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NEW CONSTRAINTS ON THE PION ELECTROMAGNETIC FORM
FACTOR AND THE CHOU-YANG MODEL

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ADDENDUM

Comparing our results with the conclusions of Chou and Yang, we have overlooked a paper published by them in Phys. Rev. 188, 2469 (1969), where a careful extrapolation of more extensive πp data was made.

The pion form factor given there is larger than that of proton and is consistent with our Ineq. (3). Of course, all our bounds remain unchanged, as they were obtained for a class of Chou-Yang form factors defined by Eq. (4).

*) On leave of absence from the Institute of Theoretical Physics,
University of Warsaw, Warsaw, Poland.

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ABSTRACT

New, strong restrictions on the pion charge radius and pion electromagnetic form factor $F(t)$ in the spacelike region, as well as on the asymptotic $\pi\pi$ scattering, are derived using the Chou-Yang model for spacelike $F(t)$ combined with experimental data on $|F(t)|$ in the timelike region and with rather weak analyticity assumptions for the pion form factor.

*) On leave of absence from the Institute of Theoretical Physics, University of Warsaw, Warsaw, Poland.

1. INTRODUCTION

The electromagnetic form factor of a pion, $F(t)$, has been extensively examined in the last few years in the modulus representation approach ^{1),2)}. As a result, quite strong restrictions on the pion charge radius and on the pion form factor in the spacelike region of momentum transfer ($t < 0$) have been obtained ¹⁾⁻⁷⁾. The input in this approach contained :

- a) experimental information on the process $e^+e^- \rightarrow \pi^+ \pi^-$;
- b) analytic properties of $F(t) \equiv |F(t)| \exp(i\psi(t))$ necessary for the existence of a dispersion relation between $\ln|F(t)|$ and $\psi(t)$.

The property b) used in Refs. 1)-7) is a very weak analyticity assumption [for a detailed discussion, see Ref. 1]; nevertheless, some cases of physical importance do not have this property, e.g., a form factor which decreases in the asymptotic spacelike region ($t \rightarrow -\infty$) as $\exp(-a\sqrt{|t|})$, and/or oscillates rapidly in the timelike region (process $e^+e^- \rightarrow \pi^+ \pi^-$). Let us recall here that the asymptotic oscillations in cross-sections for $e^+e^- \rightarrow \pi^+ \pi^-$ seem to be unavoidable whenever these cross-sections have polynomial boundedness and $F(t) \sim \exp(-a|t|^\alpha)$ for $t \rightarrow -\infty$ [cf., Ref. 8].

In this paper we shall deal with new constraints, recently derived ⁹⁾ under weaker conditions than those used up to now. We require that the form factor satisfies the following conditions :

- i) $F(t)$ is analytic in the cut t plane with a cut from $4m_\pi^2$ to ∞ ;
- ii) the behaviour of $F(t)$ at infinity is given by $\lim_{r \rightarrow \infty} r^{-1+\epsilon} \ln|F(re^{i\vartheta})| = 0$ for $0 \leq \vartheta \leq \pi$ ($t \equiv re^{i\vartheta}$, ϵ is an arbitrary small positive constant).

It was shown ⁹⁾ that the conditions i) and ii) lead to the following inequalities :

$$\int_{4m_\pi^2}^{\infty} \frac{\ln|F(t)/F(4m_\pi^2)|}{x^2} \frac{dx}{dt} dt \geq - \int_0^{\infty} \frac{\ln|F(-t)|}{x^2} \frac{dx}{dt} dt, \quad (1)$$

where $x = \frac{1}{2}\sqrt{t(t-4m_\pi^2)}$ and

$$\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\ln|F(t)/F(t_0)|}{(t-t_0)^2} dt \geq - \frac{d\psi}{dt}(t=t_0). \quad (2)$$

The conditions i) and ii) allow the behaviour $F(t) \sim \exp(bt^{1-\epsilon})$ as $t \rightarrow \infty$. Moreover, no additional assumptions about the distribution of zeros and about oscillations are made; inequalities (1) and (2) are valid even in the case when zeros of $F(t)$ have a finite limit point; i.e., in the situation generally excluded in the modulus representation approach.

