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FACTORIZATION IN A MULTIPERIPHERAL CONFIGURATION:
CAN ONE CONSTRUCT DUAL AMPLITUDES ON MORE GENERAL
FIELD THEORY VERTICES AND PROPAGATORS?

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A B S T R A C T

Observing a simple connection between the zero mode contribution in the conventional N point dual Born term for unit intercept (Virasoro case) and $\lambda \phi^3$ field theory, one wishes to study the possibility of constructing a new class of dual amplitudes based on more general field theory couplings and propagators. The resulting amplitudes would have a number of particles suppressed on the leading trajectory, the spin J ground state in a given channel being built on a two-dimensional operator formalism ("rotational" modes) that treats the integer and half-integer angular momenta in a unified way. The main part of this paper is devoted to a detailed study of factorization on a multiperipheral configuration in terms of "translational" and "rotational" modes. As a preliminary application in the context of dual models of the formalism developed we study the consequences of a simple factorization ansatz for the vertex into a pure "spin" and a harmonic oscillator part.

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1. INTRODUCTION AND SUMMARY

The relationship of the two well-studied dual models ^{1),2)} with full conformal algebra to corresponding field theories in the limit of vanishing Regge trajectory slope has been extensively studied ³⁾. There is, however, a somewhat different connection between the tree graphs of the unit intercept conventional dual model and those of $\lambda \phi^3$ field theory that may be of interest in generalizing large segments of the current dual formalism: namely, in the conventional N point Born term, the vacuum expectation value over the "zero mode" vertices ⁴⁾ generates, in the Virasoro unit intercept case ¹⁾, a Feynman amplitude corresponding to a multiperipheral graph of $\lambda \phi^3$ theory with intermediate propagators in the ground state. Feynman rules for arbitrary spin ⁵⁾ then yield a generalization of this "zero mode" contribution, thus leading to an integral representation for Feynman amplitudes based on "spin" vertices built on a two-dimensional boson calculus ("rotational" modes). This paper will thus be devoted mainly to a detailed study of factorization on a multiperipheral configuration using general couplings and vertices and to the resulting properties of the "spin" vertices. These suggest that it may not be impossible to use these spin vertices in conjunction with the harmonic oscillator modes based on the $J = \frac{1}{2} \rightarrow 0^-$ representation of the Möbius $SU(1,1)$ group ⁶⁾ or other duality generating representations. As a preliminary application of the formalism, a simple factorized ansatz for the vertex in terms of a harmonic oscillator part and a "spin" part is tried. Using a quark diagram interpretation closely related to Olesen's unequal intercept case ⁷⁾, one shows that the various operators constructed with the harmonic oscillator modes ⁸⁾ (projected twisted propagator, symmetrized vertex, gauge algebra), can be appropriately modified. However, an operational treatment of duality either fails in the spin part or leads to uncompensated ghosts. Of course, one expects the correlation between "rotational" and harmonic oscillator modes to be substantially more complicated in a satisfactory dual model than indicated by this simple ansatz. However, the formalism of Section 3, may suggest a way of incorporating spin in dual models that is radically different from the presently successful approach ^{2),6)}. It can also be of use in the treatment of other spin problems arising in multiperipheralism generally.

2. ROLE OF THE "ZERO MODE" IN THE CONVENTIONAL DUAL MODEL

In order to motivate the subsequent discussion and the formalism of Section 3, one recalls several properties of the conventional model where spin appears merely in the form of orbital excitations and Fubini and Veneziano's

"zero" mode has a definite relationship to the (spinless) ground state. Recalling the expression for the generating function of $(N+2)$ reggeon amplitudes on the leading trajectory that results from multiple factorization^{9),10)}

$$\mathcal{F} = \int d\mu(p) \prod_{i=0}^{N+1} \left(\frac{p_{i+1} - p_i}{p_i} \right)^a e^{-i\pi a} \langle 0 | \prod_{i=0}^{N+1} \mathcal{V}(p_i, k_i, \varepsilon_i(k_i)) | 0 \rangle \quad (2.1)$$

where $d\mu(p)$ is the usual integration measure given by:

$$d\mu(p) = \prod_{i=0}^{N+1} dp_i (p_{i+1} - p_i)^{-1} (dF_3)^{-1}$$

$$dF_3 = dp_a dp_b dp_c (p_b - p_a)^{-1} (p_c - p_b)^{-1} (p_a - p_c)^{-1} \quad (2.2)$$

and the vertex generating function is given by

$$\mathcal{V}(p_i, k_i, \varepsilon_i(k_i)) = e^{i\sqrt{2}k_i \cdot Q(p_i)} : e^{(\varepsilon_i(k_i) \cdot p_i \frac{d}{dp_i}) Q(p_i)} : e^{i\sqrt{2}k_i \cdot Q(p_i)} e^{i\sqrt{2}k_i \cdot Q^+(p_i)} \quad (2.3)$$

the multi-spin amplitude corresponding to $(N+2)$ leading trajectory excited states of given spins $J_0 \dots J_{N+1}$ is derived from (2.1), (2.3) by retaining terms of the form $\{\varepsilon_0\}^{J_0} \{\varepsilon_1\}^{J_1} \dots \{\varepsilon_{N+1}\}^{J_{N+1}}$ multiplied by $(\prod_i J_i!)$ where $\{\varepsilon\}^J$ denotes the wave function obtained by taking the symmetrized traceless tensor product of polarization four vectors

