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DUAL TOPOLOGY AND INCLUSIVE CROSS-SECTIONS

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ABSTRACT

We classify all dual diagrams in terms of self-energy insertions on a finite set of primitive (lowest order) diagrams, and we conjecture that the important asymptotic effects can be expressed in terms of renormalized primitive diagrams. For total cross-sections this is the familiar diffractive plus Regge (resonance) picture, but for inclusive cross-sections ($a+b \rightarrow c + \text{anything}$) one obtains a more intricate scheme with primitive graphs for diffractive dissociation, and for scaling and non-scaling of the fragmentation and pionization regions. The role of exotic channels in low-energy scaling, and the suppression of the triple pomeron vertex are clarified by this approach. With the assumption of a simple $J = 1$ diffractive pole, we calculate (on a computer) the fast fragmentation graph. Phenomenological features, and corrections to pionization and slow fragmentation in the scaling limit are discussed.

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1. INTRODUCTION

In this paper we wish to examine inclusive cross-sections from the standpoint of dual models ¹⁾⁻³⁾, [1]. We shall be concerned with the total cross-section $\sigma_T(a+b \rightarrow X)$ and the first inclusive differential cross-section $2p_{c0} d\sigma/d^3p_c(a+b \rightarrow c+X)$ where a is the projectile, b is the target, c is the detected particle of momentum p_c , and X is any hadronic state. From our experience ⁴⁾ with multiperipheral and multiregge models we may expect that if σ_T goes to a constant then $2p_{c0} d\sigma/d^3p_c$ should scale, i.e., become a function of $x = 2p_{c0}/\sqrt{s_{ab}}$ and $p_{c\perp}$ independent of $s_{ab} = (p_a + p_b)^2$ with $p_{c\parallel}$ and $p_{c\perp}$, the spacial components parallel and transverse to the incident direction. Indeed, it has been found recently for $p+p \rightarrow \pi + X$ that not only does scaling set in at accelerator energies but that the x and $p_{c\perp}$ dependence are factorizable ⁵⁾.

The inclusive cross-section is given by

$$\frac{(2\Delta^{1/2}(s_{ab}, m_a^2, m_b^2))}{2\pi} (2p_{c0} d\sigma/d^3p_c) = \sum_X \int d\varphi^{(X)} A_{ab \rightarrow cX} A_{ab \rightarrow cX}^* \quad (1.1)$$

where $\int d\varphi^{(X)}$ is the phase space integration over intermediate states in X . By using real analyticity $A_{ab \rightarrow cX}^*(s_{ab} + i\epsilon) = A(s_{ab} - i\epsilon)$, this is the exact expression for the $s_{ab\bar{c}}$ discontinuity (or z graph) that enters in the unitarity equation ⁶⁾, for the amplitude $A_{abc \rightarrow abc}(p_a, p_b, p_c; p_a, p_b, p_c)$. For the inclusive cross-section we are in the physical region for $abc \rightarrow abc$ restricting $p_a = p_a, p_b = p_b, p_c = p_c$.

Just as for Feynman perturbation theory, the full dual perturbation series ⁷⁾ is too complicated to compute directly. (For definiteness, we consider the full set of Harari-Rosner diagrams

for the dual loops with external mesons in a nonet.) Moreover, with the obvious failures of the present born term (ghosts, no fermions, etc.), one is in a position similar to $\lambda \varphi^3$ theory⁸⁾, where the general (global) properties are more significant than specific detail. In this case, we propose a classification scheme for all dual diagrams, and make specific assumptions on renormalization and diffractive scattering to compute the important high-energy contributions. Beyond certain plausibility arguments, we make no serious effort to derive these subsequent approximations. We look to experiment to confirm, modify or contradict them.

Classification of Dual Diagrams

We group all dual diagrams into classes (I, II, ...) according to the number of quark lines to which external particles are attached. Thus we can re-order the series into a sum over classes. To illustrate, let us consider first the amplitudes $A_{ab \rightarrow ab}$, whose ab discontinuity is related to the total cross-section. In Fig. 1, we show the lowest order (primitive) diagrams with s_{ab} discontinuity and with all lines attached to one (I) or two (II) quark loops $\angle 2$. All higher-order diagrams (e.g., Fig. 2) in each class⁹⁾ can be obtained by attaching planar or non-planar tadpoles $\angle 3$ (Fig. 3). Hence, we have a formal renormalization series for each topologically distinct (primitive) diagram (e.g., class I_R in Fig. 2). At least, in the limit $\alpha' \rightarrow 0$ ($\alpha'/g^2 = \lambda$) of Ref. 8), where one recovers $\lambda \varphi^3$ theory, both tadpoles contribute to mass renormalization.

Implicit in the program of loop unitarization ¹⁰⁾ of dual models is the notion that resonances will remain narrow, that smoothing at high energies of pole saturated discontinuities will result from local mass splittings and that trajectory intercepts and coupling constants will be renormalized to their physical values. Thus to the extent that we smooth over local structure in taking the high-energy discontinuity of I_R , for example, approximating the perturbation series by the first (primitive) term with renormalized parameters should be reasonable [4].

