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CHARGE INDEPENDENCE RESTRICTIONS ON SOME
HIGH ENERGY TWO-BODY PROCESSES

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A B S T R A C T

Bounds due to isospin invariance have been considered for spin effects and for the unpolarized cross-sections in some situations of experimental interest. Dynamical consequences like possible structure in the near-forward direction and some knowledge of the strength of the real part of the forward scattering amplitude follow in a model-independent way from these isospin bounds for the unpolarized differential cross-sections for $np \rightarrow np$ scattering and $K^\pm n \rightarrow K^\pm n$ scattering at high energies. Useful bounds for the differential cross-sections for high energy $\bar{p}n \rightarrow \bar{p}n$ scattering and for the high energy reactions $\pi N \rightarrow \rho N$ and $KN \rightarrow K_{890}^* N$ (and also bounds for the full observed density matrix of the vector meson) have been illustrated. We point out some simple implications of the rotation of the reference axes (about the normal to the production plane) for the resonance density matrix bounds. Some other applications are discussed.

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1. INTRODUCTION

The inequalities ^{1),2)} between the cross-sections for three (or more) processes because of relations (due to isospin-invariance or any other symmetry) between their amplitudes can be very useful in particular situations [see for example, 3)-5)]. This paper is devoted to isospin bounds in some two-body processes at high energies. In Section 2, we list these bounds some of which are applied to data in Section 3. Dynamical consequences like a possible change of slope of the near-forward peak for $np \rightarrow np$ scattering, and some knowledge of the relative strength of the real part of the forward $K^{\pm}n \rightarrow K^{\pm}n$ scattering amplitude follow by comparing (Section 3) our bounds on the unpolarized differential cross-sections due to isospin invariance alone, with the existing data. Quite restrictive bounds have been obtained for the unpolarized differential cross-sections for high energy $\bar{p}n \rightarrow \bar{p}n$ scattering and for the high energy reactions $\pi N \rightarrow \rho N$ and $KN \rightarrow K_{890}^* N$. The isospin bounds on the observed part of the resonance density-matrix have been illustrated for the case of vector-meson production. For resonance production, some simple consequences of rotating the reference frame around the normal to the production plane, and their relevance to isospin bounds, have also been illustrated for the above vector-meson production processes. This rotation provides a simple way for checking the well-known positivity requirements [see, for example, 6)] on the density-matrix. Some concluding remarks are made in Section 4.

2. THE BOUNDS

When the amplitudes T_i for the processes $i=1,2,\dots,N$ satisfy the linear relation

$$\sum_{i=1}^N T_i = 0 \quad (1)$$

due to some principle like isospin-invariance, the generalization of the usual triangular inequalities ^{1),2)} gives, for the corresponding differential cross-sections $\sigma_i \equiv |T_i|^2$, the relations

$$\sqrt{\sigma_j} \leq \sum_{i \neq j}^N \sqrt{\sigma_i} \quad ; \quad j = 1, \dots, N \quad (2)$$

which give the bounds

$$\sqrt{\sigma_M} = \sum_{i \neq j, M}^N \sqrt{\sigma_i} \leq \sqrt{\sigma_j} \leq \sum_{i \neq j}^N \sqrt{\sigma_i} \quad (3)$$

where M stands for the process with the largest cross-section. For the case of $N=3$, one has the well-known triangular inequality

$$(\sqrt{\sigma_1} - \sqrt{\sigma_2})^2 \leq \sigma_3 \leq (\sqrt{\sigma_1} + \sqrt{\sigma_2})^2 \quad (4)$$

