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MODEL AMPLITUDE ANALYSIS OF $\pi^-p \rightarrow \pi^+\pi^-n$

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A B S T R A C T

We present an amplitude analysis of the process $\pi^-p \rightarrow \pi^+\pi^-n$ at 15 GeV/c assuming spin and phase coherence of the production amplitudes. The bearing of the results on aspects of dipion production phenomenology is discussed.

We discuss a model amplitude analysis of the 15 GeV/c SLAC data on the process $\pi^- p \rightarrow \pi^+ \pi^- n$ at small momentum transfer squared t to the nucleon and for the dipion mass $M_{\pi\pi}$ in the rho band ($0.665 < M_{\pi\pi} < 0.865$ GeV)¹⁾. A complete amplitude analysis (i.e., without any model assumptions) requires the use of a polarized target^{2),3)} and we emphasize the importance of performing polarization experiments. Related vector meson production processes have also been discussed^{4),5)}. However, within the present level of statistics for experiments on dipion production, models^{6),7)} having a quite simple amplitude structure reproduce the available data at small t ($|t| < 0.2$ GeV²), where pion exchange appears to be the dominant dynamical mechanism. Here we want to investigate how strongly the data support the general structure of the amplitudes implied by these models.

We first present an amplitude analysis of the $\pi^+ \pi^-$ production process, assuming (nucleon) spin and phase coherence for the unnatural parity exchange component of the production amplitudes^{*)}. Such coherence follows strictly in any high energy model in which production occurs via elementary one-pion exchange modified by absorption (OPEA). An example is the simplified absorption model of Williams⁶⁾ to which the SLAC data have already been fitted⁸⁾. The present approach can be regarded as a generalization. Simple Regge models without absorption⁹⁾ also satisfy this coherence condition. More sophisticated models violate the coherence assumption. In particular, Reggeized absorption models entail departures from coherence mainly in the phase and we illustrate these effects.

In the following we assume that only S and P wave dipion production need be considered. This assumption is conventional, but will in future have to be abandoned to accommodate the CERN-Munich data ($\langle Y_{30} \rangle \neq 0$, $\langle Y_{31} \rangle \neq 0$)¹⁰⁾. Spin and phase coherence for the dipion production amplitudes then requires :

- i) that the ratio of nucleon helicity non-flip production to nucleon helicity flip production is independent of the dipion helicity state, and
- ii) amplitudes are all relatively real, except for an over-all S-P decay phase difference (independent of t).

In fact these coherence assumptions are only necessary for the unnatural parity exchange components^{**)}. The dipion intensity components can then be

*) A parallel investigation is being pursued by Ochs (W. Ochs, private communication).

**) This is reflected in Eqs. (1) to (6) by the natural parity exchange amplitude occurring only in the form $|F^{(+)}|^2$.

conveniently expressed in terms of the real amplitudes $F_S^{(0)}$ for S wave production, $F_P^{(0)}$ for longitudinal P wave, $F^{(+)}$ and $F^{(-)}$ for transverse P wave production, respectively, via natural and unnatural parity exchange and the relative S-P decay phase Δ :

$$(\rho_{00} - \rho_{11}) \frac{d\sigma}{dt} = F_P^{(0)2} - \frac{1}{4} [F^{(+)^2} + F^{(-)^2}] \quad (1)$$

$$\rho_{1-1} \frac{d\sigma}{dt} = \frac{1}{4} [F^{(+)^2} - F^{(-)^2}] \quad (2)$$

$$\text{Re } \rho_{10} \frac{d\sigma}{dt} = \frac{1}{2} F_P^{(0)} F^{(-)} \quad (3)$$

$$\frac{d\sigma}{dt} = F_P^{(0)2} + F_S^{(0)2} + \frac{1}{2} [F^{(+)^2} + F^{(-)^2}] \quad (4)$$

$$\text{Re } \rho_{0s} \frac{d\sigma}{dt} = F_S^{(0)} F_P^{(0)} \cos \Delta_0 \quad (\Delta_0 = \Delta) \quad (5)$$

$$\text{Re } \rho_{1s} \frac{d\sigma}{dt} = \frac{1}{2} F_S^{(0)} F^{(+)} \cos \Delta_1 \quad (\Delta_1 = \Delta) \quad (6)$$

These equations can be used in the s channel or t channel frames and we work in the high energy limit so that $F^{(\pm)}$ corresponds to $F^{m=1} \pm F^{m=-1}$. Furthermore, $t_{\min} = 0$ and the s-t helicity crossing matrix is respectively diagonal (antidiagonal) in nucleon spins for natural (unnatural) parity exchange.

