CM-P00058919

Archives

Ref.TH.1456-CERN

BOUNDS ON THE FORM FACTOR OF $K_{\ell 3}$ DECAY

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A B S T R A C T

We derive bounds on the form factor $f(t)$ of $K_{\ell 3}$ decay valid from the origin to the threshold $(m_{\pi} + m_K)^2$. If we use as input the soft pion theorem on $f_{\pm}(t)$ and a bound on the propagator $\Delta(0)$ of the divergence of the strangeness changing current at zero momentum, we obtain an upper and lower bound, which are close together but which do not fit with experiment, by allowing $\Delta(0)$ to vary as a free parameter, there is then better agreement. We also discuss some bounds for the decay parameters.

I. INTRODUCTION

The problem of deriving bounds on the $K_{\ell 3}$ decay parameters has been investigated by several authors ¹⁾⁻⁴⁾, with a particular interest in the form factor and its derivatives for zero momentum. From their analysis follows that the theoretical bounds disagree with the experimental data, but different hypotheses are included in the derivation, so that it is not clear where there is a failure in the theory. The main hypothesis is connected to the knowledge of the propagator $\Delta(t)$ of the divergence of the strangeness changing current, in its calculation the existence of a Lehmann-Källén representation is assumed, hence arises the problem of subtraction, and up to now only a dispersion relation without subtraction has been used; also the evaluation of the bounds involves an estimate on $\Delta(0)$, and the problem has been worked out ^{1),2)} assuming the validity of the chiral SW(3) model of Gell-Mann, Oakes and Renner ⁵⁾ and of Glashow and Weinberg ⁶⁾.

When comparing theoretical bounds with experimental data, it is important to point out that the information on the scalar form factor $f(t)$ is supplied by the measurement of the two quantities $f_+(t)/f_+(0)$ and $\xi(t) = f_-(t)/f_+(t)$, if the former is known with some accuracy, the latter is subjected to large errors, so the values of $f(t)/f(0)$ calculated from experiment are spread within a wide angle ⁷⁾.

Now, as we said before, bounds were calculated for zero momentum transfer, so that a comparison with experiment has been obtained by extrapolating linear or quadratic fits ⁷⁾ of the data at the origin. However, due to the large experimental uncertainty, we believe that such a procedure can easily fail to work, in particular if we hope to get the value of the successive derivatives at the origin.

In such a situation we believe that a more direct comparison with experimental data is necessary, so our purpose is to find an upper and lower bound for the scalar form factors which are valid in the region $0 \leq t < (m_K + m_\pi)^2$, i.e., including the decay region where the data are available in a model independent way.

In Section II we give a summary of the principal physical results and we explain the mathematical method we shall use in the next section.

In Section III we derive an upper and a lower bound for the normalized form factor $f(t)/f(0)$.

In Section IV we calculate numerical values for the bounds if we use as input :

- i) an estimate of $\Delta(0)$ made by Mathur and Okubo⁸⁾ assuming the validity of the chiral SW(3) model;
- ii) the theorem of Callan and Treiman⁹⁾, Mathur, Okubo and Pandit⁹⁾, giving a relation on the form factor near $t = m_K^2$.

In order to make a comparison with the present data we consider the case where $\Delta(0)$ is treated as a free parameter.

In conclusion we discuss the implications of the results on the different hypotheses we made in the derivation. An Appendix is devoted to the modifications of the bounds in the case of subtractions.

II. THE MATHEMATICAL METHOD

First, we will give a summary of the principal results obtained from the Cabibbo theory of semi-leptonic interactions. Let $V_\mu^{(4-i5)}$ be the strangeness changing current, then the two form factors $f_\pm(t)$ of the $K_{\ell 3}$ decay are defined by

$$\langle \pi^0(p) | V_\mu^{(4-i5)}(0) | K^+(k) \rangle = -(8E_p E_k)^{-\frac{1}{2}} (2\pi)^3 [(k+p)_\mu f_+(t) + (k-p)_\mu f_-(t)] \quad (1)$$

with $t = -(k-p)^2$. The matrix element of the divergence of the current allows us to define $f(t)$ and $d(t)$

$$-i(8E_k E_p)^{\frac{1}{2}} (2\pi)^3 \langle \pi^0(p) | \partial_\mu V_\mu^{(4-i5)}(0) | K^+(k) \rangle = (m_K^2 - m_\pi^2) f(t) \quad (2)$$

$$\begin{aligned} f(t) &= f_+(t) + t f_-(t) / (m_K^2 - m_\pi^2) \\ d(t) &= (m_K^2 - m_\pi^2) f(t) \end{aligned} \quad (3)$$

$d(t)$ is a real analytic function of t with a cut on the real axis at $t_0 \leq t \leq \infty$, $t_0 = (m_K + m_\pi)^2$.