We can use the same scheme to classify diagrams for $A_{abc \rightarrow abc}$. Now, there are at most three types of diagrams corresponding to the number of quark lines to which external lines are attached.

We can easily write down the full (finite) set of primitive graphs, but for simplicity we present in Fig. 4 only the graphs which contribute to $a+b \rightarrow c+X$ at high energy. Thus, we have given all graphs with (1) an $ab\bar{c}$ discontinuity and (2) a $b\bar{b}$ Regge exchange. If one simply exchanges $a, \bar{a} \leftrightarrow b, \bar{b}$ every graph has at least an $a, \bar{a}$ or $b, \bar{b}$ Regge singularity which is required if the contribution as $s_{ab} \rightarrow \infty$ is to fall off not faster than a (Regge) power. In addition, one may choose (3) to drop all graphs that are asymmetric across the $ab\bar{c}$ cut, on the grounds that they are interference terms between amplitudes that emphasize different regions of phase space (for $c+X$). Satisfying the above conditions (1)-(3) one has only the eight graphs given in this table.

	Frag. of a	Central	Frag. of b
Non-Scaling	I_R^a	II_{NS}	I_R^b
Scaling	II_S^a	III_{SC}	II_S^b
Diff. Dissociation	III_{DD}^a	---	III_{DD}^b

This simple decomposition has a good deal of intuitive appeal in the same way the Regge plus diffractive picture does for total cross-sections. Many qualitative features emerge without the drastic approximations we employ to compute these graphs. For example, I_R^a , II_{NS} and II_R^b effect the rate of approach to scaling, and they have non-exotic poles in $ab\bar{c}$ and/or ab . Consequently, these contributions support the contention of Ellis et al.¹¹⁾ that if $ab\bar{c}$ and ab are exotic scaling is approached at relatively low energies. However, the asymmetric graph (II_A), as well as other dual configurations (other discontinuities) of II_S^a break their rule (see Fig. 5). Also II_{NS} (or II_A , II_S^a) do not contribute to the triple Regge region so $ab\bar{c}$ exotic is sufficient for low energy scaling near $x \simeq 1(-1)$ when $a\bar{c}(b\bar{c})$ are non-exotic¹¹⁾. With these diagrams, one can understand both the support for and the limitations of the non-exotic criteria of Ref. 11).

Also our primitive graphs do not have a triple Pomeranchuk coupling as it is easy to show that the triple Pomeranchuk is dual to a planar insertion on the diffractive dissociation graph (II_{DD}). Therefore, we expect it to be suppressed in a "renormalized" theory, in accord with the present data.

An alternative approach taken in Ref. 1) is to classify diagrams which can contribute to $A_{ab} \rightarrow X$ without going beyond the "tree" or one quark line graph at this level. The arguments of Di Giacomo et al.¹²⁾ are then used to imply that since input resonances are narrow the tree diagrams have an approximate local unitarity already built in. This is complementary to our notion that addition of planar tadpoles will not have a gross effect. Conceptually, the present classification scheme has the advantage that crossing symmetry and unitarity are present to all orders in principle. Furthermore, primitive graphs corresponding to diffractive production (III_{DD} of Fig. 5) arise naturally, so that all of the principal mechanisms are represented in this decomposition. If we went further to look at correlations measurements ($a+b \rightarrow c+d+X$), the number of primitive graphs increases

again. Roughly speaking the additional particles "probe" more and more deeply the details of higher order topologies.

The main objection to the present approximations might be the neglect of multiple scattering due to iterating the pomeron in the s channel. Although by using the untwisted insertions of Fig. 2, it is easy to see that these are equivalent to non-planar self-energy corrections. At least, in the zero slope limit (e.g., $\lambda\phi^3$) these contribute to mass renormalization. Also the data fitting of Lovelace¹³⁾ on K^+p scattering, and the verification by Chan¹⁴⁾ of factorization for diffractive effects in inclusive processes, suggests these multiple scattering contributions are small (except possibly at large momentum transfer or large $p_{\perp}^2$).

Approximation to Pomeron

In the remainder of the paper we begin a quantitative study of these contributions. To deal with the scaling graphs, we make an extremely simple ansatz about diffractive scattering.

The diagram II_D of Fig. 1 has in part a J plane Regge cut leading with $\alpha_p(0) = \frac{1}{3}$ and slope $\frac{1}{2}$ the input trajectory. In that it has only vacuum quantum numbers in the $a\bar{a} = b\bar{b}$ channel and has an exotic continuum in the ab channel it has been identified as an embryonic pomeron satisfying the Freund-Harari hypothesis¹⁵⁾. The fact that its intercept is not 1 seems to be related to an insufficient number of degrees of freedom in the spectrum of states which may be expected to improve when baryons and exotics are added¹⁵⁾. In the present analysis we are unable to calculate this component in a meaningful way. Instead we shall assume that this singularity can be replaced by a $J = 1$ pole at $t = 0$, which couples like an ordinary (e.g., f^0) Regge pole. Hence, for high energy we replace the Pomeranchuk diagram (II_S^a) with a Born diagram (I_R^a) with a trajectory at $\alpha(0) = 1$. But in this replacement, we carry over the isospin factors and duality properties of the

Pomeron diagram. For example, if $a\bar{c}$ couple to a Pomeron, we symmetrize in (p_a, p_c) . It should be noticed that we are making no statements about the existence of exotics, since we only keep the leading ($J = 1$) cross-channel singularity. Also, we are free to choose the slope $\alpha'_p < 1$, when calculating diffractive dissociation.