The assumption of coherence is more than sufficient to determine the amplitudes and leads to constraints between experimental quantities. It turns out that the data under consideration do not furnish a very stringent test of these constraints. Two alternative procedures are adopted to derive amplitudes, a t independent analysis (TIAA) in which amplitudes are obtained separately for each t value and a t dependent analysis (TDAA) in which the data at all t values are fitted to parametric forms embodying OPE and a smooth background. The TIAA was accomplished by solving Eqs. (1), (2) and

(3) for $F_P^{(0)}$, $F^{(-)}$ and $F^{(+)}$, then (4) for $F_S^{(0)}$. Equations (5) and (6) were treated as constraints, the requirement being $\cos\Delta_0 = \cos\Delta_1 = \text{const.}$ (independent of t). The TDAA was in the main performed by fitting the experimental intensity components to the following forms of the t channel amplitudes (in $\mu\text{b GeV}^{-1}$) :

$$F_{s,p}^{(0)} = C_{s,p}^{(0)} \exp\left[D_{s,p} t/\mu^2\right] \mu \sqrt{-t}/(t-\mu^2) \quad (7)$$

$$F^{(\pm)} = C \exp\left[D^{(\pm)} t/\mu^2\right] \quad (8)$$

Here we essentially assume s channel nucleon helicity flip dominance by our choice of amplitude behaviour near $t=0$. The fitted parameters are $C_{s,p}^{(0)}$, C , $D_{s,p}$, $D^{(\pm)}$ and $\cos\Delta$.

These two procedures [in the TDAA with formulae (7) and (8) cast in the appropriate frame, using the crossing matrix for the dipion helicity] were applied to both experimental t channel and s channel intensity components to ensure that no information was lost through binning of data in t . For the s channel TDAA fits, alternative forms allowing different exponential form factors for no flip and double flip were also tried but the fits were not as good and this was not pursued. Fits were made both to all six experimental quantities (1) to (6) given equal weights (6C Fits) and with (5) and (6) essentially decoupled (4C Fits) (contribution to χ^2 multiplied by 0.1). The errors on experimental quantities were naively assumed to be uncorrelated. Also we did not include a 12% nucleon non-flip contamination from $\pi^- p \rightarrow \pi^+ \pi^- \Delta^0$ as is done in Ref. 8). Among other things, this contamination may distort density matrix elements at small t .

The results are displayed in the Table and Fig. 1, where we have allowed for the correct non-zero value of $\sqrt{-t_{\min}} = 0.14\mu$. As an example of the amplitude determinations, Fig. 1a to 1d shows the t channel amplitudes obtained by TDAA (full line) and TIAA (the points with errors), and Fig. 1e is a plot of the $\cos\Delta_0$ and $\cos\Delta_1$ values derived using Eqs. (5) and (6) as constraints. It is immediately clear that the constraints are neither seen to be violated nor strikingly vindicated. They appear to be adequately satisfied *). Figure 2 shows fits to t channel density matrix elements. Differential cross-section data were also included in the input and well reproduced by the TDAA

*) Saturation of standard Schwarz inequalities has been tested against density matrix elements from Ref. 1) in Ref. 11).

fits. Further information on $d\sigma/dt$ and s channel amplitudes is given in Ref. 12). It is of interest to remark on the emergence of the damping factor $\exp(9t/\text{GeV}^2)$ for S wave production in view of the non-attainment of the unitarity limit for the $\pi\pi$ charge exchange cross-section inferred from high energy data on $\pi^-p \rightarrow \pi^0 \pi^0 n$ ^{13),14)} if the exponential slope which best fits that data, $\exp(7t)$, is used. An exponential slope $\exp(9t)$ is still adequate and results in a $\sigma_{\pi\pi}(\text{CEX})$ which does achieve the unitarity limit ¹⁴⁾.