Assuming the existence of a Lehmann-Källén representation for the propagator

$$\Delta(t) = \frac{i}{2} \int d^4x e^{iqx} \langle 0 | (j_\mu V_\mu^{(4-i5)}(x), j_\lambda V_\lambda^{(4+i5)}(0))_+ | 0 \rangle \quad (4)$$

we write the representation

$$\Delta(t) = \int_{t_0}^{\infty} \frac{\rho(t') dt'}{t' - t} \quad (5)$$

here we suppose that $\rho(t) = \text{Im} \Delta(t)/\pi$ converges sufficiently fast so that there is no subtraction.

$$\rho(t) = \frac{1}{2} (2\pi)^3 \sum_n |\langle 0 | j_\mu V_\mu^{(4-i5)}(0) | n \rangle|^2 \delta^{(4)}(q - q_n) \quad (6)$$

Separating out the π K state from the summation over $|n\rangle$, and using the positivity of the spectral weight, Li and Pagels¹⁾ derived the inequality for $t \geq t_0$

$$\rho(t) \geq \frac{3}{64\pi^2} t^{-1} [(t-t_0)(t-t_1)]^{\frac{1}{2}} |d(t)|^2 \quad (7)$$

with $t_1 = (m_K - m_\pi)^2$. Inserting (7) into (5), we get

$$\Delta(t) \geq \frac{3}{64\pi^2} \int_{t_0}^{\infty} dt' t'^{-2} [(t'-t_0)(t'-t_1)]^{\frac{1}{2}} |d(t')|^2 \quad (8)$$

From inequality (8) we will derive bounds on $d(t)$.

At this stage, we begin with the mathematical method³⁾, which we summarize as a transformation of the integral (8) into an inner product on the Hilbert space H^2 .

Consider a mapping of the cut t plane into the interior of the unit circle $|z| < 1$ by the conformal transformation

$$(t-t_0)^{\frac{1}{2}} t_0^{-\frac{1}{2}} = (1+z)(1-z)^{-1} \quad (9)$$

If we set

$$G(z) \equiv d(t) \quad (10)$$

$G(z)$ is a real analytic function of z inside the unit circle. On the cut, define $p(\theta)$ by

$$p(\theta) = t_0 \left| \cos \frac{\theta}{2} \right| \left| \sin \frac{\theta}{2} \right|^{-3} V(t) \quad (11)$$

$$V(t) = \frac{3}{63\pi} t^{-2} [(t-t_0)(t-t_1)]^{\frac{1}{2}} \quad (12)$$

with $0 \leq \theta \leq 2\pi$, $t = t_0 / \sin^2 \theta / 2$.

As $p(\theta)$ is non-negative and is a summable function of θ on the circle, and as $\log p(\theta)$ is also summable, we can construct ¹⁰⁾ a function $\Psi(z)$ which belongs to H^2 with the representation

$$\Psi(z) = \exp \left\{ \frac{1}{4\pi} \int_0^{2\pi} d\theta \frac{e^{i\theta} + z}{e^{i\theta} - z} \log p(\theta) \right\} \quad (13)$$

moreover, it satisfies almost everywhere on the circle the boundary value condition

$$|\Psi(e^{i\theta})|^2 = p(\theta) \quad (14)$$

With the Jensen-Poisson formula, an explicit form of $\Psi(z)$ is calculated ³⁾, and Eqs. (11) and (12) lead to the expression

$$\Psi(z) = \frac{(3/\pi)^{\frac{1}{2}}}{16} (1+z) \left(b + \frac{1+z}{1-z} \right)^{\frac{1}{2}} \quad (15)$$

with $b = (1-t_1/t_0)^{\frac{1}{2}}$. Thus, from (10) and (13) the product of the two functions

$$h(z) = \Psi(z) G(z) \quad (16)$$

belongs to H^2 , and (8) is rewritten as

$$S^2 \equiv \frac{1}{2\pi} \int_0^{2\pi} d\theta |p(\theta)| |G(e^{i\theta})|^2 = \frac{1}{2\pi} \int_0^{2\pi} d\theta |h(e^{i\theta})|^2 \leq \Delta(0) \quad (17)$$

The previous equation led us to define the inner product of two analytic functions inside the unit circle $|z| < 1$ by

$$\langle g, f \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\theta g^*(e^{i\theta}) f(e^{i\theta}) \quad (18)$$

and all such analytic functions with a finite norm $\|h\| = \langle h, h \rangle^{\frac{1}{2}}$ form the Hilbert space $H^2(10)$.

