Archives

THE CROSS-SECTION FOR HIGH TRANSVERSE MOMENTUM
HADRON JET PRODUCTION IN THE PARTON MODEL

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A B S T R A C T

The cross-section for high transverse momentum hadron jet production in proton-proton collisions is calculated in the parton model. It is assumed that the high transverse momentum hadrons are produced via the "nova" states of Jacob and Slansky.

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The implications of the parton model ¹⁾ for high transverse momentum inclusive processes initiated by leptons, photons or hadrons, were investigated in a recent paper by Berman, Bjorken and Kogut ²⁾. As it was pointed out by these authors, this class of processes is sensitive to small transverse distances and constitutes therefore a basic test for our understanding of the hadron structure. In the parton model the "deep inelastic" processes proceed in two consecutive steps. First, the high transverse momentum partons are produced via the elementary parton processes (scattering or annihilation) and then the "post-reaction" parton configuration develops the observed hadron final state. The two steps are characterized by the two basic ingredients of the parton model : the probability distribution $F_{ai}(x)(xe_i^2)^{-1}$ for a hadron of type a be seen in the collision as a parton of type i (charge e_i) with longitudinal momentum fraction x , and the probability distribution $G_{ic}(x)x^{-1}$ for a hadron of type c associated with the produced parton i be observed with a fraction x of the parton's longitudinal momentum. The functions $F_{ai}(x)$ are known from deep inelastic ep scattering, whereas the main properties of $G_{ic}(x)$ were conjectured by Berman, Bjorken and Kogut on the basis of the assumptions that the evolution of the high transverse momentum parton is independent of the other partons and it is dominated by a succession of small p_T processes characteristic of "ordinary" diffraction processes.

The present letter is based entirely on the ideas developed by Berman, Bjorken and Kogut. Our additional purpose here is to calculate the cross-section for hadronic jet production which is also a measurable quantity beside the single particle distributions calculated in Ref. 2) ^{*}). We shall assume that the process leading from the "post-reaction" parton configuration to the observed hadronic final state can be described in the framework of the nova model of diffraction scattering proposed recently by Jacob and Slansky ³⁾. [For a more general class of diffraction excitation models, see also the papers of Hwa ⁴⁾.] Here only the case of the high transverse momentum proton-proton collisions will be considered.

Our main assumptions can be summarized as follows :

- i) the dynamics of the parton configuration emerging after the deep inelastic parton-parton collision is the same (or similar to) the dynamics of the diffraction processes as described in the nova model.

^{*}) The term jet means here a group of almost parallelly moving particles. An example of high transverse momentum jet would be, for instance, that $n \geq 3$ particles were observed within a small solid angle $\omega \sim 0.01$ sterad in the centre-of-mass frame with the total transverse momentum $p_T \geq 4$ GeV/c.

- ii) the transverse momentum of the high p_T parton is not (or little) affected by the "post-reaction" parton process, that is, the transverse momenta of the produced high p_T novas are the same as those of the partons themselves *).

In a diffraction process the interaction of hadrons goes mainly through the exchange of low momentum "wee" partons. Due to this exchange the parton groups representing the colliding hadrons get some excitation mass. (A part of the longitudinal motion is transformed into the excitation masses.) According to the nova model, the distribution of the excited masses (M) is energy independent and it is given by a function like

$$\varrho(M) = \frac{c}{M^2} e^{-\frac{\beta}{M}} \quad (1)$$

Here c and β denote some parameters ³⁾. (For simplicity, the mass of the incoming proton will be neglected in what follows, hence we set $M_\alpha = 0$.) In the high transverse momentum process, the "active" (high p_T) parton collects some excitation mass in a similar way and the mass distribution of the emerging high p_T nova will be roughly the same as given by Eq. (1). Then the nova decays into hadrons (mainly into pions) isotropically in its own rest system and as it had a high transverse momentum, it will be seen in the centre-of-mass frame as a "jet" of almost parallelly moving particles **).