Comparison of the TIAA and TDAA results in Fig. 1 is in general satisfactory, except for $F^{(-)}$, $F^{(+)}$ and $F_S^{(0)}$ for $|t| < 0.8 \mu^2$. The anomalous behaviour of our TIAA results for these amplitudes can be traced to the experimental results for $\text{Re } \mathcal{S}_{10}$ in Fig. 1f. The 12% contamination attributed to $\pi^-p \rightarrow \pi^+ \pi^- \Delta^0$ in Ref. 8) may account for this behaviour. Preliminary results from the high statistics CERN-Munich experiment at 17 GeV/c indicate a behaviour more in accord with our prediction ¹⁰⁾. However, it is important to note that only by the detection of kinematically anomalous behaviour in experimental quantities very close to $t=0$ (e.g., the non-vanishing of $\mathcal{S}_{00} + \mathcal{S}_{ss}$ or of $\text{Re } \mathcal{S}_{os}$) can we hope to detect any evidence for A_1 exchange in this process, from unpolarized experiments.

We now briefly discuss the assumptions of our analysis, their consequences, their relation to other people's assumptions and the extent to which the present assumptions can be said to be upheld by the data. We also discuss our result on the amount of transverse rho production at $t=0$ and conclude with some comments on further details which are of interest in experiments of this type and a plea for polarization experiments.

First our spin and phase coherence assumption. As stated earlier, this follows strictly at high energy if production is governed by elementary OPEA. A consequence of such coherence is the simplification of intensity components in terms of amplitudes which we have exploited, allowing the amplitudes to be solved for, with equations to spare (constraints). As already mentioned, the constraints of Eqs. (5) and (6) are not strongly tested in the present data (Fig. 1e). The method of Schlein ^{15),16)} assumes phase coherence but not spin coherence. In contrast, conventional absorbed Reggeism ($\pi + A_2$ exchanges with π entirely, A_2 predominantly nucleon s channel helicity flip) would readily assimilate spin coherence as a good first approximation but disfavour the concept of phase coherence ^{*}). Absorption should disturb the Regge phases

^{*}) The natural breakdowns of spin or phase coherence, respectively, allowed by these two approaches, correspond in our amplitude analysis equations (1)-(6) to making $\Delta_0 \neq \Delta_1 \neq \text{constant}$ and inserting an extra factor of $\cos(\theta_0 - \theta_1)$, where $\cos \Delta_1 = \cos \Delta \cos \theta_1$, or of $\cos(\Delta_0 - \Delta_1)$ respectively in Eq. (3).

for Reggeized OPE differently in the various amplitudes and also add in A_2 cut effects. Since the absorptive cut associated with the nucleon non-flip A_2 exchange (net helicity flip $n=1$) pole term is a small correction to an already small amplitude, it follows that spin coherence is a very good approximation for the unnatural parity exchange amplitudes.

In order to calibrate our method against Regge orthodoxy, we generated intensity components from a strongly absorbed mixture of Reggeized π and A_2 exchange chosen roughly to resemble the data. We did not attempt to make a "best fit" to the data and we neglected the nucleon non-flip A_2 exchange amplitude, which is inessential for the purposes of our discussion. We then applied our TIAA to the resulting theoretical "data" and the results are shown in Fig. 2. $\hat{F}_{S,P}^{(0)}$ and $\hat{F}^{(\pm)}$ denote the quantities inferred on our method and $\text{Re } F_{S,P}^{(0)}$, $\text{Im } F_{S,P}^{(0)}$, etc., are the input quantities. The quantities $\cos \Delta_0$, $\cos \Delta_1$ of Eqs. (5) and (6) came out to lie between 1.0 and 0.92 (a smaller spread than the typical error bar in Fig. 1e) up to and including $\sqrt{-t} = 2.5\mu$. The corresponding phase difference between $-F^{(0)}$ and $F^{(-)}$ varies from 3° to 23° . This "experiment" suggests firstly that our method may well give a reasonable determination of the moduli of the production amplitudes; secondly, it illustrates the insensitivity of the constraints to quite appreciable breakdowns of phase coherence at the present level of experimental statistical accuracy.