Let us give some examples which we shall need in the following

$$\begin{aligned} S^2 &\equiv \langle h, h \rangle \\ \langle z^n, h \rangle &= h^{(n)}(0)/n! \end{aligned} \quad (19)$$

$$\begin{aligned} \varphi_\lambda^N(z) &= z^{N+1}/(1-\lambda^*z) \quad |\lambda| < 1 \\ \langle \varphi_\lambda^N, h \rangle &= (h(\lambda) - \sum_{n=0}^N \lambda^n h^{(n)}(0)/n!)/\lambda^{N+1} \end{aligned} \quad (20)$$

$$\begin{aligned} \varphi'_\lambda(z) &= z^2/(1-\lambda^*z)^2 \\ \langle \varphi'_\lambda, h \rangle &= h'(\lambda)/\lambda - (h(\lambda) - h(0))/\lambda^2 \end{aligned} \quad (21)$$

Let $f_1, f_2, \dots, f_n$ be a set of functions, $f_n \in H^2$, and define a k -by- k matrix A_k whose elements are the inner products $a_{ij} = \langle f_i, f_j \rangle$, ($i, j = 1, \dots, n$), then the positive definiteness of A_n gives the conditions on the determinants

$$\det(A_k) \geq 0 \quad k = 1, \dots, n \quad (22)$$

the equality being possible in (22) if the f 's are not linearly independent.

One can see immediately the great flexibility of the method according to the choice of the set of functions f_n , we consider some examples :

- i) suppose that m values $f(t_1), f(t_2), \dots, f(t_m)$ of the form factor (3) are known accurately, from (16) and (20) we define the set $h(z)$,

$$\varphi_\lambda^0, \varphi_{\lambda_1}^0, \dots, \varphi_{\lambda_m}^0, 1, (\lambda_i \equiv \lambda(t_i)),$$

then the conditions (22) give a set of inequalities involving $h(\lambda)$, $h(0)$, $h(\lambda_1), \dots, h(\lambda_m)$, e.g., inequalities on the form factor in the decay region;

- ii) in the same way we obtain inequalities for the first derivative of the form factor in the decay region with the set $h(z), \varphi_\lambda', 1$;
- iii) recall the set defined by Okubo and Shi³⁾, $h(z), z^n, z^{n-1}, \dots, 1, \varphi_\lambda^N$.

We close this section by a simple remark that the set of linearly independent functions $f_n \in H^2$ is not necessarily orthogonal.

III. INEQUALITIES ON THE FORM FACTOR

In this section we derive inequalities for the scalar form factor which are valid in the physical region.

Let λ and μ be two real points inside the unit circle, with (16), (17) and (20), define the set of linearly independent functions $h(z), 1, \varphi_\mu^0(z), \varphi_\lambda^0(z)$, according to the method developed in the previous section, we construct a four-by-four matrix whose elements are made of all the inner products of the functions

$$M \equiv \begin{vmatrix} S^2 & h(0) & \mu^{-1}(h(\mu)-h(0)) & \lambda^{-1}(h(\lambda)-h(0)) \\ h(0) & 1 & 0 & 0 \\ \mu^{-1}(h(\mu)-h(0)) & 0 & (1-\mu^2)^{-1} & (1-\lambda\mu)^{-1} \\ \lambda^{-1}(h(\lambda)-h(0)) & 0 & (1-\lambda\mu)^{-1} & (1-\lambda^2)^{-1} \end{vmatrix} \quad (23)$$

