpha_2, A_3 \alpha_3} | &\equiv (\psi_k^+(x_k)_{A_1 \beta_1, A_2 \beta_2, A_3 \beta_3} | (\gamma_0)_{\beta_1 \alpha_1} (\gamma_0)_{\beta_2 \alpha_2} (\gamma_0)_{\beta_3 \alpha_3} .
 \end{aligned} \tag{18}$$

The notation $\gamma^{(k)}$ means that the γ -matrix acts on the k^{th} index of the field operator. When an electric potential A_μ is present, we replace $i\vec{\partial}_{x_k}$ by $i\vec{\partial}_{x_k} + e_k A(x_k)$ and $-i\vec{\partial}_{x_k}$ by $-i\vec{\partial}_{x_k} + e_k A(x_k)$, e_k being the charge of quark k . The first order perturbation of L_k defines then the current operator of the k^{th} quark as

$$j^{(k)}(x_k)_\mu = e_k (\tilde{\psi}_k(x_k) | 3 \gamma_\mu i \vec{\partial}_{x_k} \cdot \gamma^{(k)} - 3i \vec{\partial}_{x_k} \cdot \gamma^{(k)} \gamma_\mu + 2 G_{(k)\mu} | \psi_k(x_k)) . \tag{19}$$

Another useful form of the current operator is obtained from Eqs. (16) and (17), namely

$$j^{(k)}(x)_\mu = e_k (\tilde{\psi}(x) | \left[\exp \{ -2 D_{(k)} \cdot (\vec{\partial}_x + \vec{\partial}_x) \} \gamma_\mu^{(k)} \gamma_\nu^{(k)} (i \vec{\partial}_x^\nu + G_{(k)}^\nu) + \right. \\ \left. + (-i \vec{\partial}_x^\nu + G_{(k)}^\nu) \gamma_\nu^{(k)} \gamma_\mu^{(k)} \exp \{ -2 D_{(k)} \cdot (\vec{\partial}_x + \vec{\partial}_x) \} \right] | \psi(x)). \quad (20)$$

Actually, the current in Ref. 5 was given in a form equivalent to this latter expression. This can be seen from the definitions of $D_{(k)}$ and $G_{(k)}$ in Eqs. (13), (15) and (16).

For the vector $J_{i\mu}$ and axial-vector $J_{i\mu}^5$ current operators of $Su(3) \otimes Su(3)$ we need merely to replace $e_k \gamma_\mu^{(k)}$ in Eq. (20) by $[\lambda_i^{(k)}/2] \gamma_\mu^{(k)}$ or $[\lambda_i^{(k)}/2] \gamma_\mu^{(k)} i \gamma_5^{(k)}$ and sum over $k = 1, 2, 3$ as the total current is the sum of the currents of the individual quarks:

$$J_{i\mu}(x) = \sum_{k=1}^3 (\tilde{\psi}(x) | \frac{\lambda_i^{(k)}}{2} \left[\exp \{ -2 D_{(k)} \cdot (\vec{\partial}_x + \vec{\partial}_x) \} \gamma_\mu^{(k)} \gamma_\nu^{(k)} (i \vec{\partial}_x^\nu + G_{(k)}^\nu) + \right. \\ \left. + (-i \vec{\partial}_x^\nu + G_{(k)}^\nu) \gamma_\nu^{(k)} \gamma_\mu^{(k)} \exp \{ -2 D_{(k)} \cdot (\vec{\partial}_x + \vec{\partial}_x) \} \right] | \psi(x)) \\ J_{i\mu}^5(x) = \sum_{k=1}^3 (\tilde{\psi}(x) | \frac{\lambda_i^{(k)}}{2} \left[\exp \{ -2 D_{(k)} \cdot (\vec{\partial}_x + \vec{\partial}_x) \} \gamma_\mu^{(k)} i \gamma_5^{(k)} \gamma_\nu^{(k)} (i \vec{\partial}_x^\nu + G_{(k)}^\nu) + \right. \\ \left. + (-i \vec{\partial}_x^\nu + G_{(k)}^\nu) \gamma_\nu^{(k)} \gamma_\mu^{(k)} i \gamma_5^{(k)} \exp \{ -2 D_{(k)} \cdot (\vec{\partial}_x + \vec{\partial}_x) \} \right] | \psi(x)). \quad (21)$$