The amount of transverse rho production at $t=0$ sheds an interesting light on absorbed Reggeism and on vector meson dominance. In particular, one can contrast the prediction from absorbed one-pion exchange (OPEA) expressed via the familiar convolution integral with the prediction from what is often represented as a simplified version of this, the procedure which Fox^{17),18)} has dubbed "Poor Man's Absorption" (PMA). This latter consists of taking the helicity dependent kinematic factors of t appearing in the numerators of the input pole s channel amplitudes and replacing all factors not required by angular momentum conservation by their residues at the pion pole. These alternative procedures are illustrated symbolically in the following equations for the net helicity flip zero ($n=0$) amplitude

$$S^{n=0} = S_{abs} \otimes \frac{\sqrt{2} t}{t - \mu^2} \Phi(t) \quad - \quad \text{OPEA} \quad (9)$$

$$\rightarrow \frac{\sqrt{2} \mu^2}{t - \mu^2} \Phi(t) \quad - \quad \text{PMA} \quad (10)$$

Here the quantity $\Phi(t)$ represents all additional factors - coupling constants, form factors including all Regge factors aside from the dependence explicitly shown. The same factor appears in the other helicity amplitudes, only the explicit kinematic t factors differ. The Williams model ⁶⁾ consists in using the PMA formulae with $\Phi(t)$ as an over-all collimating factor adjusted to fit $d\sigma/dt$. If $\Phi(t)$ is a constant, then the substitution shown in Eq. (10) consists in the removal of the major $J=\frac{1}{2}$ contribution to production and can plausibly be attributed to strong central absorption. However, once $\Phi(t)$ is not a constant, as for example for Reggeized OPE, the PMA effects all partial waves and in general can extend over a larger range (in impact parameter, b) than is customarily allowed from absorption ¹⁸⁾. This depends on the form of $\Phi(t)$, which is an unfortunately underconstrained entity in Reggeism. A relevant example is illustrated in Figs. 3a and 3b, which show the results of applying the PMA prescription to OPE and the pion Regge input used in the strong absorption model (SCRAM) test of our analysis method (Fig. 2). The crucial characteristic of the OPE input (in addition to the pion pole) is an exponential collimation factor $\Phi(t) = \exp(0.04t/\mu^2)$ chosen to mimic conventional Regge collimation effects. A key factor of the output is the resulting magnitude of the transverse rho production at $t=0$ relative to the OPE signal or equivalently the size of the cut. This can be characterized by the parameter γ_1^P which we previously introduced ¹⁹⁾, defined asymptotically by $S^{n=0}(t=0) = -\sqrt{2} \gamma_1^P \Phi(t=0)$. With conventional form factors the "Poor Man's Cut" is much larger than the "Rich Man's Cut" (SCRAM π cut) and is of the correct magnitude to explain the entire observed effect from $p_{lab} = 2.7$ to 15 GeV/c [cf., Fig. 1 of Ref. 15) *).]. In contrast, the orthodox absorbed Regge approach has a substantial contribution from absorbed A_2 exchange as illustrated in Fig. 3b for the real part of the $\pi + A_2$ SCRAM calculation of Fig. 2. The detailed differences between these two approaches need careful study. The outcomes are likely not so sharply contrasting as might at first sight appear - for example the relative energy dependence of the π and A_2 contribution on the absorption model will go as $s^{\alpha_\pi(\bar{t}_\pi) - \alpha_{A_2}(\bar{t}_{A_2})}$ not as $s^{\alpha_\pi(0) - \alpha_{A_2}(0)}$ where $\bar{t}_\pi(\bar{t}_{A_2})$ is the average t value sampled by the convolution ²⁰⁾. Mass dependence does not distinguish between these alternatives. Photoproduction at small t is readily accommodated in both frameworks ^{21), 22)}.

*) Note that an over-all collimation factor is not allowed for in Ref. 15) in relating the strength of $t=0$ transverse rho production to the OPE contribution at the pion pole. For a collimation factor $\exp(Dt/\mu^2)$ with $D \sim 0.091$, the γ_1^P 's of Ref. 15) should be multiplied by 0.91.