$\varepsilon_{\mu_1}(k_1) \otimes \dots \otimes \varepsilon_{\mu_J}(k_J)$. One observes that the VEV in (2.1) splits into two separate contributions corresponding resp. to the harmonic oscillator excitations and the "zero mode" or translational degrees of freedom:

$$\langle 0 | \prod_{i=0}^{N+1} \mathcal{V}(p_i, k_i, \varepsilon_i(k_i)) | 0 \rangle = \langle 0 | \prod_{i=0}^{N+1} e^{(i\sqrt{2}k_i + \varepsilon_i(k_i) p_i \frac{d}{dp_i}) \cdot Q(p_i)} e^{i\sqrt{2}k_i + \varepsilon_i(k_i) p_i \frac{d}{dp_i} \cdot Q^+(p_i)} | 0 \rangle$$

$$\langle 0 | \prod_{i=0}^{N+1} e^{\varepsilon_i(k_i) \cdot i p_0} e^{i\sqrt{2}k_i \cdot Q(p_i)} | 0 \rangle \quad (2.4)$$

For external scalars, the "zero mode" factor yields the Feynman diagram (multiperipheral configuration) in the Virasoro ($a=1$) case with the internal lines in the ground state $\alpha_{0i} = 0$ ($i=1, \dots, N-1$). Indeed, with $p_a \equiv p_0 \rightarrow 0$, $p_b \equiv p_N \rightarrow 1$, $p_c \equiv p_{N+1} \rightarrow +\infty$ and $k^2 = a$, one obtains setting $p_i / p_{i+1} = y_i$

$$B_0 = \int \prod_{i=1}^{N-1} \left(\frac{dp_i}{(p_{i+1} - p_i)^{1-\alpha}} \right) \cdot \prod_{i=1}^{N-1} p_i^{+2k_i \cdot \sum_{j=0}^{i-1} k_j} = \int_0^1 \prod_{i=1}^{N-1} dy_i y_i^{-\alpha_i-1} (1-y_i)^{\alpha_i-1} \quad (2.5)$$

thus proving the above statement. When the harmonic excitations in (2.4) are included, (2.5) is replaced by:

$$B = \int \prod_{i=1}^{N-1} \left(\frac{dp_i}{(p_{i+1} - p_i)^{1-\alpha}} \right) \cdot \prod_{i=1}^{N-1} p_i^{+2k_i \cdot \sum_{j=0}^{i-1} k_j} \prod_{j>i}^N \left(1 - \frac{p_i}{p_j} \right)^{2k_i \cdot k_j} \quad (2.6)$$

3. "SPIN" VERTICES AND GENERALIZED "ZERO MODE" CONTRIBUTION

One should like to extend the present framework of dual models by constructing amplitudes such that spin intervenes not only as an orbital excitation of a spinless ground state but also as an intrinsic spin with its own boson calculus ("rotational" modes). (This incorporation of spin would of course be quite different from the approach of Neveu and Schwarz ²⁾). It seems further quite natural to assume that in - presumably existing - dual amplitudes of this type, the generalized "zero mode" contribution defined by analogy with (2.4) will again give rise to an integral representation for the multiperipheral tree Feynman graphs resulting from more general field theory couplings. Hence, working backward, one writes down such a representation using Feynman rules for arbitrary spin (Weinberg) and one tries to isolate a generalized "zero" mode vertex. Reconstructing satisfactory models from this limiting case presents a difficult problem. In Section 4, one will discuss, assuming for simplicity that the oscillators and rotational modes decouple (or rather are only coupled by the translational modes) how this problem could be approached. In this Section, the problem of factorization in a multiperipheral configuration in terms of "translational" and "rotational" modes will be developed in some detail.

We restrict ourselves in the following, to fields transforming under the most economical irreducible representation of the Lorentz group, namely, the $(2J+1)$ dimensional $D^{(J,0)}[\Lambda]$ representation $\vec{J} \rightarrow \vec{J}^{(J)}$, $\vec{K} \rightarrow -i\vec{J}^{(J)}$ with transformation law:

$$U[\lambda, a] \phi_{\sigma}^{(J)}(x) U^{-1}[\lambda, a] = \sum_{\sigma'} \Pi_{\sigma\sigma'}^{(J,0)}[\lambda^{-1}] \phi_{\sigma'}^{(J)}(\lambda \cdot x + a) \quad (3.1)$$

and to non-derivative couplings (as is well-known, reflection invariance requires doubling the dimensionality of the representation). When the requirements of crossing and statistics have been taken into account, these fields admit the Fourier expansion:

$$\phi_{\sigma}^{(J)}(x) = \int \frac{d^3 p}{\sqrt{(2\pi)^3 2\omega(\vec{p})}} \sum_{\sigma'} \left\{ \Pi_{\sigma\sigma'}^{(J,0)}[L(\vec{p})] \alpha(\vec{p}, \sigma') e^{i\vec{p} \cdot x} + [\Pi^{(J,0)}[L(\vec{p})] C^{-1}]_{\sigma\sigma'} \beta^*(\vec{p}, \sigma') e^{-i\vec{p} \cdot x} \right\} \quad (3.2)$$

where $C_{\sigma\sigma'}^{(J)} = (-1)^{J+\sigma} \delta_{\sigma', -\sigma}$
and

$$[\alpha(\vec{p}, \sigma), \alpha^*(\vec{p}', \sigma')]_{\pm} = \delta_{\sigma\sigma'} \delta^3(\vec{p} - \vec{p}') \quad , \dots$$

One also needs a parity conjugate field $\chi_{\sigma}^J(x)$ transforming according to $D^{(0,J)}[\Lambda]$.