An immediate question which arises is whether the relations of type (1) or (2), connecting integrals over spacelike and timelike form factors, can be of any practical importance. In the following section, we shall show that this is the case: the Chou-Yang model for the pion form factor, complemented by our relation (2), leads to many interesting restrictions on the pion charge radius and form factor, as well as asymptotic $\pi\pi$ scattering. The summary of results and conclusions are given in Section 3.

2. ANALYTICITY BOUNDS ON THE PION FORM FACTOR IN THE CHOU-YANG MODEL

The model of Chou and Yang^{10),11)} makes it possible to calculate the "hadronic stuff" distribution functions $\tilde{q}_p(t)$ or $\tilde{q}_\pi(t)$ from a knowledge of asymptotic cross-sections $d\sigma/dt$ for pp and $\pi\pi$ scattering, respectively. This, together with the hypothesis that the $\tilde{q}_p(t)$ is proportional to the electromagnetic form factor $F_{1p}(t)$, leads^{10),11)} to a striking agreement with experimental values of $F_{1p}(t)$ for $-8 \leq t \leq 0$ [here, and in what follows, we shall give the values of t in $(\text{GeV}/c)^2$]. In view of this success, it is reasonable to do the same for the pion form factor $F(t)$. In this case one does not have experimental data on the asymptotic $\pi\pi$ amplitude; Chou and Yang¹¹⁾ obtained it from the extrapolated asymptotic pp and πp differential cross-section with the help of a factorization conjecture. The resulting pion form factor was much steeper than that of a proton, which would mean that a pion is more extended than a proton. However, it has been shown elsewhere⁹⁾ that such a form factor is inconsistent with the rather weak assumptions i) and ii), unless a pronounced structure appears in the hitherto unmeasured timelike region $0 < t \leq 0.34$.

This discrepancy could easily be explained if the $\pi\pi$ asymptotic cross-section were different from that in Ref. 11). Admitting such a possibility, we shall examine the restrictions imposed by the relation (2) (taken at $t_0 = 0$)

$$I \equiv \int_{-\infty}^{\infty} \frac{\ln |F(t)|}{t^2} dt \geq 0 \quad (3)$$

upon the family of form factors (in the spacelike region $t < 0$)

$$F_{\text{Ch.Y.}}^2(t) = \sum_{n=1}^{\infty} \frac{c^n}{n^2} e^{\frac{Bt}{2n}} / \sum_{n=1}^{\infty} \frac{c^n}{n^2} \quad (4)$$

obtained in the Chou-Yang model ^{10),11)} from the asymptotic ($s \rightarrow \infty$) $\pi\pi$ scattering of the form :

$$\begin{aligned} \left(\frac{d\sigma}{dt}\right)_{el} &= \pi(Bc)^2 e^{Bt} , \\ \sigma_{el} &= \pi Bc^2 , \\ \sigma_{tot} &= 4\pi Bc , \end{aligned} \quad (5)$$

where B is the slope of the elastic differential cross-section, and $\frac{1}{4}c$ is the ratio of the elastic to the total cross-section in $\pi\pi$ collisions. There are no experimental data on B and c , but from analyses of πp and pp collisions ^{12),13)} one can expect

$$6 (\text{GeV}/c)^{-2} \leq B \leq 12 (\text{GeV}/c)^{-2}$$

and

$$0 \leq \frac{\sigma_{el}}{\sigma_{tot}} \leq 0.25 .$$

(6)

Apart from these limitations, B and c will be treated as free parameters in our analysis. Restrictions (6), used without inequalities (3), allow the values for the pion form factor to range over the large domain shown in Fig. 1.

