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SOME ASPECTS OF VECTOR MESON PHOTOPRODUCTION ON PROTONS *)

K. Schilling

CERN - Geneva

A B S T R A C T

Some recent models on diffraction dissociation of photons are reviewed that go beyond simple vector meson dominance of the final state by including non resonant diffractively produced background.

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1. INTRODUCTION

The high energy photoproduction of vector mesons near the forward direction

$$\gamma p \rightarrow V^0 p \quad ; \quad V^0 = \rho^0, \omega, \phi, \dots \quad (1)$$

belongs to the class of diffractive processes ¹⁾

$$A + B \rightarrow A + X, \quad (2)$$

which are characterized by no quantum number (Q,I,S,B,C) exchange and by a seemingly constant, non-vanishing cross-section at high energies ^{2),3)}. Here, X may be either a resonance or a non-resonating multiparticle system. Known examples of such diffraction dissociation channels are ^{4),5)}

$\pi^\pm p \rightarrow (3\pi)^\pm p$	$[A_1]$	
$\pi p \rightarrow \pi (N\pi)^\pm$	$[N^*(1470), N^*(1650)]$	
$\pi (N\pi\pi)^\pm$	"	
$K^\pm p \rightarrow (K^\pm \pi^+ \pi^-) p$	$[Q]$	(3)
$\gamma p \rightarrow (\pi^+ \pi^- \pi^0) p$	$[\omega]$	
$(\pi^+ \pi^-) p$	$[\rho^0]$	
$(K^+ K^-) p$	$[\phi]$	

These reactions are commonly thought of to proceed by Pomeron ($\mathbb{P}$) exchange, as indicated in Fig. 1. Needless to say, the diffractive channels are going to be the most important single inelastic channels in the region of several hundred GeV, where the inelastic reactions involving quantum number exchange are expected to be strongly suppressed for their lower lying trajectories.

The classical empirical approach to such few body final state reactions is to extract a quasi-two-body final state from the data and to compare this sample with theoretical models [for the diffraction dissociation processes given in (3), we have indicated the most important observed resonances in brackets]. This procedure works very nicely in the case of narrow resonances on top of a small background, like in ω or ϕ photoproduction⁵⁾, but it becomes problematical in the case of a large diffractively produced background as in the hadronic examples listed above, where the very resonance character of the A_1 , Q and $N^*(1470)$ bumps is even in dispute⁶⁾.

The 2π photoproduction is an intermediate case⁵⁾: the 2π mass spectrum is indeed strongly ρ dominated, but the ρ shape shows a dramatic deviation from the Breit-Wigner form⁷⁾. This phenomena has for long been a nuisance to anyone interested in ρ physics, and various prescriptions have been published to extract the " ρ " out of the data. Unfortunately, the resulting " ρ " cross-sections depend on the recipe by which they were cooked up. But there is some interesting physics in this annoying phenomenon, and hence one should regard it as a tool to learn something about diffraction dissociation, following Harari's surmise that the best place to learn about hadron physics is an electron accelerator.

In this talk, I shall try to review the status of those models, that have been proposed with this motivation and, therefore, treat diffractive 2π photoproduction as a whole. (These models, of course, apply, mutatis mutandis, also for $K\bar{K}$ photoproduction, for which only meager data exist; 3π photoproduction must have frightened the theorists for its complexity, and maybe the experimental status was not such a challenge to them). I shall say very little about the vector meson dominance model in $\gamma p \rightarrow Vp$ and nothing on vector meson production on nuclei and $\rho^0 - \omega$ interference which shall be covered by other speakers during the Summer Institute.

Before I come to diffractive models for $\gamma p \rightarrow \pi^+ \pi^- p$ in Section 3, let me first give a brief account on the competing, i.e., non-diffractive contributions to $\gamma p \rightarrow Vp$.

2. NON-DIFFRACTIVE CONTRIBUTIONS

1) Some SU(3) arguments about couplings.

In $\gamma p \rightarrow V^0 p$, the main competitor to $\mathbb{P}$ exchange is expected to be π^0 exchange. By SU(3) arguments with ideal mixing, one finds the $\omega\pi\gamma$ coupling to be three times stronger than the $\rho^0\pi^0\gamma$ coupling, and the $\phi\pi\gamma$ coupling zero ⁸⁾

$$\lambda_{\rho^0\pi\gamma} : \lambda_{\omega\pi\gamma} : \lambda_{\phi\pi\gamma} = 1 : 3 : 0. \quad (4)$$

On the other hand, the $\mathbb{P}$ couplings can easily be related on the assumption that $\mathbb{P}$ is an SU(3) singlet and γ a u spin scalar ⁹⁾:

$$\lambda_{\gamma\mathbb{P}\rho^0} : \lambda_{\gamma\mathbb{P}\omega} : \lambda_{\gamma\mathbb{P}\phi} = 3 : 1 : -\sqrt{2}. \quad (5)$$

Although the last ratio of Eq. (5) is badly broken experimentally ($\sigma_{\phi}^{\text{Diff}}/\sigma_{\rho^0}^{\text{Diff}} = 0.6 \text{ fb}/15 \text{ fb}!$), which can be related by quark model arguments to the SU(3) breaking observed in $\sigma_{\pi p}^{\text{tot}} \neq \sigma_{Kp}^{\text{tot}}$ ⁹⁾ in the diffraction region, Eqs. (4) and (5) nevertheless teach us that ρ^0 photoproduction has the largest diffractive contributions on one hand and the smallest π exchange contributions on the other, the inverse being true for ω photoproduction, while ϕ photoproduction has no π exchange at all and is thus the best reaction to study diffraction ¹⁰⁾ if it were not for the minute cross-section.