Clearly, a high priority is to improve on this assumption, but as seen in Chan's work, the coupling of the Pomeron, and the lower trajectories are not distinguishable in P_{11} , P_{13} dependence of the present data. Indeed, our model incorporates a version of f^0 -Pomeron proportionality suggested in the recent "dual" model of Lovelace¹³⁾ or Carlitz et al.¹⁶⁾. More elaborate assumptions are probably not of immediate phenomenological interest in inclusive processes.

2. CONTRIBUTIONS TO THE FAST FRAGMENTATION REGION

In this section we calculate the contribution of diagrams (I_R) of Fig. 2 to the forward fragmentation region. Diagrams (II_S) are treated implicitly by the simplified approximation discussed above. For this region

$$\begin{aligned} t &= (p_a - p_c)^2 \rightarrow m_a^2(1-x) + m_c^2(1-1/x) - p_{c\perp}^2/x + O(1/s) \\ u &= (p_b - p_c)^2 \rightarrow -xs + O(1/\sqrt{s}) \\ \omega &= (p_a + p_b - p_c)^2 \rightarrow (1-x)s + O(1/\sqrt{s}) \end{aligned}$$

Effectively, the $i2\pi \delta(w-m^2)$ delta function contributions to the w discontinuity from the abc states λ in $B_6(-p_c, p_a, p_b, -p_b - p_a, p_c)$ are replaced by $\int_{w(1+i\epsilon)-m_\lambda^2}^{-1} - \int_{w(1-i\epsilon)-m_\lambda^2}^{-1}$.

This is an unambiguous way of "smoothing" these contributions for w going large. Thus taking s, s' and w large along their negative real axes and following standard techniques we obtain [5]

$$B_6 \rightarrow \Gamma(\alpha_{6\bar{5}}(0)) \int_0^1 dy_1 \int_0^{1-y_1} dy_2 \left[(1-\frac{1}{y_1})(1-\frac{1}{y_2}) \right]^{\alpha(t)+1} \left[(1-y_1)(1-y_2) \right]^{-2} \\ \left[\frac{1-y_1-y_2}{(1-y_1)(1-y_2)} \right]^{-\alpha_{6\bar{5}}(0)-1} \left[-w(1-y_1-y_2) - sy_1 - s'y_2 \right]^{\alpha_{6\bar{5}}(0)} \quad (2.1)$$

There are contributions to a cut in w if we restrict $[-w(1-y_1-y_2)+sy_1+sy_2] \geq 0$ by cutting short the variables of integration. s and s' are then continued to $s(1+i\epsilon)$ and $s(1-i\epsilon)$ respectively as required by the unitarity equations. The final result after adding the other contributions $B_6(p_a, -p_c, p_b, -p_b, -p_a, p_c)$, $B_6(-p_c, p_a, p_b, -p_b, p_c, -p_a)$ and $B_6(p_a, -p_c, p_b, -p_b, p_c, -p_a)$ is [6]

$$\left(\frac{1}{\pi}\right) 2p_{c0} d\sigma/d^3p_c \rightarrow \frac{\pi s^{\alpha_{6\bar{5}}(0)-1} (1-x)^{\alpha_{6\bar{5}}(0)-2\alpha(t)}}{\Gamma(\alpha_{6\bar{5}}(0)+1)} \cdot \\ \int_0^1 dy_1 \int_0^{1-y_1} dy_2 (1-y_1-y_2)^{\alpha_{6\bar{5}}(0)} (y_1 y_2)^{-\alpha(t)-1} F$$

where

$$\begin{aligned}
 F = & \left[(1 - (1-x)y_1)(1 - (1-x)y_2) \right]^{\alpha(t) + \alpha_{a\bar{a}}(0)} \left[1 - (1-x)y_1, -(1-x)y_2 \right]^{-\alpha_{a\bar{a}}(0) - 1} \\
 & + \left[(x + (1-x)y_1)(x + (1-x)y_2) \right]^{\alpha(t) + \alpha_{c\bar{c}}(0)} \left[x + (1-x)y_1, (1-x)y_2 \right]^{-\alpha_{c\bar{c}}(0) - 1} x^{1 - \alpha_{c\bar{c}}(0)} \\
 & + 2\eta(\cos \pi \alpha(t)) \left[x - x(1-x)y_1 + (1-x)y_2 \right]^{\alpha(t) + 1} \left[(1-y_1(1-x))(x + (1-x)y_2) \right]^{-2 + 2\beta} \\
 & \left[x - x(1-x)y_1 + (1-x)y_2 \right]^{-2\beta}
 \end{aligned}$$

(2.2)

with $\beta = \left[\alpha_{ac}^-(0) + m_a^2 + m_c^2 + 1 \right]$.