Corresponding to a Feynman multiperipheral tree diagram with $(N-1)$ propagators and N vertices, one has the momentum space expression:

$$\begin{aligned} \mathcal{M}_{\text{PT}} = (2\pi)^4 \delta\left(\sum_{j=0}^{N+1} \pm k_j\right) & \left\{ \prod_{j=1}^{N-1} \frac{(-m_{\sigma_j})^{-2J_{\sigma_j}} \mathcal{F}_{\sigma_j \sigma'_{j+1}}^{(*)}(k_{\sigma_j})}{i [k_{\sigma_j}^2 + m_{\sigma_j}^2 - i\varepsilon]} \right\} \frac{\Pi_{\sigma_0 \mu_0}^{(J_0,0)}[L(\vec{k}_0)]}{\sqrt{(2\pi)^3 2\omega(\vec{k}_0)}} \\ & \left\{ \prod_{j=1}^N (\mp i g_j) \begin{pmatrix} J_{\sigma_{j-1}} & J_j & J_{\sigma_j} \\ \sigma'_{\sigma_{j-1}} & \sigma_j & \sigma_{\sigma_j} \end{pmatrix} \frac{\Pi_{\sigma_j \mu_j}^{(J_j,0)}[L(\vec{k}_j)]^{(*)}}{\sqrt{(2\pi)^3 2\omega(\vec{k}_j)}} \right\} \frac{\Pi_{\sigma_{N+1} \mu_{N+1}}^{(J_{N+1},0)}[L(\vec{k}_{N+1})]^{(*)}}{\sqrt{(2\pi)^3 2\omega(\vec{k}_{N+1})}} \quad (3.3) \end{aligned}$$

(hermitian conjugate couplings alternate as $j = 1 \dots N-1$).

In this expression the propagator numerator is

$$(-m_{oj})^{-2J_{oj}} \mathcal{P}_{\sigma_j \sigma'_{j+1}}(k_j) = \sum_{M_{oj}} \mathbb{D}_{\sigma_j M_{oj}}^{(J_{oj}, 0)} [L(\vec{k}_j)] \mathbb{D}_{M_{oj} \sigma'_{j+1}}^{(J_{oj}, 0)} [L(\vec{k}_j)]^\dagger$$

$$= \{ \exp(-2 \hat{k}_{oj} \cdot \vec{J}_{oj} \theta_{oj}) \}_{\sigma_j \sigma'_{j+1}} \quad (3.4)$$

where

$$\cosh \theta_{oj} = \omega_{oj}(\vec{k}_{oj}) / m_{oj} \quad \sinh \theta_{oj} = |\vec{k}_{oj}| / m_{oj}$$

This allows one to give a symmetrical form for the vertex in terms of Wigner's 3j symbols

$$V_j^{(0)} = -i g_j \begin{pmatrix} J_{oj-1} & J_j & J_{oj} \\ \sigma'_{j-1} & \sigma_j & \sigma_{oj} \end{pmatrix} \mathbb{D}_{\sigma'_{j-1} M_{oj-1}}^{(J_{oj-1}, 0)} [L(\vec{k}_{j-1})] \mathbb{D}_{\sigma_j M_j}^{(J_j, 0)} [L(\vec{k}_j)] \mathbb{D}_{\sigma_{oj} M_{oj}}^{(J_{oj}, 0)} [L(\vec{k}_{oj})]$$

(3.5)

leading to the expression, with the understanding that $J_{00} = J_0$, $M_{00} = M_0$, $\sigma'_{00} = \sigma_0$, $J_{ON} = J_{N+1}$, $M_{ON} = M_{N+1}$, $\sigma_{ON} = \sigma_{N+1}$:

$$\mathcal{M}_{\text{NPT}} = \frac{(2\pi)^4 \delta(\sum_{j=0}^{N+1} \pm k_j)}{\prod_{j=0}^{N+1} \sqrt{(2\pi)^3 2\omega(\vec{k}_j)}} \cdot \prod_{j=1}^{N-1} \sum_{\{M_{oj}\}} [i^{-1} \alpha' (J_{oj} - \alpha_{oj} - i\epsilon)^{-1}] \prod_{i=1}^N V_i^{(0)*} \quad (3.6)$$

Further reduction of (3.5), (3.6) requires performing summations over the $\{\sigma\}$ and $\{M\}$ indices. To separate these, use will be made of the identity:

$$\mathbb{D}_{\sigma M}^{(J, 0)} [L(\vec{k})] = v_J^M \left(\frac{\partial}{\partial \underline{x}} \right) v_J^\sigma \left(\left(\frac{k_\sigma}{m} \right)^{1/2} \underline{x} \right) \Big|_{\underline{x}=0} = \langle \Psi_0 v_J^M(\underline{a}) v_J^\sigma \left(\left(\frac{k_\sigma}{m} \right)^{1/2} \underline{a}^* \right) \Psi_0 \rangle \quad (3.7)$$

where the $v_J^M(\underline{x})$ are the homogeneous polynomial basis in two variables $\underline{x} = (x_+, x_-)$

$$v_J^M(\underline{x}) = [(J+M)!(J-M)!]^{-1/2} \cdot x_+^{J+M} x_-^{J-M} \quad (3.8)$$