The commutators of the currents in Eq. (21) are determined by the anti-commutators of the fields $|\psi(x)\rangle$ and $(\tilde{\psi}(x)|$. The quantization rules for the infinite component field $|\psi(x)\rangle$ were discussed elsewhere^{11,12)}, where also the currents equivalent to those in Eq. (20) were given, expressed through fields like $\psi(x, z_1, z_2)$. In a shorthand notation the commutation relation given in Refs. 11 and 12 can be written as

$$\{ |\psi(x)\rangle, (\tilde{\psi}(y)| \} = i \Delta(x-y; M^2) = \\ = \frac{1}{2\pi i} \varepsilon(x_0 - y_0) \left\{ \delta((x-y)^2) - \frac{M^2}{2} \theta((x-y)^2) \frac{J_1(M\sqrt{(x-y)^2})}{M\sqrt{(x-y)^2}} \right\} = \\ = \frac{1}{2\pi i} \varepsilon(x_0 - y_0) \left\{ \delta((x-y)^2) - \frac{M^2}{4} \theta((x-y)^2) + \frac{M^4}{32} \theta((x-y)^2) (x-y)^2 + \dots \right\}. \quad (22)$$

Here M^2 means the mass squared operator as defined in Eq. (13), that is the anti-commutator is an operator in the internal Hilbert space H_z^*). For the determination of light cone behaviour of the current commutators, we first prove the following statements: that near the light cone $(x - y)^2 = 0$

*) In very much the same way as, for example, the propagator is an operator in dual resonance models.

$$\exp\{-2D_{(k)} \cdot \vec{\partial}_x\} (i\vec{\partial}_x^e + G_{(k)}^e) i\Delta(x-y; M^2) \exp\{-2D_{(k)} \cdot \vec{\partial}_y\} \sim \partial_x^e \left\{ \frac{1}{2\pi} \varepsilon(x_0 - y_0) \delta((x-y)^2) \right\} \quad (23)$$

$$\exp\{-2D_{(k)} \cdot \vec{\partial}_x\} \gamma_\mu (i\vec{\partial}_x^e + G_{(k)}^e) i\Delta(x-y; M^2) \gamma_\mu (-i\vec{\partial}_y^e + G_{(k)}^e) \exp\{-2D_{(k)} \cdot \vec{\partial}_y\} \sim \varepsilon(x_0 - y_0) \delta((x-y)^2)$$

For the proof of, say, the first equality let us sandwich its left-hand side between the elements of $H_Z(z_1, z_2 |$ and $|z'_1, z'_2\rangle$, which are according to Eq. (16) eigenstates of $D_{(k)}$. Using also Eqs. (13) and (15) we have, for instance for $k = 1$:

$$\begin{aligned} & \exp\left\{-\left[(z_1 - z'_1)\frac{1}{2} + (z_2 - z'_2)\frac{1}{3}\right] \cdot \vec{\partial}_x\right\} (i\vec{\partial}_x^e + i\vec{\partial}_{z_1}^e \frac{1}{3} + i\vec{\partial}_{z_2}^e \frac{2}{3}) (z_1, z_2 | i\Delta(x-y; M^2) | z'_1, z'_2) = \\ & = \lim_{\lambda \rightarrow 1} \exp\left\{-\lambda \left[(z_1 - z'_1)\frac{1}{2} + (z_2 - z'_2)\frac{1}{3}\right] \cdot \vec{\partial}_x\right\} [i\vec{\partial}_x^e + \lambda(i\vec{\partial}_{z_1}^e \frac{1}{3} + i\vec{\partial}_{z_2}^e \frac{2}{3})] (z_1, z_2 | i\Delta(x-y; M^2) | z'_1, z'_2). \end{aligned} \quad (24)$$