For $\eta = 0$, the trajectory $\alpha(t)$ is exchange degenerate and for $\eta = \pm 1$ a pole of pure signature. Thus we get an approach to scaling like $s^{\alpha_{b\bar{b}}(0) - 1}$ and if we set $\alpha_{b\bar{b}}(0) = 1$ to approximate the diffractive component (II_S) we obtain scaling.

Although (2.2) is quite awkward it has a very simple approximation in the fast fragmentation limit x near 1 where $F \rightarrow (2 + 2\eta \cos \pi \alpha(t))$

$$\begin{aligned}
 \left(\frac{1}{\pi} \right) 2 p_{c0} d\sigma / d^3p \rightarrow & s^{2\alpha(t) - 1} W^{\alpha_{b\bar{b}}(0) - 2\alpha(t)} f(t) = \\
 & s^{\alpha_{b\bar{b}}(0) - 1} (1-x)^{\alpha_{b\bar{b}}(0) - 2\alpha(t)} f(t)
 \end{aligned}$$

(2.3)

where $f(t) = \pi \left[\Gamma^2(-\alpha(t)) / \Gamma(-2\alpha(t) + \alpha_{b\bar{b}}(0) + 1) \right] (2 + 2\eta \cos \pi \alpha(t))$.

Even though this form with $f(t)$ arbitrary seems to be a general consequence of any triple Regge pole coupling¹⁷⁾ we emphasize here a few phenomenological consequences:

- 1) in the production of low mass resonances such as $\rho, f, g, \dots$ we expect a broadening of the forward t peak as their mass w increases as observed, since the behaviour depends on w through $\exp[\sqrt{t} 2 \ln(1/w)]$,¹⁸⁾
- 2) for an inclusive process having a ρ dominated t channel (e.g., $p+p \rightarrow p+X$) there is an amusing cancellation $\alpha_p(0) - 2\alpha_\rho(0) = 0$. Since $t \rightarrow 0$ for $p_{c\perp}^2 = 0$, $x \rightarrow 1$ the x dependence of the scaling component should be flat $((1-x)^{\alpha_p(0)-2\alpha_\rho(0)} = 1)$ as observed¹⁹⁾. There is also evidence of a non-scaling triple Regge component dual to the resonances (in w) observed for $x \rightarrow 1$ ¹⁸⁾;
- 3) the fast forward spectrum with exotic exchange in t should be extremely small (e.g., $\pi^+p \rightarrow \pi^-X$);
- 4) as a general formula it indicates that the triple Pomeron coupling must be extremely small otherwise $(1-x)^{\alpha_p(0)-2\alpha_p(0)} = (1-x)^{-1}$ would cause a large increase in the spectrum near $x \rightarrow 1$.

We wish to emphasize that in general the more elaborate form Equation (2.2) is difficult to test experimentally because away from x near 1 where the simple form equation (3) breaks down diagram (III_S) will dominate it even for $p_{c\perp}^2$ small Equation (2.2). An exceptional case is $(p)+b \rightarrow (\pi)+X$. For purposes of illustration we treat this as a spinless process with a baryon t exchange intercept $\alpha_t(0) = -m_p^2$ and $m_\pi^2 = 0$. Thus $t \simeq m_p^2(1-x) - p_{c\perp}^2/x$ for $p_{c\perp}^2$ small. Hence, for $x \rightarrow 1$ we have a sharp fall-off like $(1-x)^{\alpha_{bb}(0)-2\alpha_t} \rightarrow 0$, and for $x \rightarrow 0$ we have $\alpha(t) \rightarrow 0$ which increases the cross-section by approaching the first t channel pole Equation (2.2). For these processes (I_R^a) and (II_S^a) contribute most in the slow fragmentation region and least in the fast. Most other processes behave in the opposite way, because we generally move away from the t poles as x decreases e.g., $(\pi^+)+b \rightarrow (\pi^+)+X$, $t \simeq -p_{c\perp}^2/x$.

We illustrate the above points as well as make a crude test of the factorization of x and $p_{c\perp}$ dependence by showing in Figs. 5 and 6 a numerical calculation of equation (2.2) for $p+p \rightarrow \pi^+ + X$ [9]. The dotted line in Fig. 5 is the experimental ($p_{\text{Lab}} = 19.2$ GeV/c) x dependence with the approximately universal $p_{c\perp}$ dependence factored out (the $p_{\text{Lab}} = 12.5$ and 30 GeV/c data show no change indicating that scaling is present ⁵⁾).

We note:

- 1) our x distribution for $p_{c\perp}^2 = 0$ is remarkable close to $\exp(-cx^2)$ (straight line) with $c = 7.4$;
- 2) as the figure shows factorization property is approximated for x^2 not small;
- 3) the $p_{c\perp}^2$ distribution is reasonable for small $p_{c\perp}$ but falls away too rapidly at large values, especially at small x^2 . These portions of the spectrum must be built up by corrections from diagrams (III_{SC}). In the following section we indicate how these corrections can be made.