In order to calculate the cross-section for the high transverse momentum jet production, let us first introduce the relevant kinematical variables. The four momenta of the particles will be given generally in a "light-like" basis as $p = (p_+, p_-, \vec{p})$, where

$$p_{\pm} = \frac{1}{\sqrt{2}} (p_0 \pm p_3) = \frac{1}{\sqrt{2}} (p^0 \mp p^3) \quad , \quad (2)$$

and $\vec{p}$ denotes the two-dimensional transverse momentum. The usefulness of these components lies in their simple transformation properties under longitudinal Lorentz transformations: $\vec{p}' = \vec{p}$, $p'_{\pm} = e^{\pm y} p_{\pm}$. The "left-moving" and "right-moving" incoming particles have the momenta

*) Assumption ii) is in fact less stringent than the corresponding one of Berman, Bjorken and Kogut who assume that also the longitudinal momenta of the emerging hadrons are proportional to the parton momentum (disregarding, of course, small transverse momenta characteristic of diffraction processes).

**) As a reasonable average, according to Ref. 3), about 2.5 pions will be emitted per GeV of nova mass.

$$p_L = (p_{L+}, \frac{M_L^2}{2p_{L+}}, \vec{0}) \quad p_R = (\frac{M_R^2}{2p_{R-}}, p_{R-}, \vec{0}), \quad (3)$$

respectively. (M_L and M_R denote their masses.) The j^{th} particle in the final state has the momentum $r_j = (r_{j+}, r_{j-}, \vec{r}_j)$ and mass M_j . Instead of the components $r_{j\pm}$ it is often useful to specify the quantities

$$x_{jL} = \frac{\tau_{j+}}{p_{L+}} = \frac{\tau_{j+} p_{R-}}{s_0} \stackrel{\circ}{=} \frac{\tau_j \cdot p_R}{s_0},$$

$$x_{jR} = \frac{\tau_{j-}}{p_{R-}} = \frac{\tau_{j-} p_{L+}}{s_0} \stackrel{\circ}{=} \frac{\tau_j \cdot p_L}{s_0}, \quad (4)$$

$$s_0 \equiv p_{L+} p_{R-} \stackrel{\circ}{=} p_L \cdot p_R \stackrel{\circ}{=} \frac{1}{2} s \quad ; \quad s \equiv (p_L + p_R)^2.$$

The sign $\stackrel{\circ}{=}$ means that the equality holds whenever the masses of the particles may be neglected (as in the scaling limit). The quantity x_{jL} (x_{jR}) specifies that to what extent the j^{th} particle is left-moving (right-moving). In order to fix the momentum r_j completely, it is enough to give x_{jL} , x_{jR} and the angle α_j of the half-plane $(p_R p_L r_j)$ with some fixed half-plane $(p_R p_L r_0)$. It is clear that all of these variables are invariant against longitudinal Lorentz transformations.

The centre-of-mass frame is characterized by

$$p_{R+} \stackrel{\circ}{=} p_{L-} \stackrel{\circ}{=} \sqrt{s_0}. \quad (5)$$

(The longitudinal scaling variable x_F of Feynman is therefore given by $x_{jF} \stackrel{\circ}{=} x_{jR} - x_{jL}$.) It is easy to see that we have the relations

$$\frac{d^3 \tau_j}{\tau_{j0}} = s_0 dx_{jR} dx_{jL} d\alpha_j \quad ; \quad (6.a)$$

$$4x_{jL} x_{jR} = \mu_j^2 + \vec{s}_j^2 \quad \text{if} \quad \mu_j^2 = \frac{2M_j^2}{s_0}, \quad \vec{s}_j^2 = \frac{2\vec{\tau}_j^2}{s_0}; \quad (6.b)$$

$$1 + \mu_R^2 = \sum_j x_{jL} \quad ; \quad 1 + \mu_L^2 = \sum_j x_{jR} \quad \text{if} \quad \mu_{R,L}^2 = \frac{2M_{R,L}^2}{s_0}. \quad (6.c)$$

(The last two equations express the conservation of the p_+ and p_- components of the four-momentum.)