2) Experimental checks for $\mathbb{P}$ exchange.

There are essentially three pieces of evidence that hint at the dominating role of $\mathbb{P}$ exchange in $\gamma p \rightarrow Vp$: constant cross-section, isospin zero and natural parity in the t channel (the latter being suggested by the idea of a universal pomeron).

a) Energy dependence.

Experimentally, ρ^0 and ϕ photoproduction are fairly constant above 5 GeV, while ω photoproduction has a marked energy dependence ^{5),7)}. Writing its cross-section as an incoherent sum of $\mathbb{P}$ and π exchange

$$\sigma(E_{\gamma}^{\text{lab}}) = \sigma_{\mathbb{P}} + \frac{c}{E_{\gamma}^{\text{lab}^2}}, \quad (6)$$

one can determine empirically the diffractive part, σ_p , to be $\approx 2 \mu b$ [in nice agreement with Eq. (5) and $\sigma_s \approx 15 \mu b$] and the constant c , that rules the strength of one pion exchange. In the framework of the absorption model, c can be related to the width $\Gamma_{\omega \rightarrow \pi\pi}$ and one obtains this way ¹¹⁾

$$\Gamma_{\omega \rightarrow \pi\pi} \approx 1.2 \text{ MeV}, \quad (7)$$

which is a reasonable value.

b) t channel isospin.

The t channel isospin can be studied by looking at the photoproduction on deuterons :

$$\gamma d \rightarrow d(n\pi) s^0. \quad (8)$$

The ratio of the forward production cross-sections on deuterons and protons gives an indication for the presence of t channel isospin 1 and 0 exchange amplitudes, $A^{(1)}$ and $A^{(0)}$. In the impulse approximation, using closure and neglecting spin-flip amplitudes, one finds

$$R_{DH} \equiv \left. \frac{d\sigma_D}{d\sigma_p} \right|_{0^\circ} = \frac{4|A^{(1)}|^2}{|A^{(1)} + A^{(0)}|^2}. \quad (9)$$

With Glauber corrections one would expect R_{DH} to be about 3.6 in case of pure exchange. Bubble chamber data yield a 10 - 20% lower value ¹²⁾. The situation can best be assessed by looking at Fig. 2 that shows the results of the most recent analysis of the Cornell group on their counter data ¹³⁾. Since the determination of the forward production cross-section depends rather sensitively on the method of data analysis and since one has to expect some 10 - 20% background contamination of non 2π events in the counter experiments ⁵⁾, I would see no compelling evidence for the presence of $I_t = 1$ amplitudes above 4 to 5 GeV from this data. On the other hand, a value of $|A^{(1)}|/|A^{(0)}|$ of the order of $\sim 10\%$ cannot be definitely excluded.

A direct way of looking at $I_t = 1$ exchange is, of course, to study charged s photoproduction

$$\gamma n \rightarrow s^- p. \quad (10)$$

Figure 3 shows the experimental result ¹⁴⁾, whose order of magnitude is in agreement with one-pion exchange, with a width of $\Gamma_{\rho \rightarrow \pi \gamma} = 0.13$ MeV.

c) t channel normality.

The t channel normality, defined as $N_{ex} = P \cdot \tau$, with P, τ the parity and signature of the exchanged trajectory, can be readily determined with the help of polarized photons. Given a definite normality N_{ex} in the t channel, the s channel helicity amplitudes obey the symmetry relation ¹⁵⁾

$$T_{-\lambda_V \lambda_{N'}, -\lambda_\gamma \lambda_N}^{(s)} = -N_{ex} (-)^{\lambda_V} T_{\lambda_V \lambda_{N'}, \lambda_\gamma \lambda_N}^{(s)} \quad (11)$$

to leading order in s. Exploiting this relation, one can construct combinations of density matrix elements that correspond to a pure normality N_{ex} ¹⁶⁾. For that purpose, the photon density matrix is parametrized by the Stokes parameters P_1, P_2, P_3 as

$$S^\gamma = \frac{1}{2} (I + \vec{P} \cdot \vec{\sigma}), \quad (12)$$

where I is a 2×2 unit matrix and $\vec{\sigma}$ are the usual Pauli matrices. The vector meson spin density matrix, defined by

$$S^V = T^{(s)} S^\gamma T^{(s)\dagger} \quad (13)$$

can now be expanded in the basis

$$(S^0, \vec{S}) = \frac{1}{2} T^{(s)} (I, \vec{\sigma}) T^{(s)\dagger} \quad (14)$$

with all nucleon helicities summed over. $S^0, \vec{S}$ contain the full information obtainable from experiments without nucleon polarization. It turns out, that they contain no interference terms between natural ($N_{ex} = +1$) and unnatural ($N_{ex} = -1$) parity exchange contributions.

$$S^i = S_{N_{ex}=+1}^i + S_{N_{ex}=-1}^i ; i = 0, \dots, 3 \quad (15)$$

The latter can be obtained through relations like

$$\begin{aligned} S_{\lambda\lambda'}^1 N_{ex} &= \frac{1}{2} (S_{\lambda\lambda'}^1 - N_{ex} (-)^{\lambda} S_{-\lambda\lambda'}^0) \\ S_{\lambda\lambda'}^0 N_{ex} &= \frac{1}{2} (S_{\lambda\lambda'}^0 - N_{ex} (-)^{\lambda} S_{-\lambda\lambda'}^1). \end{aligned} \quad (16)$$