3. SLOW FRAGMENTATION CORRECTIONS AND PIONIZATION

To correct the deficiencies in the scaling region of the "two quark line" graphs ($\text{II}_S^a, \text{II}_S^b$) we have only the "three quark line" graph (III_{SC}). This graph should dominate the pionization region by virtue of its double Pomeron exchange, and should dominate the large $p_{\perp}$ behaviour for any process. It will make substantial contributions to the slow fragmentation region even for small $p_{\perp}$ values, especially for exotic t channel reactions where II_S^a is missing. Before turning to the question of pionization we indicate below how corrections to slow fragmentation can be made.

Referring to Fig. 8a ($t_a = q_a^2$, $t_b = q_b^2$, $w_a = k_a^2$, $w_b = k_b^2$) we sketch the results of Silverman and Tan, Ref. 21), who argue that the phase space integration over the two-body intermediate states X_a and X_b scales. Furthermore, for s, t and u defined and approximated for the fragmentation region as in Section 2, only a small invariant mass squared w_a is channeled into X_a . For these conclusions it is necessary that there be dynamical bounds in $-t_a$ and $-t_b$ of $O(1)$. First define new scaling variables

$$y \equiv p_a \cdot q_a / p_a \cdot p_c = \frac{w_a - t_a - m_a^2}{-t + m_a^2 + m_c^2} \quad (3.1)$$

$$z \equiv p_b \cdot q_b / p_b \cdot p_c = \frac{w_b - t_b - m_b^2}{-u + m_b^2 + m_c^2} \quad (3.2)$$

After some lengthy algebra

$$\begin{aligned} & \int d^4 q_a d^4 q_b \delta^4(q_a + q_b + p_c) \rightarrow \\ & (p_{c\perp}^2 + m_c^2 + m_a^2 x^2 \theta(x) + m_b^2 x^2 \theta(-x)) \cdot \\ & \int dy \int dz \int_{-\infty}^0 dt_a \int_{-\infty}^0 dt_b \frac{\theta(-\Delta(p_{c\perp}^2, q_{a\perp}^2, q_{b\perp}^2)) \theta(q_{a\perp}^2) \theta(q_{b\perp}^2)}{\Delta^{1/2}(p_{c\perp}^2, q_{a\perp}^2, q_{b\perp}^2)} \quad (3.3) \end{aligned}$$

where Δ is the usual symmetric function. For the forward fragmentation region ($s \rightarrow \text{large}$)

$$\begin{aligned} q_{a\perp}^2 &= -t_a - (p_{c\perp}^2 + m_c^2)(z+1)y - m_b^2 x^2 y(1+y+z) + O(1/s) \\ q_{b\perp}^2 &= -t_b - (p_{c\perp}^2 + m_c^2)(y+1)z - m_a^2 x^2 z(1+y+z) + O(1/s) \end{aligned}$$

Thus, if there is a dynamical bound on $-t_a$ and $-t_b$ of $O(1)$ (i.e., the multiperipheral condition), then the conditions $q_{a\perp}^2 \geq 0$ and $q_{b\perp}^2 \geq 0$ imply that $(z+1)y \leq O(1)$ and $(y+1)z \leq O(1)$ respectively, i.e., $y \leq O(1)$ and $z \leq O(1)$. This shows that the phase space integration scales and from (3.1) implies $(k_a)^2 = w_a \leq O(1)$ and from (3.2) $w_b \rightarrow zxs$. The lower bounds on y and z result from $\Theta(-\Delta)$.

For II_{NS} and III_{SC} we can be fairly sure of diminishing contributions for $-t_a$ and $-t_b$ greater than $O(1)$. Thus there should be substantial portions of the spectrum for $p_{c\perp}$ not too large and x not too small where the ground state contribution to the w_a discontinuity should suffice. In Appendix 2 we sketch a derivation of this contribution. With no surprise we find an elaborate scaled function times $w_b \alpha_{b\bar{b}}(0) s^{-1} = (zx) \alpha_{b\bar{b}}(0) s \alpha_{b\bar{b}}(0)^{-1}$

for (III_{SC}) . As expected the result has manifest poles in the $(p_c + k_a)^2$ channel which must be extracted and arbitrarily broadened before the phase space integrals are performed over them. As x becomes smaller and $p_{\perp}$ larger more and more excited states must be included; this can be done, but the present technique rapidly becomes too cumbersome.

The above sequence of corrections can be understood from the simplest multiperipheral ideas ²²⁾ as follows: for a many-particle final state (composed of ground states) produced from a multiperipheral chain, the fastest particles (largest x) on the average come in general from the first position on the end of the chain nearest the projectile; the second fastest comes from the second position and so on. Thus, including the contribution to the w_a discontinuity above accounts for the production of the detected particle from the second position on the chain. Since the first excited state can decay into two ground states its inclusion is

analogous to accounting for production from the third position and so on.

For pionization $|x| \leq O(1/\sqrt{s})$ the double Pomeron singularity in (III_{SC}) should dominate. Our recipe would suggest that we can replace both these Pomerons with Regge poles $(III_{SC} \rightarrow I)$
 $\alpha_{aa}^-(0) = \alpha_{bb}^-(0) = 1$ and take limit u and $t \rightarrow -\infty$. This is equivalent to taking $x \rightarrow 0$ on the first term in (2.2) [as Ref. 3] points out the order of taking limits to the pionization region is reversible⁷. Since the Pomeron couples more weakly than the ordinary trajectory setting $g_p^2/g_{Regge}^2 = 1$ (see Fig. 6 and caption) we obtain an over-estimate of pionization. As Fig. 6 suggests pionization is suppressed by a small factor g_p^2/g_{Regge}^2 relative to fragmentation.