By considering the polarized differential cross-sections $(1 \pm P)\sigma$, one gets the isospin bounds ²⁾

$$(\sqrt{(1 \pm P_1)\sigma_1} - \sqrt{(1 \pm P_2)\sigma_2})^2 \leq (1 \pm P_3)\sigma_3 \leq (\sqrt{(1 \pm P_1)\sigma_1} + \sqrt{(1 \pm P_2)\sigma_2})^2 \quad (5)$$

for, for example, the processes ($i=1,2,3$)

$$1: \quad p p \rightarrow p p$$

$$2: \quad p n \rightarrow n p \quad (\text{charge-exchange})$$

$$3: \quad p n \rightarrow p n \quad (\text{elastic})$$

where the polarizations P_i are defined along the direction $(\vec{p}_1 \times \vec{p}_2)$, $\vec{p}_1$ and $\vec{p}_2$ being the momenta of the initial and final protons; here, elastic (charge-exchange) np scattering corresponds to c.m. scattering angles, between the initial and final protons, less (greater) than 90° . The relations (5) give the bounds on σ_3 (P_3) when σ_1 , σ_2 , P_1 , P_2 and P_3 (σ_3) are known. However, if σ_3 is unknown, or poorly known, one may use the bounds on P_3

$$\frac{\sqrt{\sigma_2} [\sqrt{\sigma_2} (P_2 - P_1) - x]}{\sigma_1 + \sigma_2 - y} \leq (P_3 - P_1) \leq \frac{\sqrt{\sigma_2} [\sqrt{\sigma_2} (P_2 - P_1) + x]}{\sigma_1 + \sigma_2 + y} \quad (6)$$

where

$$x = \sqrt{\sigma_1(1-P_1^2)} \left[\sqrt{(1+P_2)(1-P_1)} + \sqrt{(1-P_2)(1+P_1)} \right]$$

$$y = \sqrt{\sigma_1\sigma_2} \left[\sqrt{(1+P_2)(1+P_1)} - \sqrt{(1-P_1)(1-P_2)} \right]$$

which do not require a knowledge of σ_3 . Similarly, if P_3 is unknown or poorly known, one may use the bounds on σ_3

$$\sigma_1 + \sigma_2 - z \leq \sigma_3 \leq \sigma_1 + \sigma_2 + z \quad (7)$$

where

$$z = \sqrt{\sigma_1\sigma_2} \left[\sqrt{(1-P_1)(1-P_2)} + \sqrt{(1+P_1)(1+P_2)} \right]$$

which do not require a knowledge of P_3 . However, if only the cross-sections are known, one can get the bounds ³⁾

$$\frac{1}{2} |P_3 - P_1| \leq \sqrt{\frac{\sigma_2}{2\sigma_1\sigma_3}} \left[\sigma_1 + \sigma_3 - \frac{1}{2}\sigma_2 - \frac{(\sigma_1 - \sigma_3)^2}{2\sigma_2} \right] \quad (8a)$$

for the polarization difference $(P_1 - P_3)$.

In situations where only P_1 , σ_1 and σ_2 are known, the useful bounds for P_3 are

$$\sqrt{\frac{\sigma_2}{\sigma_1}} \left[-\sqrt{\frac{\sigma_2}{\sigma_1}} - \sqrt{(1-P_1^2)(1-\frac{\sigma_2}{\sigma_1})} \right] \leq \frac{1}{2}(P_3 - P_1) \leq \sqrt{\frac{\sigma_2}{\sigma_1}} \left[-\sqrt{\frac{\sigma_2}{\sigma_1}} + \sqrt{(1-P_1^2)(1-\frac{\sigma_2}{\sigma_1})} \right] \quad (8b)$$

where the upper bounds are applicable only when

$$\frac{(1-P_1)}{2} \frac{\sigma_1}{\sigma_2} > 1 \quad (8c)$$

and the lower bound, only when

$$\frac{(1+P_1)}{2} \frac{\sigma_1}{\sigma_2} > 1 \quad (8d)$$

The bounds (8b) can be restrictive when $\sigma_2 \ll \sigma_1$. The bounds (5), (6), (7) and (8) are only typical ones: they hold also if the polarization P is replaced by other spin effects like the D and C_{NN} parameters of nucleon-nucleon scattering, as noted by Phillips²⁾; they also hold for meson-nucleon scattering, also if the spin-parameters R and A are considered^{3),5)} instead of polarization.