Furthermore, the parity asymmetry P_{σ} can be written in terms of S^1 :

$$P_{\sigma} \equiv \frac{\sigma^{\text{nat}} - \sigma^{\text{unnat}}}{\sigma^{\text{nat}} + \sigma^{\text{unnat}}} = 2 S_{1-1}^1 - S_{00}^1. \quad (17)$$

Figure 4 shows the result of the 4.7 GeV SLAC-Tufts-Berkeley HBC experiment on $\delta p \rightarrow \rho p$ ¹⁷⁾. Within small error, P_{σ} is equal to one, i.e., $N_{ex} = +1$. For ω production, on the other hand, one has a sizeable energy dependent $N_{ex} = -1$ component ¹⁷⁾, in agreement with one-pion exchange, whereas σ^{nat} looks fairly constant between 2.8 and 4.7 GeV. So we have good evidence that 2π photoproduction is a process with a large diffractive component. Before I proceed to discuss more detailed models on $\delta p \rightarrow 2\pi p$, let me give the complete list of experimental features of this process ^{5),7)}:

- a) constant cross-section above ~ 5 GeV,
- b) strong ρ dominance of the final state,
- c) asymmetric ρ shape,
- d) the slope $A(m_{\pi\pi})$ of the proton-proton momentum transfer distribution, $\frac{d\sigma}{dt_{pp}} \sim e^{A(m_{\pi\pi})t_{pp}}$ decreases with increasing $m_{\pi\pi}$,
- e) natural parity exchange in ρ region,
- f) s channel helicity conservation (SCHC) in ρ region ^{17),18)}.

3. DIFFRACTIVE MODELS FOR $\gamma p \rightarrow \pi^+ \pi^- p$

1) The vector dominance approach.

Vector meson dominance can be applied in two stages: on the final state only and on both initial and final states. The process $\gamma p \rightarrow \rho p$ can be treated with the standard Regge machinery ¹⁹⁾. Any description of $m_{\pi\pi}$ dependent effects in this framework, however, must make use of ad hoc assumptions. For nothing is known about the behaviour of amplitudes under continuation in the masses of outside particles. E.g., a $1/m_{\pi\pi}^2$ modification to the Breit-Wigner shape can be accomplished by choosing

the kinematic singularity free t channel helicity amplitudes $m_{\pi\pi}$ independent ²⁰⁾. Apart from this, all the other observed features of our process can be naturally incorporated into the Regge framework ²¹⁾.

The second stage vector meson dominance relates our process to elastic hadronic scattering, $S p \rightarrow S p$. As already mentioned in the Introduction, this sort of approach has nothing to say about $m_{\pi\pi}$ dependent effects, the famous Ross-Stodolsky factor $1/m_{\pi\pi}^2$ ²²⁾ being again due to an ad hoc smoothness assumption. On the other hand, second stage vector meson dominance has led to a number of interesting relations between hadronic and electromagnetic phenomena that shall be discussed in the talks of Professor Gottfried and Dr. Schildknecht during this symposium.

2) The multiperipheral model.

This model is just opposite to the vector meson dominance idea : the $\pi^+\pi^-$ pair is produced in a continuum state, by dissociation of the incoming γ into $\pi + \chi$ and subsequent diffractive scattering of χp into πp , χ being either π or A_1 . The $\pi\pi$ system having $G = 1$, it can only be diffractively produced by the isovector photon, leading to isovector 2π final states with odd angular momentum. Drell originally assumed only intermediate pions ²³⁾ (see Fig. 1b). Naively, one would write for the two possible Drell diagrams, neglecting the unobserved nucleon spin

$$T = e \left\{ \frac{\varepsilon q^+}{k q^+} T_- - \frac{\varepsilon q^-}{k q^-} T_+ \right\} \quad (18)$$

with

$$T_{\pm} = i S_{\pm} \sigma_{\pi^2 p}^{\text{tot}} e^{\frac{3}{2} t_{pp}} f(t_{\pm})$$

$$S = m_{\pi\pi}^2 = (q^+ + q^-)^2, \quad S_{\pm} = (p' + q^{\pm})^2$$

$$t_{\pm} = (q^{\pm} - k)^2, \quad \bar{S} = (k + p)^2$$

$$t_{pp} = (p' - p)^2.$$

For notations see Fig. 1b. Apart from the factor $f(t_{\pm})$, which describes the possible off-shell dependence of the intermediate pions, $T_{\pm}$ is the amplitude of diffractive πN scattering, i.e., $B \approx 8 \text{ GeV}^{-2}$.

The expression Eq. (18) for T is only gauge invariant on the pion pole ; as one goes away from the pole, one has to write some gauge invariant extension, which, unfortunately, is rather arbitrary. For instance, one might replace ²⁴⁾

$$\varepsilon \cdot q^\pm \rightarrow \varepsilon \cdot q^\pm - \frac{k \cdot q^\pm}{k \cdot a} a \cdot \varepsilon \quad (19)$$

with a any of the four vectors appearing in the problem ; e.g., one might choose $a = p + p' = P^+$ which leads to an unphysical pole far off at $kP^+ \sim \bar{s} - \bar{u} = 0$ ²⁵⁾. Or one might split off a gauge invariant part and make the rest gauge invariant by extension :

$$\begin{aligned} \pi \rightarrow e \left\{ \frac{\varepsilon q^-}{k q^-} - \frac{\varepsilon q^+}{k q^+} \right\} \frac{1}{2} (\tau_+ + \tau_-) \\ + e \left\{ \frac{\varepsilon q^-}{q^- k} + \frac{\varepsilon q^+}{k q^+} - \frac{\varepsilon a}{k a} - \frac{\varepsilon a'}{k a'} \right\} \frac{1}{2} (\tau_+ - \tau_-). \end{aligned} \quad (20)$$