The parameter λ in the second expression was introduced for handling the series resulting from the expansion in Eq. (22), which we take term by term. Using $(z_1, z_2 | z'_1, z'_2) = \delta(z_1 - z'_1) \delta(z_2 - z'_2)$ and the expansions

$$[(\partial_z)^k \delta(z - z')] f(z) \sim \delta(z - z') (-\partial_z)^k f(z) + \dots \quad M^2 \sim 6\Box_{z_1} + \frac{9}{2}\Box_{z_2} + \dots \quad (25)$$

valid in Eq. (24) near the light cone, we get for the leading light cone behaviour

$$\partial_x^e \left(\frac{1}{2\pi} \varepsilon(x_0 - y_0) \delta((x-y)^2) \right) \delta(z - z') \sum_{k=0}^{\infty} (1 + 2\lambda^2) (-2\lambda^2)^k = \partial_x^e \left\{ \frac{1}{2\pi} \varepsilon(x_0 - y_0) \delta((x-y)^2) \right\} \delta(z - z'). \quad (26)$$

The series was summed here for λ^2 sufficiently small ($\lambda^2 < 1/2$) and the resulting one was "continued" back to $\lambda = 1$, as required by Eq. (24). The proof of the second equality proceeds along exactly the same lines.

From Eq. (23) the light cone commutators of the vector and axial vector currents are easily seen to coincide with the Gell-Mann, Fritzsche algebra¹⁾. The only thing to add to Eq. (23) is that the commutators of the currents of different quarks do not contribute to the leading light cone singularity. This is caused by the fact that if we look at expressions like Eqs. (23) for indices k different on the two sides of $i\Delta$ then a factor like $\exp\{(z_1 - z'_1) \cdot \vec{\partial}_x \text{ const}\}$ always survives shifting the argument of $\varepsilon(x_0 - y_0) \delta((x-y)^2)$ in a z -dependent way. Hence the z -integration involved in matrix elements of $\{|\psi(x)\rangle \langle \tilde{\psi}(y)|\}$ washes out all the δ -function

singularities from the commutator function. The commutator of two bilocal currents can be handled in the same way as the current commutators themselves, hence the whole bilocal $Su(3) \otimes Su(3)$ light cone algebra is the same also in the quark oscillator model as in the free quark model.

3. THE SCALING LIMIT OF THE STRUCTURE FUNCTIONS

The Bjorken scaling of the deep inelastic structure functions follows from the light cone algebra proved in the preceding section. The explicit form of the scaling functions, however, depends on the matrix elements of bilocal currents; therefore it is interesting to see also the exact form of scaling functions given by the model. According to the ideas underlying the parton model¹³⁾, we expect that in deep inelastic collisions the internal motion of quarks is negligible and the bound quarks behave as free particles. We shall see below that the scaling functions correspond to this expectation.

To treat the case of mesons, baryons and exotics together, we consider the oscillator bound state of N quarks. For generality we allow also the masses of quarks to be different. If the four-position of the k^{th} particle is x_k then the centre-of-mass coordinate x is, in general,

$$x = \sum_{i=1}^N \xi_i x_i \quad \sum_{i=1}^N \xi_i = 1 \quad (27)$$

Here ξ_k is the fraction of the total mass belonging to the k^{th} quark. Denoting the relative coordinates by z_ℓ ($\ell = 1, \dots, n \equiv N-1$), the general expression for the coordinate x_k is

$$x_k = x + \sum_{\ell=1}^n c_{k\ell} z_\ell \quad (k=1, \dots, N) \quad (28)$$

with some matrix $c_{k\ell}$. For instance, in the model proposed in an earlier paper¹⁴⁾ $c_{k\ell}$ is given by

$$c_{k\ell} = \begin{pmatrix} \frac{\xi_\ell}{\xi_{12}} & \frac{\xi_3}{\xi_{13}} & \dots & \frac{\xi_N}{\xi_{1N}} \\ -\frac{\xi_1}{\xi_{12}} & \frac{\xi_3}{\xi_{13}} & \dots & \frac{\xi_N}{\xi_{1N}} \\ 0 & -\frac{\xi_{12}}{\xi_{13}} & \dots & \frac{\xi_N}{\xi_{1N}} \\ \dots & \dots & \dots & \dots \\ 0 & \dots & -\frac{\xi_{1n-1}}{\xi_{1n}} & \frac{\xi_N}{\xi_{1N}} \\ 0 & \dots & 0 & -\frac{\xi_{1n}}{\xi_{1N}} \end{pmatrix} \quad (29)$$

$\xi_{ij} \equiv \xi_i + \xi_{i+1} + \dots + \xi_j$.