For resonance production processes, the isospin bounds for the combinations $\rho_{\ell\ell} \cdot \sigma$ where $\rho_{\ell\ell}$ is a (non-negative) diagonal element of the density matrix of the produced resonance, are similar to those for the polarized cross-sections $(1 \pm P)\sigma$ of $\pi N \rightarrow \pi N$ scattering. For the off-diagonal elements $\rho_{\ell m}$ which can be negative, the situation is different. For the case of only three charge-modes ($i=1,2,3$), the bounds are

$$X_1 + X_2 - Y_{12} \leq X_3 \leq X_1 + X_2 + Y_{12} \quad (9)$$

where

$$X_i = (\rho_{\ell m} \sigma)_i$$

$$Y_{12} = [\sqrt{(\rho_{\ell\ell})_2 (\rho_{mm})_1} + \sqrt{(\rho_{\ell\ell})_1 (\rho_{mm})_2}] \sqrt{\sigma_1 \sigma_2}$$

In fact, the bounds (9) hold for the off-diagonal ($\ell \neq m$), as well as for the diagonal ($\ell = m$) elements. The diagonal elements are not all independent because of the trace condition $\sum_{\ell} \rho_{\ell\ell} = 1$ which normalizes the density-matrix. For vector-meson production, for example, independently valid bounds on ρ_{00} can be obtained from the relations (9) for $\ell = m = 1$ and for $\ell = m = 0$. The bounds corresponding to (9) with Y_{12} replaced by $\sqrt{\sigma_1 \sigma_2}$ were already obtained in Ref. 3). If σ_3 is known, (9) can be treated as bounds on $(\rho_{\ell m})_3$.

If $(\rho_{lm})_3$ are unknown, or poorly known, one may use the bounds on σ_3

$$\sigma_1 + \sigma_2 - 2 Z_{12} \leq \sigma_3 \leq \sigma_1 + \sigma_2 + 2 Z_{12} \quad (10)$$

where

$$Z_{12} = \left[\sum_2 \sqrt{(\rho_{\ell\ell})_1 (\rho_{\ell\ell})_2} \right] \sqrt{\sigma_1 \sigma_2} ;$$

the relations (10) require a knowledge of the diagonal elements $\rho_{\ell\ell}$ for only the charge modes 1 and 2, but not of $(\rho_{lm})_3$. The bounds (10) are better than the bounds (4).

If one rotates the frame of axes (with respect to which the density-matrix is defined) about the normal to the production plane, one gets a continuous family of systems in each of which the bounds (9) apply. If in a certain frame of reference axes obtained by rotation through an angle θ , the diagonal element $(\rho_{\ell\ell})_2$ vanishes, one gets the simple prediction

$$(\rho_{\ell\ell})_1^\theta \sigma_1 = (\rho_{\ell\ell})_3^\theta \sigma_3 \quad (11)$$

valid in this new frame, indicated by the superscript; the matrix elements ρ_{lm}^θ are known functions of the original elements ρ_{lm} and the angle θ . Similarly, if the off-diagonal element ρ_{lm}^θ vanishes for the charge-mode 1 or/and 2, the bounds (9) for $(\rho_{lm}^\theta)_3$ get simplified. To illustrate the type of information obtained by this rotation, we consider the vector-meson production reactions (e.g., $\pi N \rightarrow \rho N$ and $KN \rightarrow K_{890}^* N$). For some of these data, $(\text{Re } \rho_{10}^\theta)_1$ and $(\text{Re } \rho_{10}^\theta)_2$ vanish for the same angle θ of rotation within errors, in which case the bound (9) becomes

$$|(\text{Re } \rho_{10}^\theta)_3 \sigma_3| \leq \left[\sqrt{(\rho_{00}^\theta)_2 (\rho_{11}^\theta)_1} + \sqrt{(\rho_{00}^\theta)_1 (\rho_{11}^\theta)_2} \right] \sqrt{\sigma_1 \sigma_2} \quad (12a)$$