This was the procedure of Kramer et al. ²⁶⁾, who chose $a = p$, $a' = p'$, thus obtaining nucleon pole terms, which are hard to interpret in terms of diffraction. Another prescription is to set the term proportional to $\tau_+ - \tau_-$ equal to zero. With $\frac{s_+ + s_-}{2} \sim \bar{s}$, this is equivalent to a Pomeron of effective spin 0^+ , which has been assumed in dual diffractive dissociation models ^{27),28)}. It might be comforting to realize that all these recipes coincide for forward production, if one chooses the Coulomb gauge either in the over-all centre-of-mass system or in the 2π rest system. For the general kinematical situation, however, one should keep in mind that this ill-definedness of the Drell amplitude has great bearing on the 2π decay distribution. We, therefore, expect the Drell model to have predictive power only on the more global properties of the process such as $d\sigma/dt_{pp}$, $d\sigma/dm_{\pi\pi}$.

3) The interference model.

The interference model ²⁹⁾ writes the 2π photoproduction amplitude as a sum of a directly produced Breit-Wigner shaped ρ plus the Drell term and thus successfully describes the ρ asymmetry by the interference of the real parts of the two terms (see Fig. 5). It also predicts the mass dependence of the t_{pp} slope, but not the ρ polarization and the absolute value of the cross-section.

It has often been objected that the interference model implies double counting. This is not so obvious in the framework of finite-energy-sum rule duality, which only connects the imaginary parts of the direct channel with the imaginary parts of the crossed channels. The success of the interference model, however, is mainly a real part effect. Yet, it is not clear, how to approximate the real part. How much of the real part of the t channel Regge exchange amplitude should be added to the s channel resonance, after the resonance has been given a real part à la Breit-Wigner ? ³⁰⁾

A more serious objection to the interference model is that it does not reproduce the $\pi\pi$ p wave scattering phase δ as predicted by the Watson theorem, which we would expect to hold in $\gamma\pi \rightarrow \pi\pi$ - although it cannot be proved here rigorously as in genuine two-body photoproduction processes. It has been suggested to achieve the expected phase by applying a factor $\cos \delta e^{i\delta}$ to the p wave part of the Drell diagram before adding it to the ρ amplitude ^{24),31)}. For small δ , this factor corresponds to final state interaction. For large δ , it can hardly be justified this way. In fact, one would expect the final state interaction to bring about an enhancement in the ρ region. This brings us directly to the final state interaction model.

4) Final state interaction model.

Kramer and Uretsky ²⁶⁾ have carried out the idea that the ρ photoproduction can be visualized as a final state interaction effect in the p wave part of the Drell amplitude. Symbolically

$$\Gamma_{\gamma\pi \rightarrow \rho\pi} = \Gamma_{\text{Drell}}^{\text{p-wave}} \times \text{FINAL STATE INTERACTION.} \quad (21)$$

In their original version, they chose an enhancement factor E to account for final state interaction

$$E(m_{\pi\pi}) = \frac{\lambda}{m_\rho^2 - m_{\pi\pi}^2 - i m_\rho \Gamma_\rho^{\text{tot}}} \quad (22)$$

The normalization λ remains an open constant. In a more refined version of this model, whose details we cannot cover here, Kramer and Quinn ²⁶⁾ set up and approximately solved the N/D problem for the 2π p wave. Again, the over-all normalization is undetermined ³²⁾. It is related to the unknown behaviour of the p wave amplitude for large $m_{\pi\pi}$. Their result is essentially

$$\left. \frac{d^2\sigma}{dt_{pp} dm_{\pi\pi}^2} \right|_{\text{p-wave}} = \frac{\pi m_\rho \Gamma(m_{\pi\pi})}{(m_\rho^2 - m_{\pi\pi}^2)^2 + m_\rho^2 \Gamma_\rho^2(m_{\pi\pi})} \cdot \left(\frac{m_\rho^2 - t_{pp}}{m_{\pi\pi}^2 - t_{pp}} \right)^2 \frac{4d^2\lambda^2}{8s_{\pi\pi}^{3/2} 4u} \frac{d\sigma_{\pi\pi}}{dt_{pp}} \quad (23)$$

with $\Gamma(m_{\pi\pi})$ the Jackson p wave width.

Figure 6 to Fig. 8 show that this formulation describes very well the shape of $d\sigma/dm_{\pi\pi}$ and $d^2\sigma/dt_{pp} dm_{\pi\pi}^2$. This is essentially due to the factor $1/(m_{\pi\pi}^2 - t_{pp})^2$, whose origin is the pion propagator which can be written as

$$\frac{1}{t_{\pm} - m^2} = \frac{1}{2q^2 k} \sim \frac{1}{m_{\pi\pi}^2 - t_{pp}} \cdot \frac{1}{1 \pm b \cdot \cos \Theta_{\pi\pi}} \quad (24)$$

in the 2π rest system, with b a smooth function. This factor is a good old friend by now : except for the t_{pp} term, it is the Ross-Stodolsky factor. The appearance of t_{pp} is mainly responsible for the successful description of the slope variation with mass, $A(m_{\pi\pi})$, (see Fig. 9). s channel helicity conservation, on the other hand, is not predicted by this model. It can be built in, however.