Let us now come to the details of our analysis. The integration region from inequality (3) is divided into two parts : $(-\infty, 0.34)$, $(0.34, \infty)$, and the appropriate contributions to the integral I are calculated as follows :

- 1) $-\infty < t \leq 0.34$. Here we put $F(t) = |F_{\text{Ch.Y.}}(t)|$ for $T \leq t \leq 0$ and assume $|F(t)| \leq |F_{\text{Ch.Y.}}(T)|$ for $t \leq T$. In the small range $0 \leq t \leq 0.34$ we interpolate $F(t)$ smoothly between its values for $t < 0$ and $t > 0.34$, putting

$$|F(t)|_{\text{int}} = |F_{\text{Ch.Y.}}(t)| e^{bt^2}$$

with

(7)

$$b = \frac{1}{0.34^2} \ln \left| \frac{F(0.34)}{F_{\text{Ch.Y.}}(0.34)} \right| (\text{GeV}/c)^{-4} .$$

The pion form factor is therefore continuous at $t=0$ and $t=0.34$ while its derivative is continuous at $t=0$ ^{*}). The experimental value $|F(0.34)|_{\text{exp}}^2 = (5.1 \pm 2.2)$ and we consider further two cases ^{**}) :
 I) $|F(0.34)|^2 = 5$, II) $|F(0.34)|^2 = 7$. The contribution I_1 is then maximized by

$$I_1 \equiv \int_{-\infty}^{0.34} \frac{\ln |F(t)|}{t^2} dt \leq \int_{-\infty}^T \frac{\ln |F_{\text{Ch.Y.}}(T)|}{t^2} dt + \int_T^{-0.34} \frac{\ln |F_{\text{Ch.Y.}}(t)|}{t^2} dt + \int_{-0.34}^0 \frac{\ln (|F_{\text{Ch.Y.}}(t)| |F(-t)|_{\text{int}})}{t^2} dt. \quad (8)$$

The results given later correspond to two values of T , namely $-2(\text{GeV}/c)^2$ and $-6(\text{GeV}/c)^2$.

- 2) $0.34 \leq t < \infty$. In this case we use the existing experimental data ¹⁴⁾⁻¹⁷⁾ on the pion form factor ($|F(t)| = |F(t)|_{\text{exp}}$) in the range $0.34 \leq t \leq 4.41$ and assume that $|F(t)| < 1$ for $t > 4.41$. The contribution to I is then maximized by

$$I_2 \equiv \int_{0.34}^{\infty} \frac{\ln |F(t)|}{t^2} dt \leq \int_{0.34}^{4.41} \frac{\ln |F(t)|_{\text{exp}}}{t^2} dt = (2.96 \pm 0.13) (\text{GeV}/c)^{-1} \quad (9)$$

and in further calculations we take $I_2 \leq 3.1(\text{GeV}/c)^{-1}$.

Once the upper bound on I_2 is fixed, I_1 may not be too negative, i.e., $F_{\text{Ch.Y.}}(t)$ cannot decrease too rapidly at small $|t|$ and, e.g., at fixed B the value of c may not be too small. In this way we get the lower bounds on the pion form factor shown in Fig. 2, and the upper bounds on the pion charge radius shown in Fig. 4. Figure 3 gives, for comparison, the charge form factor of the proton $F_{1p}(t)$, its isovector part, $F_{1v}(t)$ (normalized to 1 at $t=0$), as well as the pion form factor of the ρ pole model, $F^{(\rho)}(t)$, and the pion form factor, $F^{(\text{pred})}(t)$, predicted by Chou and Yang from their πp and pp analysis under the specific factorization assumption ¹¹⁾. The results obtained in this paper prove that, if our weak assumptions about the analyticity of the pion form factor are taken into account, then $F^{(\text{pred})}(t)$ cannot be accepted.

^{*}) Other smooth interpolation formulae (e.g., with higher derivatives at $t=0$ being continuous as well) imply even stronger restrictions.

^{**}) For $|F(0.34)|^2 = 3$ our restrictions would become stronger.