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APPENDIX 1

We consider here the high energy limit of the ground state $X_a \equiv d$ contribution of (II_{NS}) for the forward fragmentation region of a . We start with $B_8(-p_c, -k_a, p_a, p_b, -p_b-p_a, k_a, p_c)$ expressed in the standard Bardakçi-Ruegg variables $(x_1 y_1 z y_2 x_2)$ for the configuration of Fig. 8b.

$$\begin{aligned}\hat{t} &= (p_c + k_a)^2 = -t + t_b - t_a + m_a^2 + m_c^2 + \mu^2 \\ \hat{w} &= (p_a + p_b - k_a)^2 = w_b + t_a - t_b - \mu + m_c^2 + m_b^2\end{aligned}$$

where $k_a^2 = w_a \xrightarrow{\text{5}} \mu^2$ the mass squared of the ground state. We can write B_8 as

$$\int_0^1 du_1 \int_0^1 dv_1 \int_0^1 du_2 \int_0^1 dv_2 \int_0^1 dz J_1 z^{-\alpha(w_b)-1} (1-z)^{-\alpha(t_b(0)-1} \\ u_1^{-\alpha(\hat{w})+\alpha(s)+\alpha(t_a)} u_2^{-\alpha(\hat{w}')+\alpha(s')+\alpha(t_a)} v_1^{-\alpha(s)-1} v_2^{-\alpha(s')-1} \cdot G$$

where

$$y_1 = \left(\frac{1-v_1}{1-v_1 z} \right), y_2 = \left(\frac{1-v_2}{1-v_2 z} \right), x_1 = \left(\frac{1-u_1}{1-u_1 z} \right) \left(\frac{1-v_1 z}{1-v_1} \right), x_2 = \left(\frac{1-u_2}{1-u_2 z} \right) \left(\frac{1-v_2 z}{1-v_2} \right),$$

$$J_1^{-1} = (1-u_1 z)^2 (1-u_2 z)^2 (1-u_1 z) (1-v_2 z) (1-v_1) (1-v_2) (1-z)^{-4}$$

(A.1)

and

$$\begin{aligned}
 G(x_1, y_1; x_2, y_2; z) = & \\
 & (1-x_1)^{-\alpha(t_a)-1} (1-x_1 y_1 z y_2)^{\alpha(t_a) + \alpha_{ab}\bar{\sigma}(0)} \\
 & \cdot (1-x_2)^{-\alpha(t_a)-1} (1-x_2 y_2 z y_1)^{\alpha(t_a) + \alpha_{ab}\bar{\sigma}(0)} \\
 & \cdot x_1^{-\alpha(\hat{t})-1} x_2^{-\alpha(\hat{t})-1} y_1^{-\alpha(t_b)-1} y_2^{-\alpha(t_b)-1} \\
 & \cdot (1-x_1 y_1 z y_2 x_2)^{-\alpha_{cb}(0) - \alpha_{ab}\bar{\sigma}(0)} \\
 & \cdot (1-y_1 z y_2)^{-\alpha_{cb}(0) - \alpha_{ab}\bar{\sigma}(0)} (1-y_1 z)^{\alpha_{cb}(0)-1} (1-y_2 z)^{\alpha_{cb}(0)-1}
 \end{aligned}$$

(A.1)
ctd.

We give a somewhat detailed derivation of the high energy limit using a notation analogous to Ref. 2). w_0 , s , and s' are taken large along their negative real axis away from their regions of poles $\sqrt{t_a}$ and t_b are held fixed; $\alpha(\hat{w}) - \alpha(s) - \alpha(t_a) \rightarrow w_b - u - s$. Rewriting with new variables of integration

$$u_1 = e^{-p\bar{u}_1}, u_2 = e^{-p\bar{u}_2}, v_1 = e^{-p\bar{v}_1}, v_2 = e^{-p\bar{v}_2}$$

$$\text{and } z = e^{-p(1-\bar{u}_1-\bar{u}_2-\bar{v}_1-\bar{v}_2)}$$

$$\begin{aligned}
 & \int_0^\infty d\rho \int_0^1 d\bar{u}_1 \int_0^1 d\bar{u}_2 \int_0^1 d\bar{v}_1 \int_0^1 d\bar{v}_2 \Theta(1-\bar{v}_1-\bar{v}_2-\bar{u}_1-\bar{u}_2) J_1 J_2 \\
 & \cdot e^{p[(1-\bar{u}_1-\bar{u}_2-\bar{v}_1-\bar{v}_2)w_b + \bar{u}_1(w_b-u-s) + \bar{u}_2(w_b-u'-s') + v_1 s + v_2 s']} \\
 & \cdot [1 - e^{-p(1-\bar{u}_1-\bar{u}_2-\bar{v}_1-\bar{v}_2)}] \alpha_{cb}(0)-1 G
 \end{aligned}$$