5) Veneziano approach

One might try to extend the idea of duality³³⁾ to diffraction dissociation by a B_4 description³⁴⁾ of the bubble $\gamma + p \rightarrow \pi^+ \pi^-$ ²⁸⁾ (see Fig. 1a). With the simplest choice of $J^p = 0^+$ for the pomeron, the amplitude for $\gamma p \rightarrow \pi\pi$ is

$$A = \{(\epsilon Q^+)(k Q^-) - (\epsilon Q^-)(k Q^+)\} V(s, t, u) \quad (25)$$

with $Q^\pm = q^+ \pm q^-$, $s = m_{\pi\pi}^2$, $t = t^+$, $u = t^-$. Insisting on parent-parent duality, we write a Veneziano form for the invariant amplitude V

$$V(s, t, u) = \frac{\beta}{\alpha_t + \alpha_u} \{ B(1 - \alpha_s, -\alpha_t) + (t \rightarrow u) + \eta B(-\alpha_t, -\alpha_u) \}, \quad (26)$$

$$B(x, y) = \frac{\Gamma(x) \Gamma(y)}{\Gamma(x+y)}$$

with α_t, α_u the pion trajectories in the t, u channels and α_s the direct channel s trajectory (all of slope 1 GeV^{-2}) with an imaginary part linear in s , reproducing the s width. This form has the correct pion poles and the correct Regge behaviour. The amplitude for the full process $\gamma p \rightarrow \pi^+ \pi^- p$ is then written

$$T = i \bar{s} e^{i B t_{pp}} A(s, t, u) \quad (27)$$

with $e^{\frac{1}{2} B t_{pp}}$ coming from the (ppP) form factor, i.e., $B = 5 \text{ GeV}^{-2}$. There are two parameters in this model, β and η . It turns out that $\eta = 0$ is empirically the best value, which implies strong $\pi - A_1$ exchange degeneracy (there is no theoretical reason, however, for such exchange degeneracy in this model). If β is fitted to the total reaction cross-section $\sigma(\gamma p \rightarrow \pi^+ \pi^- p)$, the $m_{\pi\pi}$ distribution and differential cross-section are nicely reproduced (see Figs. 10, 11). In particular, $A(m_{\pi\pi})$ decreases from a large value to 5 GeV^{-2} for large $m_{\pi\pi}$, with a minimum due to the resonances in between, as can be seen in Fig. 12. Such structure is also seen in the SLAC data, which also yield a value of 5 GeV^{-2} for the t_{pp} slope of s production¹⁷⁾, in agreement with the present model (see Fig. 12b).

As is well known, the Veneziano model predicts higher lying vector mesons ³⁴⁾ on daughter trajectories (see Fig. 13 for the situation in diffractive photoproduction). With the trajectories chosen, the ratio of the $\mathbf{S}'$ peak at 1250 MeV to the $\mathbf{S}$ peak in $d\sigma/dm_{\pi\pi}$ is about 1:20. This suppression depends sensitively on the B_4 form chosen, as can be recognized from the ratio of residues

$$\frac{\text{Res } \{B(1-\alpha_s, -\alpha_t) + (t \rightarrow u)\} |_{\mathbf{S}'}}{\text{Res } \{B(1-\alpha_s, -\alpha_t) + (t \rightarrow u)\} |_{\mathbf{S}}} = \frac{1}{4}, \quad (28)$$

but

$$\frac{\text{Res } \{B(1-\alpha_s, 1-\alpha_t) + (t \rightarrow u)\} |_{\mathbf{S}'}}{\text{Res } \{B(1-\alpha_s, 1-\alpha_t) + (t \rightarrow u)\} |_{\mathbf{S}}} = 2. \quad (29)$$

On the other hand, the factor $1/\alpha_t + \alpha_u \sim 1/t_{pp} - m_{\pi\pi}^2$ implies ancestors in the t and u channels. If one insists on full crossing symmetry for $\pi\pi \rightarrow \pi^+\pi^-$ ³⁵⁾, with no ancestors in any channel, one has to give up the idea of parent-parent duality, obtaining forms like

$$V(s, t, u) \sim B(1-\alpha_s, 1-\alpha_t) + (t \rightarrow u) + \gamma \frac{1}{\alpha_t + \alpha_u} B(-\alpha_t, -\alpha_u), \quad (30)$$

which have no $\pi-\mathbf{S}$ B_4 duality, and a very large $\mathbf{S}'$ contribution, as can be seen from Eqs. (28) and (29). As a direct consequence of the 0^+ assumption for the pomeron, the model predicts t channel helicity conservation for the $\mathbf{S}$ pole ¹⁸⁾, which is meanwhile ruled out by experiment ¹⁷⁾. Outside the $\mathbf{S}$ band, the 2π decay distribution agrees nicely with the model (see Fig. 14). The significance of the data for $m_{\pi\pi}$ above 1 GeV is not clear, however, since one expects here reflections from Δ production.

It is interesting to note that the interference model is a good approximation to the B_4 model in the vicinity of $\mathbf{S}$:

$$B(1-\alpha_s, -\alpha_t) \approx \frac{1}{1-\alpha_s} + \frac{1}{-\alpha_t}, \quad (31)$$

as can be seen from Fig. 17. In that figure the $\mathbf{S}$ mass has been taken at 765 MeV. The resulting peak is at 745 MeV.