Indeed, expanding the right-hand side of (3.7)

$$v_J^M \left(\frac{\partial}{\partial \tilde{x}} \right) v_J^\sigma \left(\left(\frac{k \cdot \sigma}{m} \right)^{1/2} \tilde{x} \right) \Big|_{\tilde{x}=0} = \left[(J+M)! (J-M)! (J+\sigma)! (J-\sigma)! \right]^{-1/2} \frac{\partial^{2J}}{\partial x_+^{J+M} \partial x_-^{J-M}} \left[x_+^{J+\sigma} x_-^{J-\sigma} \right]_{\tilde{x}=0}$$

where

$$\tilde{x}(k) = \left(\frac{k \cdot \sigma}{m} \right)^{1/2} \tilde{x} = \left[(\hat{k} \cdot \sigma) \sinh \frac{\theta}{2} - 1 \cosh \frac{\theta}{2} \right] \cdot \tilde{x}$$

and the hyperbolic angle θ has been previously defined. The obtained expression is readily compared with the $O(3)$ matrix element for a rotation $\mathcal{D}$ around the z axis:

$$d_{\sigma M}^J(\mathcal{D}) = (-1)^{J-\sigma} \left[\frac{(J+M)!}{(J-M)! (J+\sigma)! (J-\sigma)!} \right]^{1/2} \cdot \left[\sin \frac{\theta}{2} \right]^{-M+\sigma} \left[\cos \frac{\theta}{2} \right]^{-M-\sigma} \cdot \left[\left(\frac{d}{dt} \right)^{J-M} t^{J+\sigma} (1-t)^{J-\sigma} \right]_{t=\cos^2 \frac{\theta}{2}}$$

We shall mainly use the representation (3.7) in terms of the two-dimensional boson operators $[a_z, a_z^*] = \delta_{zz}$, describing elementary spin $\frac{1}{2}$ excitations. Summation over the $\{M_{oj}\}$ indices is carried out using completeness of the states $|\Psi_J^M\rangle = v_J^M(a^*) |\Psi_0\rangle$; summation over the $\{\sigma\}$ indices is done using the unique isotropic invariant built out of three finite dimensional (spinorial) representations

$$\begin{aligned} \sum_{\{\sigma\}} \begin{pmatrix} J_{oi-1} & J_i & J_{oi} \\ \sigma'_{oi-1} & \sigma_i & \sigma_{oi} \end{pmatrix} v_{J_{oi}}^{\sigma'_{oi}}(\tilde{a}_{(oi)}^{*'}) v_{J_i}^{\sigma_i}(\tilde{x}_i) v_{J_{oi-1}}^{\sigma_{oi-1}}(\tilde{a}'_{(oi-1)}) \\ = N_i^{-1/2} (\tilde{x}_i \wedge \tilde{a}_{(oi)}^{*'})^{J_i+J_{oi}-J_{oi-1}} (\tilde{a}_{(oi)}^{*'} \wedge \tilde{a}'_{(oi-1)})^{J_{oi}+J_{oi-1}-J_i} (\tilde{a}'_{(oi-1)} \wedge \tilde{x}_i)^{J_{oi-1}+J_i-J_{oi}} \end{aligned} \quad (3.9)$$

where

$$N_i = (J_{oi-1} + J_i + J_{oi} + 1)! (J_{oi-1} + J_i - J_{oi})! (J_{oi-1} + J_{oi} - J_i)! (J_i + J_{oi} - J_{oi-1})!$$

and the momentum dependence has been taken into

$$\tilde{a}_{(oi)}^{*'} = \left(\frac{k_{oi} \cdot \sigma}{m_{oi}} \right)^{1/2} \tilde{a}^{*}, \dots$$

A treatment closely similar to the one of Section 2 allows one to incorporate the spin carrying propagator denominators inside the vertices:

$$\int \prod_{i=1}^{N+1} dy_i y_i^{-1+J_i-\alpha_{0i}} (1-y_i)^{-1+a} = \lim_{\substack{p_0 \rightarrow 0 \\ p_{N+1} \rightarrow 0 \\ p_{N+1} \rightarrow +\infty}} \int \frac{\prod_{i=0}^{N+1} dp_i / dF_3}{\prod_{i=0}^{N+1} (p_{i+1} - p_i)} e^{i\pi a \sum_{i=0}^{N+1} \left(\frac{p_{i+1} - p_i}{p_i} \right)^a \prod_{i=0}^{N+1} p_i^{J_i - J_{i-1}}} \prod_{i=0}^{N+1} p_i^{+2k_i \cdot k_{0i} - k_i^2} \quad (3.10)$$

Besides the easily recognized translational mode factor, one has now an extra factor corresponding to the "rotational" modes. Since $(2J)$ is the total number of spin $\frac{1}{2}$ deviations, one has in terms of the boson calculus introduced above:

$$J_{(0i)} = \frac{1}{2} (n_+ + n_-)_{(0i)} = \frac{1}{2} \sum_S a_S^* a_S \quad (3.11)$$