The determination of the parameter γ_1^P from $\pi^+\pi^-$ production is important for testing the vector meson dominance relation between transverse rho production and isovector charged pion photoproduction at small t . Assuming a rho photon coupling constant of $\gamma_\rho^2/4\pi = 0.5$, the present fits would indicate a prediction of $200 \pm 50 \mu\text{b}(\text{GeV})^2$ for the forward value of $(s-m_N^2)^2(d\sigma/dt)$ for charged pion photoproduction, to be compared with the observed value of $280 \pm 30 \mu\text{b}(\text{GeV})^2$ - cf., Ref. 23) where the authors of Ref. 1) make their own comparison. Also our fits do not give such a very sharp forward peak as obtained in Ref. 23), but are in more accord with ones expectations¹⁸⁾ for the s channel amplitudes. The transverse component of the rho production is of course a relatively small effect and furthermore one which has to be disentangled from S wave production.

We have seen that, although compatible with the small t ($|t| < 10 \mu^2$) data, the model assumptions used in our amplitude analysis are not stringently tested and sizeable breakdowns of phase coherence by $\sim 20^\circ - 30^\circ$, as possible in the SCRAM model, cannot be ruled out. Data of increased statistical precision will further tie down the production amplitudes. In particular the density matrix element ρ_{1-1}^H in the s channel frame, being governed by the product of the no net flip $S^{n=0}$ and double flip $S^{n=2}$ amplitudes (assuming predominant nucleon flip) is an especially informative quantity. It is also of great interest to extend the study of the production amplitudes to larger $|t|$ values. The behaviour of $\rho_{00}^H(d\sigma/dt)$, $\rho_{10}^H(d\sigma/dt)$ and S wave dipion production near $t = -0.6 \text{ GeV}^2$ is of particular interest. As pointed out by Fox¹⁸⁾, the underlying unnatural parity exchange is presumably nucleon helicity flip and corresponds to an "unhappy" net flip $n=1$ amplitude for longitudinal rho or S wave production - i.e., the expected absorption zero in $\text{Im} S^{n=1}$ does not coincide with a wrong signature zero ($\alpha_\pi = -1$). In addition to detailed t dependence, it will be interesting to peruse $M_{\pi\pi}$ and dipion spin dependence. The Williams model has analogous predictions for D wave production with the predominant background again in $m=1$ helicity states.

However, substantial progress towards the goal of a complete amplitude analysis must await production experiments on polarized targets^{2),3)}. Dipion production is a very attractive field for such work. In fact, the eight underlying complex amplitudes can be essentially determined up to one relative phase^{4),5)} *) (in addition to the usual over-all phase ambiguity) from experiments with a target polarized perpendicular to the beam^{**)}. Sign ambiguities in this solution are removed by experiments with a longitudinally polarized

*) A nucleon spin correlation experiment analogous to the A and R measurement in πN elastic scattering would determine this relative phase.

**) For S and P wave dipion production, nine extra measurables are obtained.

target ⁴⁾ (or, perhaps more realistically, from recoil polarization). With the predominant exchanges already in part understood a big advance in our understanding can be looked for from transversely polarized target experiments alone. Targets polarized perpendicular to the production plane ^{*)} are particularly sensitive for detecting the exchange of A_1 quantum numbers. Conversely if A_1 exchange and the net helicity flip $n=1$ A_2 cut are negligible, only one of the six measurable quantities is non-zero at high energies ³⁾. On the other hand, targets transversely polarized in the plane of production allow the measurement of three nucleon flip, non-flip interference terms between unnatural and natural parity exchanges, which can be sizeable. In our simplified $\pi + A_2$ SCRAM model calculation of Fig. 2, the transversely polarized target dipion density matrix elements all vanish, since we have neglected helicity non-flip amplitudes. However, we note that sizeable effects are obtained with longitudinal target polarization in this example [e.g., the three dipion density matrix elements describing the dipion decay off a longitudinally polarized target, defined analogously to Ref. 3), are all greater than 0.1 in magnitude around $\sqrt{-t} = 3\mu$]. A model calculation of various transversely polarized target observables has been given by Fox in Ref. 3).