This simple model was modified by Dewey and Humpert ²⁵⁾ to include the double Regge limit, absolute normalization at the pion pole and to reproduce SCHC at the $\mathbf{s}$ pole in the limit t_{pp} fixed, $\bar{s} \rightarrow \infty$, giving up the 0^+ assumption for the pomeron. Their ansatz is :

$$\begin{aligned} T = i \cdot 2 e \sigma_{\pi N}^{tot} e^{i B t_{pp}} & \left[P q^+ \{ q^- \epsilon - \frac{q^+ k}{P k} P \epsilon \} B(1-\alpha_s, -\alpha_t) - \right. \\ & \left. - P q^- \{ q^+ \epsilon - \frac{q^+ k}{P k} P \epsilon \} B(1-\alpha_s, -\alpha_u) \right], \end{aligned} \quad (32)$$

with $P = p + p'$. In the forward direction, this form coincides on the $\mathbf{s}$ pole with the previous ansatz, so the main features of its predictions are the same as in the previous model (see Fig. 16). Again, strong πA_1 exchange degeneracy is implied. The decay angular distributions in the $\mathbf{s}$ region are now very nicely reproduced at 4.7 GeV (see Fig. 17). The details of the $\mathbf{s}'$ prediction, however, are very doubtful, since the ansatz Eq. (32) has ancestors in the direct channel, due to the $P q^+$, $P q^-$ factors, which are first order polynomials in the cosine of the $\pi^\pm$ direction. Hence the ancestors show up at odd spins on a trajectory one unit higher than the $\mathbf{s}$ trajectory. There is a way out, of course, as indicated by Dorren et al. ³⁶⁾ : take (e.g., for the $s - t$ term) :

$$(P q^+) B(2-\alpha_s, -\alpha_t) + b \bar{s} B(1-\alpha_s, 1-\alpha_t). \quad (33)$$

Again, this means to give up parent-parent duality and implies the need of satellites to reduce the large $\mathbf{s}'$ contribution arising from the form Eq. (33).

In conclusion, the B_4 approach is a first step to incorporate duality ideas into $\gamma \rightarrow 2\pi$ diffraction dissociation and is still on a rather crude phenomenological level : in order to gain predictive power, one has so far to live with ancestor problems.

Up to now, we have taken for granted that the π and $\mathbf{s}$ are dual to each other. It has frequently been stated, however, that the pion might fall outside duality, since it contributes mainly to the real part of the amplitude which is not connected by finite energy sum rules to the direct channel resonances. We now turn to a discussion of this question in 2π photoproduction.

6) F.E.S.R. approach.

In the following, we want to use semi-local duality plus resonance saturation to predict the $\mathbf{s}$ and $\mathbf{s}'$ production rates from the well-known size of the Drell graph ³⁷⁾. This attempt is far less ambitious than the B_4 model in the sense that we do not want to derive such details as the $\mathbf{s}$ shape and the t_{pp} distributions. In view of the difficulties in formulating the Drell model for non-forward production, we restrict our considerations to exact forward production.

We start with the "amplitude" for $\gamma R \rightarrow \pi^+ \pi^-$

$$T_4 = [(\epsilon q^+)(k q^-) - (\epsilon q^-)(k q^+)] F(s, t, u), \quad (34)$$

that is based on the (fairly reasonable) assumption that the pomeron behaves like O^+ for forward production (see the discussion in 3.2). The π exchange Born term reads

$$A_{\text{Born}} = \frac{-4e g_{\pi\pi\pi}}{(t-\mu^2)(u-\mu^2)}. \quad (35)$$

The Reggeized pion exchange is taken to be

$$R_{\pm} = -\pi e \frac{1+e^{-i\pi d_{\pi}(t_{\pm})}}{\sin \pi d_{\pi}(t_{\pm})} \frac{d_{\pi}(t_{\pm})+2}{2} \left(\frac{m_{\pi}^2}{v_0} \right)^{d_{\pi}(t_{\pm})} \quad (36)$$

with $s, t, u = m_{\pi}^2$, t_+ , t_- . Then, the zeroth moment F.E.S.R. at fixed t yields a trivial relation between the t and u channel pion poles :

$$\frac{1}{2} \int_{-N}^{+N} dv \left\{ \frac{2\pi e g_{\pi\pi\pi}}{t-\mu^2} \delta(v-v_{\text{Born}}) + \text{Im} A_{\text{Born}}(v, t) \right\} = \frac{\pi e g_{\pi\pi\pi}}{d_{\pi}(t)} \frac{d_{\pi}+2}{2} \left(\frac{N}{v_0} \right)^{d_{\pi}(t)} \quad (37)$$

with $v = 1/2(s-u)$, which equals $v_{\text{Born}} = 1/2(s-\mu^2)$ at the u pole. A connection between the S and π couplings can be obtained from the first moment sum rule

$$\int_{N_1}^{N_2} dv \left(\frac{v-v_{\text{Born}}}{v_0} \right) \text{Im} A(v, t) = \frac{\pi e g_{\pi\pi\pi}}{d_{\pi}+1} d_{\pi}' \frac{d_{\pi}+2}{2} \left(\frac{v}{v_0} \right)^{d_{\pi}+1} \Big|_{N_1}^{N_2}. \quad (38)$$

Saturating the left-hand side by S , one finds at $t = \zeta^2$

$$m_s^2 g_{\gamma\pi\pi} g_{S\pi\pi} = e g_{\pi\pi\pi} d_{\pi}' (N_2 - N_1). \quad (39)$$

After replacing $g_{\pi\pi\pi} \rightarrow i\bar{s} \sigma_{\pi p}^{\text{tot}}$, one deduces

$$\frac{d\sigma}{dt_{pp}} \Big|_{0^0} = \frac{\frac{3}{4} e^2 (\sigma_{\pi p}^{\text{tot}})^2 (N_2 - N_1)^2 d_{\pi}'^2}{384 \pi^2 m_s^2 \Gamma_s}. \quad (40)$$

The integration should cover the resonance spacing. If the next higher resonance is assumed to be at $m_{\rho} = 1.5$ GeV, as suggested by the nuclear experiments described below ^{38),39)}, and d_{π}^{ρ} is set equal to 1 GeV^{-2} , one has $d_{\pi}^{\rho} (N_2 - N_1) = 1.4$ and Eq. (40) yields

$$\frac{d\sigma}{dt_{pp}} \Big|_{0^\circ} \approx 50 \mu\text{b}/\text{GeV}^2, \quad (41)$$

which is in fair agreement with the experimental value of $80 - 140 \mu\text{b}/\text{GeV}^2$ ⁵⁾. Thus, in contrast to the wide-spread belief, the pion is a perfectly acceptable dual partner in the F.E.S.R. sense to the direct channel ρ .