Setting, with (15),

$$g(\mu) = k_1 \Psi(\mu) / \Psi(0) \quad (24)$$

and $k_2 = \Psi(0)(m_K^2 - m_\pi^2)$, we get

$$h(\mu) = k_2 g(\mu) f_+(0) \quad (25)$$

$$h(0) = k_2 f_+(0) \quad (26)$$

Following Eq. (22), the successive determinants extracted from the matrix M yield the inequalities :

$$(In.1) \quad [f_+(0)]^2 < \frac{S^2}{k_2^2}$$

$$(In.2) \quad [f_+(0)]^2 < \frac{S^2}{k_2^2} \left[1 + \frac{1-\mu^2}{\mu^2} (1-g(\mu))^2 \right]^{-1}$$

$$(In.3) \quad \frac{\Psi(0)}{\Psi(\lambda(t))} [1 - X_+] \leq \frac{f(t)}{f(0)} \leq \frac{\Psi(0)}{\Psi(\lambda(t))} [1 - X_-]$$

where $X_{\pm}$ and D are given by the expressions

$$X_{\pm} = \frac{(1-\mu^2)\lambda}{(1-\lambda\mu)\mu} (1-g(\mu)) \pm D^{\frac{1}{2}} \quad (27)$$

$$D = \frac{\lambda^2(\lambda-\mu)^2}{(1-\lambda\mu)^2(1-\lambda^2)} \left[\frac{S^2}{h(0)^2} - 1 - \frac{(1-\mu^2)(1-g(\mu))^2}{\mu^2} \right] \quad (28)$$

These inequalities are not independent : (In.2) corresponds to a better inequality than (In.1), also (In.2) is the expression of the positivity condition of D in (28). The inequalities (In.3) give practical bounds to the normalized form factor as a function of $f_+(0)$ in the region $0 \leq t < t_0$, once we know the value of the form factor at one point in the interval and an upper estimate of S^2 , but in most cases S^2 depends itself on $f_+(0)$ so that our bounds are dependent on $f_+(0)$. Note that with the same assumption we can derive bounds on $df(t)/dt$ in the previous interval; the corresponding set of functions is : $h(z)$, 1 , $\varphi_{\lambda}^0(z)$, $\varphi_{\lambda}'(z)$, but in the absence of accuracy on the data to set precise values on the derivatives, we do not give the expressions of the bounds.

IV. THE RESULTS

In order to give a numerical estimate of the bounds (In.2) and (In.3) we must define the input. A knowledge of the value of the form factor at one point is required in the calculation, for this purpose we recall that a theorem of Callan and Treiman⁹⁾, Suzuki⁹⁾, Mathur, Okubo and Pandit⁹⁾, states the following relation for the form factor $f_{\pm}(t)$ at $t=m_K^2$

$$f_+(m_K^2) + f_-(m_K^2) = \frac{f_K}{f_{\pi}} + O(\varepsilon_2) \quad (29)$$

where, in the approximation $m_K^2/(m_K^2 - m_{\pi}^2) \sim 1$, we get a value of the form factor at $t=m_K^2$; a more accurate version of the theorem corresponds to shift the value of t to the point $(m_K^2 - m_{\pi}^2)$ ¹¹⁾.

The second input is an estimate of $\Delta(0)$ implied by the inequality (17). Here we use an inequality derived by Okubo¹²⁾ assuming the validity of the chiral SW(3) model^{5),8)}

$$(\Delta(0))^{\frac{1}{2}} \leq 2^{-\frac{1}{2}} (m_K f_K - m_{\pi} f_{\pi}) \quad (30)$$

Now, making use of the experimental value $f_K/f_{\pi} = 1.27 f_+(0)$, and from (In.2) we obtain the lower bound

$$f_+(0) \geq 0.25 \quad (31)$$

The numerical calculation of the bounds (In.3) on $f(t)/f(0)$ is presented in the Figure (solid line), with the normalization $f_+(0)=1$ owing to the dependence of $\Delta(0)$ on $f_+(0)$. We see the disagreement with the experimental data (which lie mostly under the value 1), also we remark that the bounds join smoothly the soft pion point, and that the input yields bounds close together, which could correspond to a good approximation to the form factor if there were a better agreement with the data. We have checked the dependence of the bounds when $f_+(0)$ varies, in the range $0.3 \leq f_+(0) \leq 1.2$; the variation of the bounds is small and cannot be seen on the plot. Also a comparison of the results using the soft pion point or the point m_K^2 show a small effect.