This allows one to introduce the ground state spin dependence inside the vertex; upon using the ordered operator identities:

$$\rho^{\pm \frac{1}{2} (a \cdot \tilde{a})} = : \exp (a^* (\rho^{\pm \frac{1}{2}} - 1) \cdot \tilde{a}) : = \rho^{\mp \frac{1}{2}} : \exp (\tilde{a} \cdot (1 - \rho^{\mp \frac{1}{2}}) \cdot a^*) :_\tau \quad (3.12)$$

one can commute the ρ factors on both sides to act on the vacuum inside the vertex:

$$\begin{aligned} \langle \Psi_0 | a_S^r \rho^{-\frac{1}{2} (a \cdot \tilde{a})} &= \rho^{-\frac{1}{2}} \langle \Psi_0 | a_S^r : e^{(1-\rho^{-1/2}) a \cdot \tilde{a}} :_\tau = \langle \Psi_0 | (\sqrt{\rho} a_S^r) \\ \rho^{+\frac{1}{2} (a \cdot \tilde{a})} (a_S^r)^\dagger | \Psi_0 \rangle &= \rho^{+\frac{1}{2}} : e^{(1-\rho^{+1/2}) a \cdot \tilde{a}} :_\tau^\dagger a_S^r | \Psi_0 \rangle = (a_S^r / \sqrt{\rho})^\dagger | \Psi_0 \rangle \end{aligned} \quad (3.13)$$

One sees that $\rho^{\pm \frac{1}{2}}$ powers are clearly associated with each "rotation" mode creation or annihilation operator a_S, a_S^* resp. The next step is to eliminate all dependence upon the internal four-momenta, thus enabling one to rewrite the whole integrand as a vacuum expectation value over a product of vertices, where the relevant vacuum $|\Psi_0\rangle$ is the direct product of the translational $\langle 0 | p_0 = p_0 | 0 = 0 \rangle$ and spin vacua $(a_S | \Psi_0 \rangle = \langle \Psi_0 | a_S^* = 0)$.

The outmost vertices in such a chain satisfy LSZ type boundary conditions ⁹⁾

$$\lim_{p_0 \rightarrow 0} p_0^{-J_0} V^{(0)}(p_0; J_0 M_0 k_0) |\Psi_0\rangle = |J_0 M_0 k_0\rangle$$

$$\lim_{p_{N+1} \rightarrow +\infty} \langle \Psi_0 | p_{N+1}^{+J_{N+1}} V^{(0)}(p_{N+1}; J_{N+1} M_{N+1} k_{N+1}) = \langle J_{N+1} M_{N+1} k_{N+1} |$$

(3.14)

The purpose of working out these properties is, of course, that, since similar properties have been shown to hold in the conventional dual model, it appears possible to recouple the harmonic oscillator modes on top of the translational-rotational modes (cf., next Section).

For later purposes, one also gives here the generating function of all "spin" vertices $V^{(0)}(J_i J J_f)$ ¹¹⁾. Multiplying by the homogeneous polynomial in three variables

$$v_{J_i J J_f}(\vec{\alpha}) = N(\{J\}) \alpha_i^{J+J_f-J_i} \alpha^{J_i+J_f-J} \alpha_f^{J_i+J-J_f}$$

$$N(\{J\}) = \left[(2J+1)! \right]^{-1/2} \left[(J+J_f-J_i)! (J_i+J_f-J)! (J_i+J-J_f)! \right]^{-1/2}$$

(3.15)

and summing over all $\{J\}$ values compatible with the triangular inequalities, one gets:

$$\pi V^{(0)}(\vec{\alpha}) = e^{+\alpha_f \left(\frac{\underline{a}}{\sqrt{p}} \left(\frac{\underline{k} \cdot \underline{\sigma}}{m_0} \right) \cdot \underline{c}^T \cdot \underline{x} \right)} : e^{-\underline{a} \cdot \left(\alpha \left(\frac{\underline{k} \cdot \underline{\sigma}}{m} \right) \cdot \underline{c}^T - 1 \right) \cdot \underline{a}} + \alpha_i (\underline{x} \cdot \underline{c}^T \cdot \underline{a} \sqrt{p}) : e^{ik(q_0 + ip_0 \log p)}$$

(3.16)

i.e., a canonical form for the generating function. In writing (3.16), one has inserted a normal ordering and eliminated the explicit dependence of the spin boson operators upon the channel considered.

4. A SIMPLE APPLICATION OF THE FORMALISM OF SECTION 3

Although the formalism obtained in the last section to describe the propagation of spin excitations and four-momentum along a multi-peripheral chain in terms of coherent states could have other applications, we wish to discuss briefly here whether it could lead to new dual amplitudes. The discussion will be further restricted to the rather trivial case where the "rotational" modes and the usual harmonic oscillator modes couple only through the translational mode resulting in a simple multiplicative form for the vertex. The propagator denominators being provided by the functions:

$$B(\{J_{ik}\}) = \int \prod_{i=1}^{N-1} [dy_i y_i^{J_{0i}-\alpha_{i,i-1}} (1-y_i)^{J_{2,i+1}-\alpha_{2,i+1}-1}] \prod_{i=1}^{N-1} \prod_{k=i+2}^N (1 - \frac{k-1}{N} y_m)^{k_i \cdot k_k - \epsilon_{ik}} \quad (4.1)$$