The high p_T hadron production in proton-proton scattering is given by the photon exchange diagram (Fig. 1) as the Drell-Yan parton-antiparton annihilation mechanism was shown in Ref. 2) to be negligible in this reaction. The momenta of the protons and of the partons are defined on Fig. 1. R is the four-momentum of the nova associated to the parton $\hat{p}$. This nova is composed of a part of the particles outgoing from the left upper blob in the diagram. In what follows, the masses of the protons and partons will be neglected in contrast to the mass M of the nova which is kept finite. [This is in the spirit of the nova model where scaling is due to the energy independence of the nova excitation function $Q(M)$.] The partons carry some fraction of the proton's momentum, that is

$$\hat{p}_L = x p_L, \quad \hat{p}_R = x' p_R. \quad (7)$$

Hence from Eq. (4) follows $\hat{q}_0 = x x' q_0$ (quantities with a hat refer always to the parton scattering), and from Eqs. (6.b), (6.c) : $1 = \hat{x}_L + \hat{x}_R$. According to our assumption ii), the transverse momentum of the nova $\vec{R}$ is equal to $\vec{\hat{p}}$, therefore using again Eqs. (6.b) and (7) :

$$\vec{q}^2 \equiv \frac{2 \vec{R}^2}{q_0} = 4 x x' \hat{x}_L (1 - \hat{x}_L) \leq 1. \quad (8)$$

From this relation

$$\hat{x}_L = \frac{1}{2} \left(1 \pm \sqrt{1 - \vec{q}^2 (x x')^{-1}} \right). \quad (9)$$

The upper sign corresponds to "more leftwards" scattering of the parton $\hat{p}$, the lower sign to "less leftwards" scattering.

The cross-section corresponding to the diagram in Fig. 1 can be easily written down from the rules of Ref. 2) :

$$d\sigma = \int dx dx' \frac{F(x) F(x')}{x x'} d\hat{x}_R d\hat{x}_L \delta(1 - \hat{x}_L - \hat{x}_R) \frac{\alpha^2 \pi}{\hat{q}_0} \cdot \frac{(1 + \hat{x}_L^2)}{\hat{x}_R^2} \quad (10)$$

and hence, using Eqs. (8) and (9), the differential cross-section for the nova production is [$\pm$ corresponding to the two signs in Eq. (9)] :

$$\frac{d\sigma_{\pm}}{d\vec{q}^2} = \frac{\pi \alpha^2}{4 q_0} \int_{\vec{q}^2 \leq x x'} dx dx' \frac{F(x) F(x')}{x x'} \cdot \frac{6 x x' - \vec{q}^2 \pm 2 \sqrt{x x' (x x' - \vec{q}^2)}}{(2 x x' - \vec{q}^2 \mp 2 \sqrt{x x' (x x' - \vec{q}^2)}) x x' \sqrt{x x' (x x' - \vec{q}^2)}} \quad (11)$$

The function $F(x)$ gives the probability distribution of the partons inside the proton as measured in deep inelastic ep scattering. An analytic expression for $F(x)$ is [see Ref. 5] :

$$F(x) = 1.1 (1-x)^3 + 0.3 (1-x)^{\frac{3}{2}} . \quad (12)$$

In order to take into account the fact that the total nova production is the sum of the nova production from the left-moving and right-moving protons, one has simply to add the two contributions corresponding to the subscripts $\pm$. Going over from $d\sigma/d\vec{q}^2$ to $d\sigma/d|\vec{q}|$ the result is

$$\frac{d\sigma}{d|\vec{q}|} = \frac{d\sigma_+}{d|\vec{q}|} + \frac{d\sigma_-}{d|\vec{q}|} = \frac{\pi\alpha^2}{q_0 |\vec{q}|^3} \int_{x x' \geq |\vec{q}|^2} dx dx' \frac{F(x)F(x')}{x x'} \frac{(16x^2 x'^2 - 12 x x' |\vec{q}|^2 + |\vec{q}|^4)}{x x' \sqrt{x x' (x x' - |\vec{q}|^2)}} \quad (13)$$