If in a certain frame, $(\rho_{00}^{\theta})_1 = 0$, it would mean $(\text{Re } \rho_{10}^{\theta})_1 = 0$ and, therefore,

$$(\text{Re } \rho_{10}^{\theta})_2 - \sqrt{\frac{1}{2} \sigma_1 \sigma_2 (\rho_{00}^{\theta})_2} \leq (\text{Re } \rho_{10}^{\theta})_3 \leq (\text{Re } \rho_{10}^{\theta})_2 + \sqrt{\frac{1}{2} \sigma_1 \sigma_2 (\rho_{00}^{\theta})_2} \quad (12b)$$

If, however, $(\rho_{11}^{\theta})_1 = 0$, it means $(\rho_{00}^{\theta})_1 = 1$, $(\text{Re } \rho_{10}^{\theta})_1 = 0$ and

$$(\text{Re } \rho_{10}^{\theta})_2 - \sqrt{\sigma_1 \sigma_2 (\rho_{11}^{\theta})_2} \leq (\text{Re } \rho_{10}^{\theta})_3 \leq (\text{Re } \rho_{10}^{\theta})_2 + \sqrt{\sigma_1 \sigma_2 (\rho_{11}^{\theta})_2} \quad (12c)$$

In the relations (12b, c), the normalization

$$\rho_{00} + 2 \rho_{11} = 1 \quad (12d)$$

has been used.

The observed part of the density-matrix, after rotation is given in terms of the original matrix elements by

$$\begin{aligned} \rho_{00}^{\theta} &= (\rho_{11} - \rho_{1-1}) \sin^2 \theta + \rho_{00} \cos^2 \theta - \sqrt{2} (\text{Re } \rho_{10}) \sin 2\theta \\ - 2 \rho_{1-1}^{\theta} &= (\rho_{11} - \rho_{1-1}) \cos^2 \theta + \rho_{00} \sin^2 \theta + \sqrt{2} (\text{Re } \rho_{10}) \sin 2\theta - (\rho_{11} + \rho_{1-1}) \\ - \sqrt{2} \text{Re } \rho_{10}^{\theta} &= \frac{1}{2} (\rho_{11} - \rho_{1-1}) \sin 2\theta - \frac{1}{2} \rho_{00} \sin 2\theta - \sqrt{2} (\text{Re } \rho_{10}) \cos 2\theta \\ \rho_{00}^{\theta} + 2 \rho_{11}^{\theta} &= \rho_{00} + 2 \rho_{11} = \text{constant} \\ \rho_{11}^{\theta} + \rho_{1-1}^{\theta} &= \rho_{11} + \rho_{1-1} = \text{constant} \end{aligned}$$

The relations (9) in the rotated frame can give useful bounds on the density-matrix elements $(\rho_{lm})_3$ of the original frame, even in situations where the relation (9) cannot be used in the original frame because of inadequate data. For example, the extraction of the off-diagonal element $(\text{Re } \rho_{10})_{1,2}$ can become difficult because of a poor knowledge of the azimuthal dependence in the angular distribution; one cannot then use the relations (9), in the original frame, for getting bounds on $(\text{Re } \rho_{10})_3$. However, the relations (9), expressed in the rotated frames for suitable values of θ , can give bounds on certain linear combinations of $(\rho_{lm})_3$ - even if only incomplete information on $(\rho_{lm})_2$ and $(\rho_{lm})_1$ is available. For example, the relation (9) for $(\rho_{00}^\theta \sigma)_3$, $\tan^2 \theta = 2$ gives

$$(\sqrt{A_1} - \sqrt{A_2})^2 \leq A_3 \leq (\sqrt{A_1} + \sqrt{A_2})^2 \quad (14)$$

where $A_i = \frac{1}{3}(1 \mp 4 \text{Re } \rho_{10} - 2 \rho_{1-1})_i \sigma_i$, ($i=1,2,3$); this can be used to get information on $(2 \text{Re } \rho_{10} \pm \rho_{1-1})_3$ even if $(\rho_{00})_1$, $(\rho_{00})_2$, $(\rho_{11})_1$ and $(\rho_{11})_2$ are not known; the normalization (12d) has been used in (14). Using the expressions (13), one can similarly construct other examples where the relations (9) in a suitably rotated frame can provide useful bounds even if this relation in the original frame cannot be used for data reasons.