where

$$J_2 = p^4 u_1 u_2 v_1 v_2 z (1-\bar{u}_1-\bar{u}_2-\bar{v}_1-\bar{v}_2)$$

The high-energy behaviour comes from the region

$p = 0$ ($u_1 = u_2 = v_1 = v_2 = z = 1$). Thus, we have

$$\begin{aligned} & \mathcal{I}(-\alpha_{\delta\delta(0)}) \int_0^1 d\bar{u}_1 \int_0^1 d\bar{u}_2 \int_0^1 d\bar{v}_1 \int_0^1 d\bar{v}_2 \theta(1-\bar{v}_1-\bar{v}_2-\bar{u}_1-\bar{u}_2) \\ & \cdot [-w_b (1-v_1-v_2) - (-u-s) \bar{u}_1 - (-u-s') \bar{u}_2]^{\alpha_{\delta\delta(0)}} \\ & \cdot (1-\bar{u}_1-\bar{u}_2-\bar{v}_1-\bar{v}_2)^{-\alpha_{\delta\delta(0)}} \mathcal{T}_1 \mathcal{T}_2 G \Big|_{p=0} \end{aligned}$$

This is analogous to Eq. (1). The $\left[\right]$ $\alpha_{b\bar{b}}(0)$ factor contributes to a cut in w_b with discontinuity

$$(2i \sin \pi \alpha_{\delta\delta(0)}) w_b^{\alpha_{\delta\delta(0)}} [1-\bar{v}_1-\bar{v}_2-\bar{u}_1-\bar{u}_2]^{\alpha_{\delta\delta(0)}}$$

provided we restrict the region of integration to

$$[1-\bar{v}_1-\bar{v}_2-\bar{u}_1-\bar{u}_2] \geq 0$$

where

$$\bar{u}_1 = \left(-\frac{u-s}{w_b}\right) \bar{u}_1 \quad \text{and} \quad \bar{u}_2 = \left(-\frac{u-s'}{w_b}\right) \bar{u}_2$$

s and s' are now continued to $s(1+i\epsilon)$ and $s(1-i\epsilon)$ as before. The final result is

$$\begin{aligned} & \frac{\pi}{\mathcal{I}(\alpha_{\delta\delta(0)}+1)} w_b^{\alpha_{\delta\delta(0)}-1} \left(\frac{zx}{1-x}\right)^2 \int_0^1 d\bar{u}_1 \int_0^1 d\bar{u}_2 \int_0^1 d\bar{v}_1 \int_0^1 d\bar{v}_2 \theta(1-\bar{u}_1-\bar{u}_2-\bar{v}_1-\bar{v}_2) \\ & \cdot \frac{[1-\bar{v}_1-\bar{v}_2-\bar{u}_1-\bar{u}_2]^{\alpha_{\delta\delta(0)}} (1-\bar{u}_1-\bar{u}_2-\bar{v}_1-\bar{v}_2)^{1-\alpha_{\delta\delta(0)}}}{\bar{v}_1 \bar{v}_2 (1-\bar{u}_1-\bar{u}_2-\bar{v}_1)(1-\bar{u}_1-\bar{u}_2-\bar{v}_2) (1-\bar{v}_1-\bar{v}_2-\bar{u}_1)^2 (1-\bar{v}_1-\bar{v}_2-\bar{u}_2)^2} \\ & \cdot G(x_1, y_1; x_2, y_2; 1) \end{aligned}$$

(A.2)

where $\bar{u}_1 = \left(\frac{-zx}{1-x} \right) \bar{u}_1$, $\bar{u}_2 = \left(\frac{-zx}{1-x} \right) \bar{u}_2$

and

$$x_1 = \frac{\bar{u}_1 (1 - \bar{u}_1 - \bar{u}_2 - \bar{v}_1)}{\bar{v}_1 (1 - \bar{v}_1 - \bar{v}_2 - \bar{u}_1)} , \quad x_2 = \frac{\bar{u}_2 (1 - \bar{u}_1 - \bar{u}_2 - \bar{v}_2)}{\bar{v}_2 (1 - \bar{v}_1 - \bar{v}_2 - \bar{u}_1)} , \quad y_1 = \frac{\bar{v}_1}{(1 - \bar{u}_1 - \bar{u}_2 - \bar{v}_2)} , \quad y_2 = \frac{\bar{v}_2}{(1 - \bar{u}_1 - \bar{u}_2 - \bar{v}_1)} .$$

Thus we find an elaborate scaled function times $\alpha_{bb}^{-}(0)$ =
 $(zx) \alpha_{bb}^{-}(0) \alpha_{bb}^{-}(0)$.

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FOOTNOTES

- [1] During the course of our investigation we have received several preprints (Ref. 1,2,3) which have in part similar results. We shall emphasize those aspects peculiar to our approach.

- [2] For simplicity we rule out external singlets and thus do not allow quark lines with only one particle attached.

- [3] Non-orientable tadpoles have quark-quark mesons, and therefore have vanishing isospin factors. It is very useful to first remove all twist by using the diagrams of Fig. 2, since all duality relations are then quickly derived by just "sliding lines". (=). Iterations of the Pomeron in Fig. 2 (with twist) lead to orientable diagrams, equivalent to self energy insertions.