Before proceeding with the discussion, we wish to extend the numerical analysis made here by taking the bound (30) as a free parameter, with (27) and (28), the inequalities (In.3) show that any increase of the value of (30) corresponds to an enlargement of the domain included between the upper and lower bounds of $f(t)/f(0)$, and such an enlargement allows the experimental

points to fall down in the new domain. Let us recall that with the inequality (30) and $f_+(0) = 1$, we have :

$$s^2/h^2(0) \leq \Delta(0)/h^2(0) \leq 5.85 \quad (32)$$

A new contour (dashed line) is plotted on the Figure, with arbitrary value

$$\Delta(0)/h^2(0) \sim 1300. \quad (33)$$

The corresponding bounds on $\Lambda_0 = m^2 d'(0)/d(0)$ are

$$-0.072 \leq \Lambda_0 \leq 0.10$$

In order to improve the bounds, there is a possibility to include in the derivation more values of the form factor in addition to the soft point value, but this latter is the only theoretical value known today, and the experimental points are too badly distributed to be inserted in the derivation.

Another numerical search concerns the soft pion theorem, we suppose that $f(m_K^2 - m_\pi^2) \equiv F$ is a free parameter, then a value of F less than (29) must improve the bounds because in that case if $F < 1$, we observe a smooth decrease of the bounds to F . We mention that (In.2) (giving the positivity of D) can be used to find the lowest bound to $f(t)/f(0) = \alpha(t)$ when $\Delta(0)/h^2(0)$ is known; we have plotted on the Figure $\alpha(t)$ (dotted line) with the value (32).

V. CONCLUSION

From the results of the previous section, our bounds do not fit with the experimental data using the present input, however, we can draw two general features, the input value of the form factor controls the increasing or decreasing shape of the bounds, the upper estimate (17) to s^2 controls the width of the domain included between the upper and lower bounds, i.e., the quality of the bounds.

The main difficulty to reconcile our bounds with the data comes from the soft pion theorem, which determines the input value of the normalized form factor to be greater than unity, with such a value it would be necessary to have a dip mechanism to explain the fall-off of the data, but this is not the case. Also, inequality (30) is too stringent to give a domain including most of the experimental points, and in the present state of the analysis only

an increase of its value seems to give a better agreement. In fact, the effects of the two input parameters are related and they show that their values are not compatible, in the sense that if we take one for granted, the other one has to be changed [for a review of the physical content of the input, see Ref. 13].

Now the problem we want to mention is that of the subtractions. In the derivation we made the simplifying assumption that $\Delta(t)$ satisfies an unsubtracted dispersion relation (5), but it must be borne in mind that the effects of subtractions open a possibility for modifying the bounds and an improvement is not completely ruled out. As a preliminary, we give in the Appendix the modifications to the bounds corresponding to one and two subtractions. We also specify the new input parameters coming into the problem.

ACKNOWLEDGEMENTS

I am indebted to Professor A. Martin for suggesting this study and for helpful discussions. I wish to thank Dr. M.K. Gaillard for a careful reading of the manuscript and for relevant comments. I also wish to acknowledge the CERN Theoretical Study Division for its hospitality.

A P P E N D I X

This Appendix is included in view of a possible future treatment of the problem of subtractions. We shall show the corresponding modification to our previous formula and the new input parameters coming into the problem. At present we have very few indications on the number of necessary subtractions. So, as a first attempt, we will consider the problem with one and two subtractions.

In deriving the bounds, we started with the lower inequality (7) on the spectral weight of the propagator (derived by just retaining the π K intermediate state), here we suppose that the effect of the subtraction can be included in $d(t)$ and we keep the kinematical factor to be the same

$$\rho(t) \geq g_1(t) = \frac{3}{64\pi^2} t^{-1} [(t-t_0)(t-t_1)]^{\frac{1}{2}} |d(t)|^2 \quad (\text{A.1})$$

Subtractions are implied when $d(t)$ does not decrease sufficiently fast at infinity. First, suppose that $|d(t)|^2$ behaves like a constant as t goes to infinity, then we write

$$\Delta(t) = \Delta(t_s) + \frac{t-t_s}{\pi} \int_{t_0}^{\infty} \frac{\rho(t') dt'}{(t'-t)(t'-t_s)} \quad (\text{A.2})$$

In the following, we restrict ourselves to $t=0$ and $t_s < t_0$. The positivity of $g_1(t)$ gives the inequalities

$$I_1^2 = \int_{t_0}^{\infty} \frac{\rho(t') dt'}{t'(t'-t_s)} \geq \int_{t_0}^{\infty} \frac{g_1(t') dt'}{t'(t'-t_s)} \geq 0 \quad (\text{A.3})$$

We set

$$J_1^2 \equiv \int_{t_0}^{\infty} \frac{g_1(t') dt'}{t'(t'-t_s)} \quad (\text{A.4})$$

Equation (A.3) implies that

$$(\Delta(t_s) - \Delta(0)) \frac{\pi}{t_s} = I_1^2 \geq J_1^2 \quad (\text{A.5})$$

as a consequence $\Delta(t_s) \geq \Delta(0)$.