7. Why is ρ' photoproduction suppressed?

A number of counter experiments of symmetric dipion production on nuclei have been performed to search for the photoproduction of high mass vector mesons in the coherence region ^{38),39)}. It has been found that $d\sigma/dm_{\pi\pi}$ drops by two orders of magnitude between m_{ρ} and $m_{\rho'} = 1.5$ GeV, with a broad shoulder between 1.3 and 1.8 GeV, which might or might not be the ρ' (or the ρ'').

In the following we want to show that such a dramatic suppression of the ρ' (with $m_{\rho'} = 1500$ MeV) production rate is to be expected on the basis of the Reggeized Drell mechanism and F.E.S.R. The argument goes as follows: the Reggeized Drell amplitude for $\gamma p \rightarrow \pi^+ \pi^- p$, in forward direction, is given by

$$T = e \{ (\epsilon q^-) R_- T_+ - (\epsilon q^+) R_+ T_- \} \quad (42)$$

with $T_{\pm}, R_{\pm}$ defined by Eqs. (18) and (36), respectively. It contains only one open parameter, ν_0 . Choosing $\nu_0 = 0.7 \text{ GeV}^2$, and normalizing the SLAC data ³⁸⁾, measured on Be, to the HBC data, one obtains a good average description of the experimental curve by the theoretical curve, calculated from Eq. (42) and

$$\frac{d\sigma}{dt_{pp} dm_{\pi\pi} d\Omega} = \frac{9\pi}{256\pi^4 (\bar{s} - m_{\rho'}^2)^2} \frac{1}{2} \sum_{\lambda_0} |T|^2 \quad (43)$$

Ω is the solid angle of the π direction in the 2π rest system (here $\Omega = 90^\circ$!). See Fig. 18.

Since the ρ' is a broad object, $\Gamma_{\rho'} \approx 300$ MeV, we would not expect it to protrude much over its background. This is also the answer from a semiquantitative application of F.E.S.R. to the ρ' region, which we define to be the region of the shoulder mentioned above ($m_{\rho'} \approx 1500$ MeV). The method is clearly less reliable here

than in the case of the ρ region, because one has to project out the p wave contributions of the Drell amplitude before one can relate the ρ' residue to the π exchange. The final answer is ³⁷⁾

$$\frac{d\sigma}{dt_{pp} dm_{\pi\pi} d\Omega} \bigg|_{\substack{m_{\pi\pi} = 1.5 \text{ GeV} \\ t_{pp} \approx 0 \\ \theta = 90^\circ}} = \frac{.015}{[\Gamma_{\rho'}^{\text{tot}} \text{ in GeV}]^2} \text{ } \mu\text{b} / \text{GeV}^3 \text{ ster.} \quad (44)$$

Normalizing this to the full Regge-Drell prediction of $0.14 \mu\text{b}/\text{GeV}^3$ ster, one arrives at an enhancement factor expressible in terms of the width

$$E_{\rho'} \approx \frac{.11}{[\Gamma_{\rho'}^{\text{tot}} \text{ in GeV}]^2}, \quad (45)$$

which confirms our earlier expectation.

We want to stress once more : these considerations refer to the p wave part and assume the existence of ρ' around 1500 MeV. Experimentally we do not know at all what partial waves are involved. In particular, the nature of the shoulder is completely open. All that we wanted to demonstrate is the fact that F.E.S.R. suggests strong p wave suppression at higher dipion masses.

The smallness of ρ' production is in no contradiction to the fact that the $1/t_{pp}^2$ behaviour of the proton electromagnetic form factor calls for a strong contribution (ρ' or non-resonating) of opposite sign to the ρ contribution ⁴⁰⁾. In particular, it does not demand $g_{\rho'p}/g_{\rho p}$ (which are defined at $k^2 = m_\rho^2, m_{\rho'}^2$) to be small. For such conclusion, one would require strong assumptions on mass continuations, on completely unknown quantities as $\sigma_{\rho'p}^{\text{tot}}$ as well as the diffractive transition $\sigma(\rho'p \rightarrow \rho p)$.

4. SUMMARY AND CONCLUSIONS

In an attempt to gain some insight into the diffraction dissociation dynamics by concentrating on the relation between resonant and non-resonant 2π photoproduction, we discussed three models, all of which describe the gross features of the empirical $m_{\pi\pi}$ and t_{pp} distributions below $m_{\pi\pi} = 1 \text{ GeV}$ equally well : the interference model (IM), the final-state interaction model (FIM) and the B_4 model (BM). IM obtains the observed deviation from a Breit-Wigner shaped ρ by an additive, FIM by a multiplicative modification of the Breit-Wigner expression, while BM is somewhere in between, but closely related to IM insofar as it is well approximated by its π and ρ pole contents. So the interference model (in the form ρ pole + $\pi^\mp$ poles) is in no immediate conflict with duality. In fact, it might be regarded as an approximation to the manifestly dual B_4 formulation.