- [4] We do not assume that the higher orders diagrams (with new thresholds) are negligible, but that in certain limits controlled by leading J singularities they are accounted for by renormalization. In other limits (e.g., wide angle) the higher order effects are more difficult to "guess".

- [5] Note that we set $\alpha_{ca\bar{a}}(m_c^2) = 0$ for example, since it is a trajectory with the quantum number of the external particle c which we treat spinless. For details, we refer the reader to Ref. 2), or Appendix I where a more elaborate calculation of this type is given.

[6] As a strict consequence of duality for the Pomeron graph II_S^a , one has a signature symmetry under the exchange $a \leftrightarrow \bar{c}$ and $\bar{a} \leftrightarrow c$. Imposing this symmetry leaves only one free parameter (γ). Note that we obtain the usual signature factor $\frac{1+z e^{i\alpha(t)}}{1+z e^{-i\alpha(t)}}$ where $z = 2x/(1+x^2)$ only near $x = 1$.

[7] We are warned that this should be the case from previous calculations based on the multiregge calculations of Caneschi and Pignotti ²⁰⁾ (CP).

[8] The CP calculations result in the form of Eq. (3) with $f(t) = e^{At}$ which is to be applied for all x and neglects the near by nucleon pole.

[9] We do not intend this to be understood as a "fit to the data" in that we are treating the process pp less and ignoring tachyon problems in unphysical channels as well as approximating the Pomeron in a crude way. We take $m_p^2 = -\alpha_t(0) = +0.88$, $m_\pi^2 = 0.02$, $\alpha_{b\bar{b}}(0) = 1$, $\alpha_{aa}(0) = \alpha_{cc}(0) = 0.5$. Even if we take $\alpha_{aa}(0) = \alpha_{cc}(0) = 1$ to approximate III_{SC} for $|x| < 1$, we obtain the same qualitative conclusions. (See Section III) (Note that finite kinematics are used in the calculation and not the scaled form of Eq. 2.2).

FIGURE CAPTIONS

Fig. 1 : Lowest order (primitive) graphs for $a+b \rightarrow a+b$ with all external lines attached to one (class I) or two (class II) quark loops. Only I_R (Regge) and II_D (Diffractive) contribute to Regge behaviour of total cross-sections ($a+b \rightarrow X$) as $s_{ab} \rightarrow \infty$. Graphs to the right are obtained by permuting legs on the topological graph given to the left of the arrow.

Fig. 2 : The "renormalization" series for the I_R primitive graph.

Fig. 3 : The non-planar tadpole and the "orientable" Pomeron exchange drawn without twists. When this Pomeron is inserted into an orientable diagram, it yields an orientable diagram.

Fig. 4 : All primitive graphs with $ab\bar{c}$ discontinuity and $b\bar{b}$ Regge exchange $\Delta\phi = 1 + \frac{1}{2}$ is the signaturized propagator. Graphs asymmetric across the $ab\bar{c}$ cut should be reflected across this line (----) to get all asymmetric contributions. Exchanging $a, \bar{a}$ with $b, \bar{b}$ gives all graphs contributing to $a+b \rightarrow c+X$, which fall off no faster than a power as $s_{ab} \rightarrow \infty$. Aside from the asymmetric (interference) graphs, there are only the eight contributions listed in the Table.

Fig. 5 : The discontinuity exhibited on the left scales, but the discontinuity exhibited on the right does not. This lower-order contribution (in s) can have $ab\bar{c}$ and ab exotic and consequently breaks the rules of Ref. 11).

Fig. 6 : The contribution of (II_S^a) in the fragmentation region (solid line) compared with 19.2 GeV/c data for $p+p \rightarrow \pi^+ + X$ at $p_\perp = 0$ (dotted line). The fall off in x^2 is $\exp(-7.4 x^2)$ as compared with the experimental value of $\exp(-8.5 x^2)$. Assuming

$\varepsilon_{\text{Regge}} = \varepsilon_{\text{Pom}}$, the arrow gives the pionization contribution of (III_{SC}) .

Fig. 7 : The $p_{\perp}^2$ distribution of (II_S^a) normalized at $p_{\perp}^2 = 0$, compared with data for $p+p \rightarrow \pi^+ + X$ at 19.2 GeV/c. At high $p_{\perp}^2$ higher-order effects are apparently important.

Fig. 8 : a) Kinematics for the contribution of

$$a(p_a) + b(p_b) \rightarrow c(p_c) + X_a(k_a) + X_b(k_b)$$

to II_{NS}^a and III_S^a .

b) For calculation in the Appendix for the fragmentation of a ($x > 0$), the sum over X_a has been approximated by the ground stated.

FIG.1

$$\text{Diagram with shaded circle} = g^2 \text{Diagram with rectangle} + g^4 \text{Diagram with rectangle and bubble} + g^6 \left[\text{Diagram with two bubbles} + \text{Diagram with bubble and triangle} \right] + \dots$$

FIG.2

FIG.3

FIG.4

FIG. 5

FIG. 6

FIG.7

FIG.8a

FIG.8b