where one has:

$$\begin{aligned} \epsilon_{ik} &= \epsilon_{ki} = \epsilon_{ik}^{(0)} - \epsilon_{ik}^{(J)} \\ \epsilon_{ik}^{(0)} &= \alpha_{ik}^{(0)} - \alpha_{i+1,k}^{(0)} - \alpha_{i,k-1}^{(0)} + \alpha_{i+1,k-1}^{(0)} \\ \epsilon_{ik}^{(J)} &= J_{ik} - J_{i+1,k} - J_{i,k-1} + J_{i+1,k-1} \end{aligned}$$

the requirement of factorizability in the unequal intercept case leads to the solution ⁷⁾:

$$\alpha_{ik}^{(0)} = (b_i + b_{k+1}) - \frac{1}{2} \left(\sum_{j=i}^k d_j \right)^2$$

guaranteeing factorization of the exponent in terms of the conserved M dimensional extra momenta:

$$\epsilon_{ik}^{(0)} = - d_i \cdot d_k \quad \sum_{i=0}^{N+1} d_i = 0$$

By analogy with the "quark" intercept values b_i , one introduces "circular" angular momenta ν_i ($i=0, \dots, N+1$) such that $J_{ik} = \nu_i + \nu_{k+1}$, hence $\epsilon_{ik}^{(J)} = 0$. Let $\bar{p} = (k, \bar{q})$, one rewrites, defining $b'_i = b_i - \nu_i$ (note:

$$\frac{1}{2} \bar{p}_i^2 = b'_i + b'_{i+1} \quad , \quad J_i = \alpha_i^{(0)} - \frac{1}{2} k_i^2 = \nu_i + \nu_{i+1} \quad , \quad 2\nu_i = J_i + J_{0,i-1} - J_{0,i} \quad)$$

$$B_{N+2}(\bar{p}_i, v_i) = \int \prod_{i=1}^{N-1} [dy_i] y_i^{-(b'_0 + b'_{i+1}) + \frac{1}{2} \bar{p}_{0i} - 1} (1-y_i)^{2b'_{i+1} - 1} \prod_{i=1}^{N-1} \prod_{k=i+1}^N (1 - \prod_{m=i}^{k-1} y_m)^{\bar{p}_i \cdot \bar{p}_k} \quad (4.1')$$

Defining the propagator by:

$$\Pi(s_{0i}) = \int_0^1 dz \ z^{L_0 - (b_0 + b_{i+1}) + J_{0i} - 1} (1-z)^{2b_{i+1} - J_{0i} + J_i - 1} \quad (4.2)$$

and the harmonic oscillator part of the vertex by

$$V_{H_0}(\bar{p}; \bar{A}) = \exp(\bar{A}^\dagger \circ \bar{p}_i) \exp(\bar{A} \circ \bar{p}_i) \quad (\bar{A} \circ \bar{p}_i) = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \bar{A}_n \cdot \bar{p}_i \quad (4.3)$$

the projector onto the non-spurious sector is

$$P_{ij} = 1 - W_{ij}^{\dagger(1)} (W_{ij}^{(1)} W_{ij}^{\dagger(1)})^{-1} W_{ij} \quad \text{satisfying} \quad P_{ij}^2 = P_{ij}, \quad W_{ij}^{(1)} P_{ij} = P_{ij} W_{ij}^{(1)} = 0 \quad (4.4)$$

The projected twisted propagator is found to be symmetric in the quark lines:

$$\mathcal{D}_{ij} = \mathcal{D}_{ij}^\dagger = \mathcal{D} \Omega P = P^\dagger \mathcal{D} \Omega P = \int_0^1 dz \ z^{L_0 - (b_i + b_{j+1}) + \frac{1}{2} (\underline{a}^\dagger \cdot \underline{a}) - 1} \Omega (1-z)^{L_0 - L_- + (b_i + b_{j+1}) - \frac{1}{2} (\underline{a}^\dagger \cdot \underline{a}) - 1} \quad (4.5)$$

whereas the symmetrical vertex is obtained by taking partial derivatives with respect to the $\underline{a}$ variables in the generating function [cf., Eq. (3.16)].

$$V_S(k_1, k_2, k_3; A, \underline{a}, \vec{d}) = \langle \exp [A_1^\dagger \circ k_2 + (A_1^\dagger, A_2)_- + \underline{a}_3 \cdot C^T(k_1, k_2) \cdot \underline{a}_2 + \text{cyclic}] \rangle$$

where

$$C^T(k_1, k_2) = \left(\frac{k_1 \cdot \sigma^T}{m_1} \right)^{1/2} \cdot C^T \cdot \left(\frac{k_2 \cdot \sigma}{m_2} \right)^{1/2} \quad C^T = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (4.6)$$

It is further possible to modify the conformal algebra appropriately:

$$W_{o,i-1}^{(n)'}(\bar{p}_{o,i-1}) = L_o(\bar{p}_{o,i-1}) - L_-^{(n)}(\bar{p}_{o,i-1}) - (b'_o + b'_i)$$

$$W_{o,i}^{(n)}(\bar{p}_{o,i}) = L_o(\bar{p}_{o,i}) - L_-^{(n)}(\bar{p}_{o,i}) - (b'_o - (2n-1)b'_{i+1})$$

(4.7)