In this specific model the k^{th} quark oscillates with a frequency ω_k around the centre-of-mass of the first k quarks. We note, however, that for our purposes the explicit form of $c_{k\ell}$ is not needed; we use only the relation^{*)}

$$1 + \sum_{\ell=1}^n \frac{c_{k\ell}^2}{\mu_\ell} = \frac{1}{\xi_k} \quad \mu_\ell \equiv \frac{\xi_{1\ell} \xi_{\ell+1}}{\xi_{1\ell+1}} . \quad (30)$$

In order to write down the infinite component field equation of the system, we define the creation operator of the ℓ^{th} oscillator mode as

$$a_\ell^+ = \frac{1}{2} \sqrt{\omega_\ell \mu_\ell} z_\ell + \frac{1}{\sqrt{\omega_\ell \mu_\ell}} \frac{\partial}{\partial z_\ell} . \quad (31)$$

The infinite component field equation describing the motion of the whole system is then

$$\left(\square_x - \sum_{\ell=1}^n \omega_\ell a_\ell^+ a_\ell + m_0^2 \right) |\psi(x)\rangle \equiv (\square_x + M^2) |\psi(x)\rangle = 0 . \quad (32)$$

This equation contains Eq. (6) for the baryons as a special case, when $N = 3$, $\omega_1 = \omega_2 = \omega$, $\xi_1 = \xi_2 = \xi_3 = 1/3$. The equation of motion of the k^{th} quark is obtained in the same way as Eq. (14) above:

$$\left\{ \frac{1}{\xi_k} \square_{x_k} + M^2 - 2i \frac{\partial}{\partial x_k} \cdot G_{(k)} \right\} |\psi_k(x_k)\rangle = 0 . \quad (33)$$

The quantities $G_{(k)}$ are given by

$$G_{(k)} = \sum_{\ell=1}^n c_{k\ell} \frac{i}{2} \sqrt{\frac{\omega_\ell}{\mu_\ell}} (a_\ell^+ - a_\ell) . \quad (34)$$

The electromagnetic current operator of the whole system has now the following form:

$$j^{(x)}_\mu = \sum_{k=1}^N e_k \langle \tilde{\psi}(x) | \left[\exp \left\{ \left(1 - \frac{1}{\xi_k} \right) D_{(k)} \cdot (\vec{\xi}_x + \vec{\partial}_x) \right\} \gamma_\mu^{(k)} \gamma_\nu^{(k)} (i \vec{\partial}_x + G_{(k)}) + \right. \right. \\ \left. \left. + (-i \vec{\partial}_x + G_{(k)}) \gamma_\nu^{(k)} \gamma_\mu^{(k)} \exp \left\{ \left(1 - \frac{1}{\xi_k} \right) D_{(k)} \cdot (\vec{\xi}_x + \vec{\partial}_x) \right\} \right] | \psi(x) \right\rangle , \quad (35)$$

where $D_{(k)}$ is defined as

$$D_{(k)} = \frac{\xi_k}{1 - \xi_k} \sum_{\ell=1}^n c_{k\ell} \frac{1}{\sqrt{\omega_\ell \mu_\ell}} (a_\ell^+ + a_\ell) ; \quad [G_{(k)\mu}, D_{(k)\nu}] = i g_{\mu\nu} . \quad (36)$$

*) μ_ℓ is the reduced mass belonging to the ℓ^{th} oscillator mode. Equation (30) assumes the diagonalization of the kinetic energy in the variables $x, z_1, \dots, z_n$.