On the other hand, it is not so obvious why one should start from a B_4 form that includes the pion pole. The answer to this question, which concerns the real parts in an essential way, is outside of duality and it goes much beyond the problem of $\pi - \rho$ duality in the finite energy sum rule sense. Within the FESR framework, the ρ production rate can reasonably well be predicted from the known size of Reggeized pion exchange and, therefore, the ρ might well be considered as dual to the π . In other words, the imaginary part of the Reggeized π exchange (with $\alpha'_\pi = 1$) is of normal strength, while the real part is abnormally large.

Both IM and BM violate the Watson theorem for $\gamma\gamma \rightarrow \pi\pi$, i.e., they violate unitarity. This is related to the well-known difficulty of how to pass from the narrow resonance approximation to the real case of finite width resonances. FIM tackles this problem from the other side, imposing the correct phase from the very beginning. Both ends should meet as soon as there is progress in unitarization of dual amplitudes.

Further progress in the understanding of 2π photoproduction will come from more detailed studies of structures in the $m_{\pi\pi}$ dependence of observable quantities like the t_{pp} slope around the ρ or by performing experiments with reasonable statistics in the higher dipion mass region, which is still rather unexplored but is very important in the study of non-resonant 2π production (e.g., everybody somehow believes in the Drell mechanism, but nobody has seen multiperipheralism at its best, namely when all subenergies are large). As it stands, the process $\gamma p \rightarrow \rho p$ is just a Gedankenexperiment, which is not known to better accuracy than the models that we have on $\gamma p \rightarrow \pi^+ \pi^- p$.

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FIGURE CAPTIONS

Figure 1 : a) Example of a diffraction dissociation process [Eq. (2)], with $A = p$, $B = \chi$, $\chi = \pi^+ \pi^-$.

b) Drell diagram for $\chi p \rightarrow \pi^+ \pi^- p$, showing our notation.

c) Vector meson dominance graph.

Figure 2 : The ratio of deuteron to hydrogen cross-sections [Eq. (9)], taken from Ref. 13), as a function of E_χ^{lab} .

Figure 3 : Cross-section for $\chi n \rightarrow p \pi$ for $|t_{pp}| < 1.1 \text{ GeV}^2$ as a function of E_χ^{lab} , taken from Ref. 14). The solid curve is the prediction of π exchange with $\Gamma_{\pi\pi\chi} = 0.13 \text{ MeV}$. The triangles give an upper limit of the cross-section for all $|t_{pp}|$.

Figure 4 : P_π and Σ in the reaction $\chi p \rightarrow p \pi$ for two photon energies. $P_\pi = \Sigma = 1$ means pure natural parity in the t channel. Figure taken from Ref. 17).

Figure 5 : The three contributions to $d\sigma/dm_{\pi\pi}$ in the interference model. Figure taken from Ref. 25).

Figure 6 : Prediction of final-state interaction model on $d\sigma/dm_{\pi\pi}$. Data from Ref. 41). Figure taken from Ref. 26b).

Figure 7 : Prediction of final-state interaction model on $d\sigma/dt_{pp}$, in various $m_{\pi\pi}$ bins. Figure taken from Ref. 26b).

Figure 8 : Same as Fig. 7, continued.

Figure 9 : Variation of the t_{pp} slope with $m_{\pi\pi}$ in the final-state interaction model. Figure taken from Ref. 26b).

Figure 10 : The predicted $d\sigma/dm_{\pi\pi}$ distribution of the dual model of Ref. 28).

Figure 11 : $d\sigma/dt_{pp}$, as predicted by the dual model of Ref. 28). Data from Ref. 41).

Figure 12 : Variation of the t_{pp} slope with $m_{\pi\pi}$:
 (a),(b) : Data from Ref. 17),
 (c) : prediction of dual model of Ref. 28).

Figure 13 : The 2π spectrum of particles predicted to be diffraction produced in the dual model of Ref. 28) for the reaction $\gamma p \rightarrow \pi^+ \pi^- p$.

Figure 14 : The π decay distributions, plotted versus t_+ and the cosine of the Jackson angle for various $m_{\pi\pi}$ intervals. Data from Ref. 41), theoretical curves predicted by the dual model of Ref. 28).

Figure 15 : Comparison between the prediction of the B_4 function and its approximate pole form [Eq. (32)] on $d\sigma/dm_{\pi\pi}$.

Figure 16 : $d\sigma/dm_{\pi\pi}$, as predicted by the dual model of Ref. 25). Data taken from Ref. 17).

Figure 17 : The prediction of the dual model of Ref. 25) (full histogram) against the experimental results of Ref. 17) (dotted histogram) on the π angular distribution in the helicity frame ($|t_{pp}| < 0.4 \text{ GeV}^2$, $0.65 < m_{\pi\pi} < 0.85 \text{ GeV}$).

Figure 18 : Plot of $d^2\sigma/dt_{pp} dm_{\pi\pi} d\Omega$ versus $m_{\pi\pi}$ for $t_{pp} = t_{\min}$, Jackson angle $\theta = 90^\circ$. The solid curve is the Reggeized Drell model. The broken curve is the contribution from the p wave of the imaginary part. The curves are calculated for $E_{\text{lab}} = 15 \text{ GeV}$ on a proton target with the free parameter $\nu_0 = 0.7 \text{ GeV}^2$. The data points are taken from Ref. 38), which is on beryllium and which gives no normalization; we have applied a normalization appropriate for a proton target ⁵⁾ of $d^2\sigma/dt_{pp} dm_{\pi\pi} d\Omega = 48 \mu\text{b/GeV}^3 \text{ster}$ at the ρ peak for $\theta = 90^\circ$. Figure from Ref. 37).

(c)

(b)

(c)

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