A start on polarization experiments has been made by Sonderegger and Bonamy ¹³⁾ with measurements on $\pi^- p(\uparrow) \rightarrow \pi^0 \pi^0 n$. They report a considerable asymmetry indicative of A_1 exchange, which is an important experimental result to be checked. A substantial A_1 coupling could of course disrupt the spin coherence assumption but, as in the case of phase coherence discussed above, a sizeable breaking could pass unnoticed in unpolarized experiments. One can even devise special, but by no means implausible, coupling schemes for A_1 to $\rho\pi$ and $\epsilon\pi$ such that there is no breaking of spin coherence. Polarization experiments on $\pi^- p \rightarrow \pi^+ \pi^- n$ will be of the greatest interest, allowing us to settle these points and many others.

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*) These measure nucleon flip, non-flip cross-terms between amplitudes of the same naturality exchange.

ANALYSIS	$c_P^{(0)}$	c	$c_S^{(0)}$	D_P	$D^{(-)}$	$D^{(+)}$	D_S	$\cos \Delta$	χ^2/NDF	χ^2_1	$c_S^{(0)}/c_P^{(0)}$
4C Fit to t channel data	34.2	9.34	12.78	0.091	0.106	0.071	0.092	0.997	1.65	0.90	0.374
6C Fit to t channel data	34.3	9.25	12.88	0.092	0.104	0.069	0.096	1.000	1.46	0.89	0.375
4C Fit to s channel data	34.10	8.61	14.69	0.086	0.094	0.074	0.101	0.883	1.53	0.82	0.431

TABLE : Parameters from representative t dependent amplitude analyses
[Cf. Eqs. (7) and (8)].

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FIGURE CAPTIONS

Figure 1 : Plots of t channel production amplitudes and density matrix elements.

- a) $F_P^{(0)}$ longitudinal P wave production;
- b) $F_P^{(-)}$ unnatural parity exchange transverse P wave production;
- c) $F_P^{(+)}$ natural parity exchange transverse P wave production;
- d) $F_S^{(0)}$ S wave production;
- e) results for the constraint testing quantities $\cos \Delta_0$, (circles), $\cos \Delta_1$ (squares) of Eqs. (5) and (6) for the TIAA (the horizontal dashed line indicates the lower end of the range of fitted TDAA $\cos \Delta$ values);
- f) TDAA fits to density matrix elements.

i) For a)-d) the full lines denote the results derived by TDAA and the points with error bars are the results of TIAA (not data).

Figure 2 : Test using theoretical "data" (SCRAM $\pi+A_2$) of analysis method based on assumption of spin and phase coherence. $\text{Re } F_P^{(0)}$, $\text{Im } F_P^{(0)}$, etc., are the input amplitudes (full lines); $\hat{F}_P^{(0)}$, etc. (dashed lines) is the TIAA output.

Figure 3 : a) "Poor Man's Absorption" (PMA) versus "Rich Man's Absorption" (RMA) in impact parameter space (b space). The example illustrated concerns the (elementary) OPE contribution to the no-net flip s channel amplitude $S^{n=0}$. The full line denotes the OPE input (complete with exponential form factor); the dashed line shows the same quantity modified by PMA; and the dotted line depicts a typical RMA modification [Worden's rounded-off square cut version of SCRAM ²²] - note that the wiggles are harmless and come from the assumed form of S_{abs} .

b) Contrasts of absorption prescriptions in t space for the $S^{n=0}$ amplitude. In addition to the t space transforms of the quantities shown in a), the corresponding cut contributions ($S_{\text{abs}} \otimes \text{OPE} - \text{OPE}$) are shown as dashed lines. Also shown are some key contributions to the $\pi+A_2$ SCRAM example plotted in Fig. 2, $\text{Re}[S_{\text{abs}} \otimes \text{Reggeized OPE}]$ (dash-dot line) and $\text{Re}[S_{\text{abs}} \otimes (\pi+A_2)]$.

FIG.1

FIG. 2

FIG.3