The condition for (presumably) complete ghost cancellation would then be:

$$b'_i = b_i - \gamma_i = \frac{1}{2} \text{ implying } \alpha_{ij}(0) - J_{ij} \geq 1 \text{ and at least a tachyon.}$$

Turning next to the duality condition in operator form ⁸⁾:

$$\left\{ V_s(1,2,s;A,a) \mathcal{D}(A,a;s) V_s(3,4,s;A,a) - V_s(2,\bar{3},t;A,a) \mathcal{D}(A,a;t) V_s(4,\bar{1},t;A,a) \right\} \prod_{i=1}^4 P_i^\dagger = 0 \quad (4.8)$$

one may start with usual case differential form and observe that multiplication by a crossing symmetric factor, namely $x^{(b'_1 + b'_3)}(1-x)^{(b'_2 + b'_4)}$ yields the correct modified expression involving the propagators and harmonic oscillator part of the vertices:

$$S_{12}'(x;k_1) V_s(1,2,s) dD(x) V_s(3,4,s) S_{34}'(x;k_4) = \Omega(k_2) S_{34}'(1-x;k_3) V_s(2,3,t)$$

$$dD(1-x) V_s(t,1,4) S_{14}'(1-x;k_4) \Omega^\dagger(k_4) \quad (4.9)$$

where

$$S_{ij}'(x;k) = (1-x) W_{ij}'(k)$$

Turning to the "spin" expression involving the external wave function, propagator numerator and "spin" vertices that factor out because of our simple ansatz, one can evaluate the S channel expression

$$\{S\} = g(J_1 J_2 J) g(J_3 J_4 J) \sum_{\mu} \sum_{\mu'} \begin{pmatrix} J_1 & J_2 & J \\ \sigma_1 & \sigma_2 & \sigma \end{pmatrix} \begin{pmatrix} J_3 & J_4 & J \\ \sigma_3 & \sigma_4 & \sigma' \end{pmatrix} \Pi_{\sigma_1 \mu_1}^{(J_1,0)} [L(\vec{k}_1)] \Pi_{\sigma_2 \mu_2}^{(J_2,0)} [L(\vec{k}_2)]$$

$$\Pi_{\sigma_3 \mu_3}^{(J_3,0)} [L(\vec{k}_3)] \Pi_{\sigma_4 \mu_4}^{(J_4,0)*} [L(\vec{k}_4)] \Pi_{\sigma_3 \mu_3}^{(J_3,0)*} [L(\vec{k}_3)] \Pi_{\sigma_4 \mu_4}^{(J_4,0)*} [L(\vec{k}_4)]$$

(4.10)

using the technique of Section 3, (4.10) yields the operator expression to be taken between two-particle rotational states:

$$\{S\} = g(v_1 v_2 v_3) g(v_3 v_4 v_1) \left\langle (a'_1 \wedge a'_2)^{2v_3} (a'_3 \wedge a'_4)^{2v_4} (a'_4 \wedge a'_1)^{2v_1} \right\rangle_{34}$$

$$_{12} \left\langle (a'_1 \wedge a'_2)^{2v_2} (a'_2 \wedge a'_3)^{2v_3} (a'_3 \wedge a'_1)^{2v_1} \right\rangle_a$$

(4.11)

Using the generating function for the spin vertices in symmetrical form ¹²⁾:

$$\begin{aligned} \{S\} &= g^2 \cdot \frac{\delta^{4(v_1+v_3)+2(v_2+v_4)}}{\gamma_{\alpha_1}^{2v_3} \gamma_{\alpha_2}^{2v_1} \gamma_{\alpha_3}^{2v_1} \gamma_{\alpha_4}^{2v_3} \gamma_{\alpha'}^{2v_4} \gamma_{\alpha''}^{2v_2}} \left\langle \pi^{(0)} \left(\begin{matrix} \alpha_3 & \alpha_4 & \alpha' \\ \underline{\alpha}_3 & \underline{\alpha}_4 & \underline{\alpha}' \end{matrix} \right) \pi^{(0)} \left(\begin{matrix} \alpha_1 & \alpha_2 & \alpha'' \\ \underline{\alpha}_1 & \underline{\alpha}_2 & \underline{\alpha}'' \end{matrix} \right) \right\rangle_a \\ &= g^2 \cdot (\underline{\alpha}_4'' \cdot \underline{\alpha}_1'')^{2v_1} (\underline{\alpha}_3'' \cdot \underline{\alpha}_2'')^{2v_3} (\underline{\alpha}_3'' \cdot \underline{C}^T \cdot \underline{\alpha}_4'')^{2v_4} (\underline{\alpha}_1'' \cdot \underline{C}^T \cdot \underline{\alpha}_2'')^{2v_2} \end{aligned} \quad (4.12)$$

In obtaining (4.12), one has defined:

$$\underline{\alpha}_h'' = \underline{\alpha}_h' \left(\frac{k_s \cdot \sigma}{m_s} \right)^{1/2}$$

and used:

$$\left(\frac{k_s \cdot \sigma^T}{m_s} \right)^{1/2} \cdot C^T \cdot \left(\frac{k_s \cdot \sigma}{m_s} \right)^{1/2} = C^T$$