The two Donohue-Högaasen frames ⁷⁾ in which $\text{Re } \rho_{10}^\theta = 0$ correspond to the angles of rotation ⁷⁾

$$\tan 2\theta = \frac{-2\sqrt{2} \text{Re } \rho_{10}}{\rho_{00} - \rho_{11} + \rho_{1-1}} \quad (15)$$

In fact, the density-matrix elements ρ_{00} , ρ_{11} and ρ_{1-1} obtain extremum values in these frames ⁸⁾. One gets

$$\begin{aligned} (\rho_{00})_{DH} &= \frac{1}{2}(\rho_{00} + \rho_{11} - \rho_{1-1}) \mp \frac{1}{2}[(\rho_{00} - \rho_{11} + \rho_{1-1}) + 8(\text{Re } \rho_{10})^2]^{\frac{1}{2}} \\ (\rho_{1-1})_{DH} &= \frac{1}{4}(\rho_{11} - \rho_{00} + 3\rho_{1-1}) \mp \frac{1}{4}[(\rho_{00} - \rho_{11} + \rho_{1-1}) + 8(\text{Re } \rho_{10})^2]^{\frac{1}{2}} \end{aligned} \quad (16)$$

the upper and lower signs corresponding respectively to the two values of θ in (15). We shall refer to the frames in which ρ_{00}^{θ} (and also ρ_{1-1}^{θ}) attain their maximum and minimum values as DH1 and DH2, respectively, and the angles of rotation to these frames as $(\alpha - \frac{\pi}{2})$ and α respectively.

3. APPLICATIONS TO SOME HIGH ENERGY TWO-BODY PROCESSES

A. NUCLEON-NUCLEON SCATTERING

A1. Differential cross-sections

If the cross-section σ_2 is much smaller than σ_1 , the upper and lower bounds (4) on σ_3 lie close to each other; this is the case if the indices 1, 2 and 3 stand respectively for pp→pp, charge-exchange pn→np and elastic np→np scatterings at high energies. Figure 1 compares these bounds on the np→np elastic differential cross-section at 12, 19.2 and 24 GeV/c with the available data ⁹⁾⁻¹¹⁾. The input pp data are from Ref. 12); for the np→pn charge exchange cross-sections, instead of interpolating ^{*)} the available data, we have used a phenomenological fit ¹³⁾ which reproduces very well the data of Ref. 14) and assigned generous (30%) errors to it ^{**)}. In this Figure and in Figs. 2, 3, 4, the optical points have been calculated by using the total cross-sections from Ref. 15).

Before drawing possible conclusions from the comparison of our bounds of Fig. 1 with the data, we make a few simple remarks. The extraction of np→np data from pd scattering ^{9),16)} which usually employs the Glauber theory and some further simplifying assumptions is not a completely unique procedure. On the other hand, the angular distribution of the np→np

*) Whenever the data on the different charge modes were not available at the same t (the invariant momentum-transfer squared), we did some simple interpolations; this applies to all our isospin bounds. Also, we have treated the errors as if they were uncorrelated; hence our errors for the density-matrix element calculations of the subsection D below cannot be taken literally.

**) The bounds are not very sensitive to small changes in the charge-exchange cross-section and, therefore, our procedure is quite adequate. In fact, the 19 and 24 GeV/c charge-exchange data exist only at $|t| < 0.4 \text{ GeV}^2$; but we feel that for larger $|t|$ also, this phenomenological fit [which gives a very good representation at those $|t|$ values at lower energies, 8 and 10 GeV/c] is good enough for our purposes.

data obtained with neutron beams should be quite reliable; there may be normalization difficulties. In the neutron-beam experiment of Ref. 10), the normalization is fixed by extrapolating to the optical point and assuming the relative strength of the real part of the forward amplitude to be known. This strength is extracted from pd data¹⁶⁾ and may be ambiguous. This normalization is non-unique also because of the possibility of a change of the diffraction slope for very small $|t|$ values; such a change of slope has been previously reported¹⁷⁾ for $pp \rightarrow pp$ scattering.