In Figs. 5 and 6 we give the lower bounds on the values of σ_{el}/σ_{tot} and σ_{tot} for $\pi\pi$ collisions, which follow from Eq. (5). If σ_{tot} for $\pi\pi$ collisions is not larger than 20 mb (which is the quark model prediction) then the slope B may be at most from $6(\text{GeV}/c)^{-2}$ to $8(\text{GeV}/c)^{-2}$, hence the charge radius of a pion has to be in the range (0.50 to 0.65)fm (cf., Figs. 6 and 4). This value of $\langle r^2 \rangle^{\frac{1}{2}}$ implies smaller dimensions for a pion than for a proton, in contradiction with the conclusion of Chou and Yang¹¹⁾.

3. CONCLUSIONS

It has been demonstrated that relation (3), following from our analyticity assumptions i) and ii), is strongly restrictive when applied to the Chou-Yang model for the pion form factor. For each value of the slope B [cf., Eq. (5)] a lower bound on σ_{el}/σ_{tot} and on σ_{tot} for $\pi\pi$ collisions can be established (Figs. 5, 7). This leads to lower bounds for space-like form factors (Fig. 2) and to upper bounds for the charge radius of a pion (Fig. 4) for each value of B .

Comparison of Figs. 2a' and 3 leads to the conclusion that if : a) the Chou-Yang model works as far as $t = -6$ and b) $|F(0.34)|^2 \leq 5$, then the lower bound for the pion form factor $F(t)$ is close to the charge form factor of a proton $F_{1p}(t)$, in contradiction with the result of Ref. 11). On the other hand, if, in addition, σ_{tot} for $\pi\pi$ collisions is not larger than 20 mb, then $B \leq 6.8(\text{GeV}/c)^{-2}$ and consequently the charge radius of the pion has to be in the range (0.50 to 0.60)fm (see Figs. 6 and 4).

Let us say now a few words about inequality (1), which was not used in our analysis. Inequality (1), unlike inequalities (2) and (3), connects the physical regions (processes $e + \pi \rightarrow e + \pi$, $e^+ + e^- \rightarrow \pi^+ + \pi^-$) only. Once the $e^+ + e^- \rightarrow \pi^+ + \pi^-$ colliding beam experiment is performed for $4m_\pi^2 \approx 0.08 \leq t = -4E^2 \leq 0.34$, then inequality (1) should be used rather than the somewhat simpler inequalities (2) or (3). In such a case the necessity of interpolating in the low energy region will not arise.

Finally, let us emphasize that the real asymptotic cross-sections in $\pi\pi$ collisions may happen to be of a different form (even for small values of $|t|$) than that given by Eq. (5); in fact, the ISR results indicate the existence of two different slopes in the range of $|t| < 0.4$ in pp scattering¹⁸⁾. If such a behaviour is confirmed at still higher energies, then one will probably have to replace Eqs. (4) and (5) by more complicated forms in order to obtain reliable values of the form factor.

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FIGURE CAPTIONS

- Figure 1 : Upper and lower bounds on the pion form factor in the Chou-Yang model, resulting from inequality (6) only - input bounds.
- Figure 2 : Lower bounds on the pion form factor in the Chou-Yang model after taking inequality (3) into account - output bounds.
- Figure 3 : The charge form factor of a proton and some predictions on the pion form factor.
- Figure 4 : Input (dashed lines) and output (solid lines) bounds on the charge radius of a pion.
- Figure 5 : Input (dashed lines) and output (solid lines) bounds on the ratio $\sigma_{\text{el}}/\sigma_{\text{tot}}$ for $\pi\pi$ collisions.
- Figure 6 : Input (dashed lines) and output (solid lines) bounds on σ_{tot} for $\pi\pi$ collisions.

FIG.1

FIG. 2a'

FIG. 2b'

FIG.3

FIG.5a

FIG.5b

FIG. 6a

FIG. 6b