The corresponding t channel expression is found to be identical but for a different arrangement and momentum dependence of the metric tensors C^T , $(p \cdot \sigma / m)$. As a possible remedy, one may try to use higher dimensional (J, J) type representations for the propagator numerator. For a process such as $1(J) + 2(J') \rightarrow 3(J) + 4(J')$, the S and T channels can thus be identified on account of the generalized Fierz identity:

$$\sigma^{(\mu\nu)(\lambda)}_{(\mathcal{J}'; \mathcal{J})} \bar{\sigma}_{(\mu\nu)(\lambda)}(\mathcal{J}; \mathcal{J}') \dot{\beta}'^{\alpha'} = 2^{2M} \delta_{\alpha}^{\alpha'} \delta_{\dot{\beta}}^{\dot{\beta}'}$$

where the $\sigma^{(\mu\nu)(\lambda)}$ are the generalized Pauli matrices defined by Williams ¹³⁾ and the indices α , α' and $\dot{\beta}$, $\dot{\beta}'$ run over $(2J'+1)$ and $(2J+1)$ values respectively. This corrective action will however generate ghosts uncanceled by the modified conformal algebra (4.7), e.g., for the spin 1 case:

$$\delta_{\mu\nu} = \left(\delta_{\mu\nu} + \frac{k_{3\mu} k_{3\nu}}{m^2} \right) - \frac{k_{3\mu} k_{3\nu}}{m^2}$$

one has besides the $(\frac{1}{2}, \frac{1}{2})$ spin 1 state a $(0,0)$ ghost.

It is, of course, possible that a more elaborate ansatz concerning the interplay of the "rotational" modes and harmonic oscillator modes will lead to more satisfactory amplitudes.

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REFERENCES AND FOOTNOTES

- 1) M.A. Virasoro, Phys.Rev. D1, 2933 (1970).
- 2) A. Neveu and J. Schwarz, Nuclear Phys. B31, 86 (1971).
- 3) J. Scherk, Nuclear Phys. (to be published);
A. Neveu and J. Scherk, Orsay preprint 71/37, 71/43.
- 4) S. Fubini and G. Veneziano, Ann.Phys. 63, 12 (1971), where this
concept is introduced.
- 5) S. Weinberg, Phys.Rev. 133B, 1318 (1964).
Boson calculus for SU(2) or SU(1,1) finite dimensional representations
is described e.g., in
V. Bargmann, Revs.Mod.Phys. 34, 829 (1962);
J. Schwinger, NYO 30-71 Rept (1952, unpublished);
A. Barut, Boulder Lectures (1966);
W.J. Holman and L.C. Biedenharn, Ann.Phys. 39, 1 (1966).
- 6) L. Clavelle and P. Ramond, Phys.Rev. D2, 973 (1970). For a comprehensive
review see
P. Ramond "Group Theoretical Properties of Dual Resonance Models",
NAL-THY 15 (July 1971);
V. Alessandrini and D. Amati, CERN preprint TH.1425 (1971).
- 7) P. Olesen, Nuclear Phys. B18, 459, 473 (1970);
E.M. Gordon and C.S. Lam, Nuovo Cimento 5A, 495 (1971).
- 8) D.J. Gross and J.H. Schwarz, Phys.Rev.Letters 25, 406 (1970);
M. Ida, H. Matsumoto and S. Yazaki, Progr.Theor.Phys. Vol. 44, 456 (1970).
- 9) P. Campagna, E. Napolitano, S. Sciuto and S. Fubini, Nuovo Cimento 2A,
911 (1971).
- 10) C. Rosenzweig and P. Sukhatme, Nuovo Cimento 3A, 511 (1971).
- 11) One also has the following algebra relating the "spin" vertices with
fixed external spin J corresponding to a given external leg. Let

$$\alpha'^* = \left(\frac{p \cdot \sigma}{m} \right) \cdot \alpha_{(oi)}^* \quad , \quad \alpha = \alpha_{(oi)}$$

$$\mathcal{L}_+ = \frac{1}{\sqrt{2}} (\alpha'^* \cdot \alpha'^*) \quad \mathcal{L}_- = \frac{1}{\sqrt{2}} (\alpha' \cdot \alpha) \quad \mathcal{L}_0 = \frac{1}{2} [(\alpha'^* \cdot \alpha') - (\alpha' \cdot \alpha)]$$

one has:

$$[L_+, V^{(0)}(J_f, J, J_i)] = - \frac{\rho}{\sqrt{2}} \left(\rho \frac{d}{d\rho} + J+1 \right) V^{(0)}(J_f + \frac{1}{2}, J, J_i - \frac{1}{2})$$

$$[L_-, V^{(0)}(J_f, J, J_i)] = + \frac{\rho^{-1}}{\sqrt{2}} \left(\rho \frac{d}{d\rho} + J+1 \right) V^{(0)}(J_f - \frac{1}{2}, J, J_i + \frac{1}{2})$$

$$[L_0, V^{(0)}(J_f, J, J_i)] = \left(\rho \frac{d}{d\rho} \right) V^{(0)}(J_f, J, J_i)$$

(the right-hand side vanishes when the triangular inequalities are not satisfied).

12) One has set:

$$g' \equiv g'(v_1 v_2 v_3) = g(v_1 v_2 v_3) \left[(2 \sum_i v_i + 1)! \prod_i v_i! \right]^{-1/2}$$

13) D. Williams, Boulder Symposium on the Lorentz group (1965).