Figure 1 shows that the neutron-beam data of Ref. 10) lie consistently above the upper bound, and there is an indication of the data having a flatter slope than the upper bound. If the normalization of the data is shifted to produce good agreement with our bounds, one sees that for the smallest $|t|$ values, $d\sigma/dt(np \rightarrow np) < d\sigma/dt(pp \rightarrow pp)$; and also, the data of Ref. 10) extrapolate to a point below the optical point^{*)}. With the above assumptions, isospin invariance would, therefore, imply a change of slope in $np \rightarrow np$ scattering at small $|t|$ values. Moreover, because of the near-equality of the pp and np total cross-sections, this change of slope should probably be bigger than the corresponding change seen in pp elastic scattering. It would be interesting to have $np \rightarrow np$ experimental results with neutron beams for the small $|t|$ region. We should remark, however, that if one compares our bounds with another neutron beam experiment¹¹⁾, our conclusion of a possible change of slope becomes weaker; unfortunately, the quoted errors on the data of this latter experiment are larger.

A2. Polarization

The relations (5) provide bounds on polarization (P_3) in elastic $pn \rightarrow pn$ scattering if cross-sections for all the three modes and polarizations for the other two modes are known. Because of the above-mentioned normalization difficulties for the $np \rightarrow np$ elastic cross-section (σ_3), use of the relations (6) which do not require a knowledge of σ_3 may be better, in order to obtain bounds on P_3 . In practice, the improvement obtained by using the bounds (6) or the bounds (8b), instead of the bounds (8a) which do not require as much knowledge of polarizations, is negligible as compared to the errors on the bounds. At 10 GeV/c, near the forward direction, we find typically, $|P_1 - P_3| \leq \text{about } 0.2$, where P_1 is the polarization in pp elastic scattering. Because $P_1 \sim 0.1$ experimentally, this bound is insufficient to determine the sign of P_3 . One may give a duality argument that, unlike the case of πN scattering⁵⁾, the bounds (8a) on

*) Such a phenomenon has recently been observed at Serpukhov energies in $\pi^- p \rightarrow \pi^- p$ scattering¹⁸⁾ also.

$|P_3 - P_1|$ are not saturated: the point is that at low energies, $P_1 \approx P_3$; and because of the lack of dibaryon resonances, global duality^{*)} expects this near equality to continue to high energies also. Unfortunately, this duality argument can be, at best, qualitative.

Comparison of the polarizations P_1 and P_3 can give useful information on the relative strength of the isoscalar and isovector t channel exchanges; only the isovector contribution changes sign in going from pp to np elastic scattering. At high energies, to a good approximation

$$P_{1,3} \sim (\operatorname{Re} S \operatorname{Im} N)_{1,3}$$

where S and N stand for the s channel single- and zero-flip helicity amplitudes; $\operatorname{Im} N_1 \simeq \operatorname{Im} N_3$, but $\operatorname{Re} S_1$ and $\operatorname{Re} S_3$ differ by the amount of isovector exchange in them. If isovector exchange dominates the amplitude S , $P_1 \simeq -P_3$ and the above-mentioned bound $|P_1 - P_3| \leq 0.2$ obtained above would be almost saturated, because $P_1 \sim 0.1$. However, if the amplitude S is dominated by isoscalar exchanges, $P_1 \simeq P_3$ and this bound would not be saturated.