Let us now calculate the structure functions W_1 and W_2 of the ground state of the system $|p\rangle$ (p is the total momentum of the state). First, W_1 and W_2 are defined in the usual way:

$$W_{\rho\sigma} = \frac{1}{4\pi} \int d^4y e^{iqy} \langle p | j(y)_\rho j(0)_\sigma | p \rangle =$$

$$= \left(\frac{q_\rho q_\sigma}{q^2} - g_{\rho\sigma} \right) W_1(q^2, \nu) + \left(p_\rho - \frac{\nu}{q^2} q_\rho \right) \left(p_\sigma - \frac{\nu}{q^2} q_\sigma \right) \frac{W_2(q^2, \nu)}{m_0^2} \quad (37)$$

$$p^2 = m_0^2, \quad \nu = p \cdot q.$$

The excited states of the system are characterized by the occupation numbers n_ℓ of the oscillator modes. According to Eq. (32) the mass squared of the excited states is $m_{(n)}^2 = m_0^2 + \sum_{\ell=1}^n \omega_\ell n_\ell$; therefore the structure function $W_{\rho\sigma}$ has the form

$$W_{\rho\sigma} = \frac{1}{2} \sum_{n_1 \dots n_n=0}^{\infty} \delta(k^2 - m_{(n)}^2) W_{\rho\sigma}^{n_1 \dots n_n}; \quad k = p + q, \quad (38)$$

where $W_{\rho\sigma}^{n_1 \dots n_n}$ is the contribution of the excited state $n_1 \dots n_n$ put in between the two currents in Eq. (37). As the wave functions of the excited states are all explicitly known^{11,12,14)} the calculation of $W_{\rho\sigma}^{n_1 \dots n_n}$ is straightforward. The result is

$$W_1^{n_1 \dots n_n} = \sum_{k, k'=1}^N e_k e_{k'} \exp \left\{ \frac{q^2}{2} \sum_{\ell=1}^n \frac{c_{k\ell}^2 + c_{k'\ell}^2}{\omega_\ell \mu_\ell} \right\} \cdot$$

$$\prod_{\ell=1}^n \left\{ \frac{1}{n_\ell!} \left(-q^2 c_{k\ell} c_{k'\ell} \frac{1}{\omega_\ell \mu_\ell} \right)^{n_\ell} \right\} \sum_{\ell=1}^n \left\{ \frac{n_\ell \omega_\ell^2}{-q^2} + \frac{-q^2}{\omega_\ell \mu_\ell} \right\}, \quad (39)$$

$$W_2^{n_1 \dots n_n} = 4m_0^2 \sum_{k, k'=1}^N e_k e_{k'} \exp \left\{ \frac{q^2}{2} \sum_{\ell=1}^n \frac{c_{k\ell}^2 + c_{k'\ell}^2}{\omega_\ell \mu_\ell} \right\} \prod_{\ell=1}^n \left\{ \frac{1}{n_\ell!} \left(-q^2 c_{k\ell} c_{k'\ell} \frac{1}{\omega_\ell \mu_\ell} \right)^{n_\ell} \right\}.$$

The δ -functions appearing in Eq. (38) are characteristic for zero width resonance models, as the virtual photon can produce from the ground state of the system only the excited states with definite mass. Hence the missing mass squared $W^2 = k^2$ is restricted to the well-defined values belonging to the different excitations. This causes some difficulties in defining the scaling limit of W_1 and W_2 . These are usually circumvented (see for example Ref. 6) by assuming that the effect of the decay channels open to the excited states is to smear out the δ -function peaks in such a way that the integral of $W_{\rho\sigma}$ over an interval in k^2 remains the

same. Assuming that the resonances overlap strongly in the deep inelastic region, $W_{\rho\sigma}$ may be replaced in this region by its average over a small number of resonances. In the case of more than one internal oscillator mode, different occupation number sets can contribute in a given interval of k^2 . Therefore, the contributions must be averaged over the different possible choices of occupation numbers in order to receive the contribution of an "average" resonance. Thus the smeared-out structure functions are defined by

$$\langle W_{\rho\sigma} \rangle = \frac{1}{2\omega_0} \int dn_1 \dots dn_n W_{\rho\sigma}^{n_1 \dots n_n} \delta(n_0 - \sum_{l=1}^n n_l b_l) \quad (40)$$

Here k^2 is put equal to $m_0^2 + \omega_0 n_0 \equiv m_0^2 + \sum_{l=1}^n \omega_0 n_l b_l$, that is by definition $\omega_l \equiv \omega_0 b_l$ with ω_0 as an arbitrary parameter.