The method developed in Section II is still valid, and we replace (11) by

$$P(t) = \frac{3}{64\pi} t^{-2} (t-t_s)^{-1} [(t-t_0)(t-t_1)]^{\frac{1}{2}} \quad (\text{A.6})$$

and

$$J_1^2 \equiv \frac{1}{\pi} \int_{t_0}^{\infty} P(t') |d(t')|^2 dt' \quad (\text{A.7})$$

From (13) and the expression of $P(t)$ a new function $\phi_s(z)$ can be constructed, and $h_s(z)$ is the function

$$h_s(z) = \phi_s(z) G_s(z) \quad (\text{A.8})$$

Inequality (17) is replaced by

$$J_1^2 \equiv \frac{1}{2\pi} \int_0^{2\pi} d\theta |h_s(e^{i\theta})|^2 \leq \frac{\pi}{t_s} (\Delta(t_s) - \Delta(0)) \quad (\text{A.9})$$

In Section III, inequalities (In.1)-(In.3) remain valid with the modifications $\Psi(z) \rightarrow \phi_s(z)$, $h(z) \rightarrow h_s(z)$ and $S^2 \rightarrow J_1^2$.

If two subtractions are necessary, let t_0 and t_1 be the subtraction points ($t_0 < t_1$), then (A.4) and (A.5) lead to

$$J_2^2 \equiv \int_{t_0}^{\infty} \frac{g_2(t') dt'}{t'(t'-t_0)(t'-t_1)} \quad (\text{A.10})$$

$$J_2^2 \leq \frac{\pi}{t_1 t_0} \cdot \frac{t_0 (\Delta(t_1) - \Delta(0)) - t_1 (\Delta(t_0) - \Delta(0))}{t_1 - t_0} \quad (\text{A.11})$$

with the inequality $\Delta(t_1) \geq \Delta(t_0)$. The corresponding changes to be made in the previous formula are straightforward.

Now the important problem is to find an upper bound to $\Delta(t)$ at the subtraction points in order to compute numerically the new bounds on $f(t)$.

R E F E R E N C E S

- 1) L.F. Li and H. Pagels, Phys.Rev. D3, 2192 (1971); D4, 255 (1971).
- 2) S. Okubo, Phys.Rev. D3, 2807 (1971).
- 3) S. Okubo and I-Fu Shih, Phys.Rev. D4, 2020 (1971).
- 4) R. Acharya, preprint CPT 118 ORO 39 2-69 (1971).
- 5) M. Gell-Mann, R.J. Oakes and B. Renner, Phys.Rev. 175, 2195 (1968).
- 6) S.L. Glashow and S. Weinberg, Phys.Rev.Letters 20, 224 (1968).
- 7) For example, L.M. Chounet, J.M. Gaillard and M.K. Gaillard, Leptonic decays of hadrons (to be published in Physics Reports).
- 8) V.S. Mathur and S. Okubo, Phys.Rev. D1, 3468 (1970).
- 9) C. Callan and S. Treiman, Phys.Rev.Letters 16, 153 (1966);
M. Suzuki, Phys.Rev.Letters 16, 212 (1966);
V.S. Mathur, S. Okubo and L.K. Pandit, Phys.Rev.Letters 16, 371 (1966).
- 10) K. Hoffmann, "Banach Spaces of Analytic Functions", (Prentice Hall, Englewood Cliffs, N.J., 1962).
- 11) R. Dashen and M. Weinstein, Phys.Rev.Letters 22, 1337 (1969);
V.S. Mathur and S. Okubo, Phys.Rev. D2, 619 (1970).
- 12) S. Okubo, Phys.Rev. D4, 725 (1971).
- 13) M.K. Gaillard, Talk presented at the winter meeting of the American Physical Society, Cambridge, Mass., 27-29 December 1971.

FIGURE CAPTION

Bounds on the form factor. Solid line with inequality (30),
dashed line with the value (33), dotted line $\alpha(t)$.
Compilation of data of Ref. 7).

FIG.1