The numerical values of $d\sigma/d|\vec{q}|$ are depicted in Fig. 2a and the corresponding differential cross-sections in the transverse momentum $d\sigma/d|\vec{R}|$ are given in Fig. 2b. The values of $2q_0 = s = 500-3000 \text{ GeV}^2$ are typical for ISR measurements. The $d\sigma/d|\vec{q}|$ curves are practically straight parallel lines on the logarithmic plot, whereas the $d\sigma/d|\vec{R}|$ curves show a slight flattening out as q_0 increases.

The nova model is able to specify also the more detailed properties of the produced jets. As an example, the double differential cross-section $d^2\sigma/(d|\vec{q}|dM)$ is given by

$$\frac{d^2\sigma}{d|\vec{q}|dM} = q(M) \frac{d\sigma}{d|\vec{q}|} , \quad (14)$$

where $q(M)$ is taken from Eq. (1). It is normalized to unity here. Therefore

$$c = \frac{\beta}{2} \left(e^{-\frac{\beta}{M_{\max}}} - e^{-\frac{\beta}{M_{\min}}} \right)^{-1} \sim \frac{\beta}{2} \left(1 - e^{-\frac{\beta}{M_{\min}}} \right)^{-1} . \quad (15)$$

$M_{\min}$ is the minimum mass for the novas (say, 1 GeV) and $M_{\max}$ is the maximum mass proportional to $\sqrt{q_0}$. (hence, the second, approximate expression for c holds in the limit of high enough q_0 .) As the average number of the decay

products of a nova is proportional to its mass M , the form of the function $q(M)$ determines the prong number distribution of the jets ^{*)}.

All of our calculations are based on the assumption that the deep inelastic processes proceed via electromagnetic interactions. As it was pointed out by Berman, Bjorken and Kogut, if deep inelastic processes involving strong parton vertices are present then the above values may be multiplied roughly by a factor of $\alpha^{-2} \sim 10^{+4}$. In any case the expected magnitude of cross-sections is in the range of experimental feasibility ^{**)}.

In accordance with our assumption ii) in the present paper we calculated the differential cross-section only as a function of the transverse momentum and integrated over the longitudinal momenta of jets. The reason for this is that in the framework of the nova model the longitudinal momenta of the novas seem to be dependent on the details of the dynamics (on the eventually produced longitudinal novas, for instance, or on the mechanism resolving the puzzle of the fractionally charged parton, separated from the other partons due to its large transverse momentum). It is also possible that the mean mass of the transverse novas is for some dynamical reason essentially higher than that of the diffractive (longitudinal) novas. This results in a higher multiplicity and consequently in lower average transverse momentum of the individual hadrons reducing to a large extent the high p_T particle yield. The higher multiplicity of the decay products of novas does not prevent, however, their observation in the form of hadronic jets.

In conclusion, in parallel with the measurement of single particle distributions, the experimental determination of the cross-section for jet production will certainly give valuable information concerning the mechanism of high transverse momentum particle production.

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*) Using the detailed information of the nova model about the decay properties of the novas, it is also possible to calculate by Monte Carlo methods the cross-section for specific jet-like events satisfying definite experimental criteria, say, n charged particles be observed within the solid angle ω in the centre-of-mass frame with the total transverse momentum p_T . The cross-section formulae included in the present letter are intended as a guide for planning of experiments.

***) From the point of view of the observation of jets, the measurements at the CERN ISR are in a privileged position as it is probably much easier to identify the jets in experimental conditions where the centre-of-mass frame is at rest. I thank Dr. P. Darriulat for useful discussions on this.

R E F E R E N C E S

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FIGURE CAPTIONS

Figure 1 : The photon exchange diagram to the high p_T hadron production.

Figure 2 : The differential cross-sections for high p_T jet production.

FIG.1

FIG. 2b.

$$\frac{d\sigma}{dp_T} \equiv \frac{d\sigma}{d|\vec{R}|}$$