In the Bjorken limit the scaling functions resulting from Eqs. (39) and (40) are

$$\begin{aligned} F_1(\xi) &= \lim_{-q^2 \rightarrow +\infty, \xi \text{ fixed}} W_1(q^2, \nu) = F_2(\xi) \frac{1}{2\xi} ; & \xi &\equiv -\frac{q^2}{2\nu} , \\ F_2(\xi) &= \lim_{-q^2 \rightarrow +\infty, \xi \text{ fixed}} \frac{1}{m_0^2} W_2(q^2, \nu) = \sum_{k=1}^N e_k^2 \xi_k \delta(\xi - \xi_k) . \end{aligned} \quad (41)$$

The first equality here is the Callan-Gross relation¹⁵⁾ which is the consequence of the spin $\frac{1}{2}$ character of the quarks.

It can be seen from Eq. (41) that the scaling functions are obtained as incoherent sums of the contributions of different quarks in the bound state. This is consistent with the basic assumptions underlying the parton picture¹³⁾. The fraction of longitudinal momentum ξ_k carried by quark k is determined in our case by the fraction of mass belonging to quark k [compare with Eq. (27)!]. This shows that, unlike in field theoretic bound states (obtained, for example, from Bethe-Salpeter equations), in our bound states the momentum fraction of the constituents effective in high-energy collisions are fixed^{*)}. This is the consequence of the simple potential picture underlying the model.

4. CONCLUDING REMARKS

The main result of the present paper is that, giving an infinite component field theory formulation to the relativistic quark model of Feynman, Kislinger and Ravndal, we wrote down the $Su(3) \otimes Su(3)$ current operators and proved in the model the validity of the Gell-Mann-Fritzsch light cone algebra. As the infinitely deep oscillator potential does not allow the production of free quarks from the

*) This does not mean, of course, that the constituents are on the mass shell. This means only that the amount by which one of the constituents is off the mass shell is compensated on an average by the other constituents going also off the mass shell.

bound states this model is an explicit example of field theories without free quarks but with the validity of the free quark bilocal algebra near the light cone. The light cone algebra is not restricted to $Su(3) \otimes Su(3)$ bilocal currents as the derivation in Section 2 is valid for an arbitrary Dirac operator $\Gamma^{(k)}$ instead of $\gamma_\mu^{(k)}$. Hence the 144-dimensional generalization of the $Su(3) \otimes Su(3)$ algebra is also possible.

The main shortcoming of the bound state quark model, mentioned also in Ref. 5, is the presence of ghosts and parity partners. [These latter are due to the absence of the "Dirac-like" equation corresponding to Eq. (8).] In Ref. 5 the presence of superfluous, unphysical states was taken into account by the introduction of an adjustment factor in the matrix elements. As we can see, the unphysical states do not influence, however, the algebraic properties of the currents near the light cone. Therefore, our main emphasis is on the light cone commutation relations and not on the explicit form of scaling functions we arrived at in Section 3. Besides the unphysical features of the spectrum a further motivation for this is provided by the fact that we actually used the Gaussian form factors of the model, not only near $q^2 \sim 0$ but also for $q^2 \rightarrow -\infty$. This is obviously an oversimplification of the real situation. The strong interactions resulting, for instance, in the (strong) decays of the excited states essentially influence also the high q^2 behaviour of form factors [for an explicit example see, for instance, an earlier paper¹⁶⁾]. The high q^2 behaviour of form factors, in turn, is intimately related to the observed scaling functions. This explains the trivial scaling functions we arrived at in the preceding Section.*)

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*) A possibility to use Eq. (41) for model building is to write it not for "constituent" quarks but for quarks considered as being partons. In that case the observed scaling functions are due to the fact that the partons have varying fractions of the total mass of the bound state.

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