B. NUCLEON-ANTINUCLEON SCATTERING

Here again, one expects to get good bounds on elastic $\bar{p}n \rightarrow \bar{p}n$ scattering because of the smallness of the charge exchange ($\bar{p}p \rightarrow \bar{n}n$) scattering as compared to elastic ($\bar{p}p \rightarrow \bar{p}p$) scattering. Our isospin bounds are shown in Fig. 2 for 5.7 and 8 GeV/c, to be compared with future data. In fact, our bounds may be a bit too conservative because there is some evidence²¹⁾ that the charge-exchange data (which we have used as they stand) may be somewhat too highly normalized. The input $\bar{p}p \rightarrow \bar{p}p$ data are from Ref. 22) and the charge exchange data from Ref. 23).

*) The lack of a qualitative change in the experimental P_2 (the polarization in charge-exchange $pn \rightarrow np$ scattering) as one goes to higher energies supports this duality argument. Measurements 19), 20) of P_2 exist up to 5.6 GeV/c, while P_3 measurements 20) exist only at low energies.

C. KAON-NUCLEON SCATTERING

As in the previous subsections, since the high-energy cross-section for the charge-exchange process $K^-p \rightarrow \bar{K}^0 n$ is much smaller than that for the elastic process $K^-p \rightarrow K^-p$, one can get quite good isospin bounds on the cross-section for $K^-n \rightarrow K^-n$ scattering. The resulting bounds are shown in Fig. 3a at 7.1, 9.5 and 12.3 GeV/c, using input $K^-p \rightarrow K^-p$ data from Refs. 24), and the charge-exchange data from Ref. 25). Similarly, the charge exchange $K^+n \rightarrow K^0 p$ is much weaker than elastic scattering $K^+p \rightarrow K^+p$ and one expects restrictive bounds for $K^+n \rightarrow K^+n$ scattering. The bounds at 12 GeV/c shown in Fig. 3b were obtained by using input K^+p data from Ref. 26), and the charge exchange data from Ref. 27). Since the charge-exchange processes $K^+n \rightarrow K^0 p$ and $K^-p \rightarrow \bar{K}^0 n$ have equal (or nearly equal) cross-sections in the near-forward direction at 12 GeV/c, and since also the elastic $K^\pm p$ cross-sections are quite similar, the bounds at 12.3 and 12 GeV/c in Figs. 3a and 3b are very similar. There is, however, a qualitative difference in the extrapolation of the lower bounds to the forward direction where the known $K^\pm n$ total cross-sections¹⁵⁾ provide the lower bounds, via the optical theorem. Barring a drastic variation with angle, this extrapolation indicates the need of a large real part of the forward amplitude for the $K^+n \rightarrow K^+n$ case; for the $K^-n \rightarrow K^-n$ case, the extrapolation in t is larger and does not necessarily call for a real part of the forward amplitude. This is consistent with the expectations of an exchange-degenerate dual Regge model [with trajectory intercepts $\alpha_\rho(0) = \alpha_{A_2}(0) = \alpha_{\rho_1}(0) = \alpha_\omega(0) = 0.5$] which makes the contribution of the secondary trajectories to the forward $K^+n \rightarrow K^+n$ amplitude purely real and the forward $K^-n \rightarrow K^-n$ amplitude purely imaginary. It is interesting to be able to get such qualitative conclusions from just isospin invariance alone.

As an application of the bounds (8b) on the polarization P_3 when only P_1 , σ_1 and σ_2 are known, we show in Fig. 4 the bounds on polarization in $K^-n \rightarrow K^-n$ scattering at 10 GeV/c as a function of t . The input $K^-p \rightarrow K^-p$ data (P_1 and σ_1) are from Ref. 24) and the charge exchange $K^-p \rightarrow \bar{K}^0 n$ data (σ_2) are from Ref. 25). Because of the smallness of σ_2/σ_1 , the bounds of Fig. 4 are quite restrictive.

If charge symmetry holds, $K^+(K^-)n$ scattering is equal to $K^0(\bar{K}^0)p$ scattering, and because of their relation

$$2 T(K_L p \rightarrow K_S p) = T(K^0 p \rightarrow K^0 p) - T(\bar{K}_0 p \rightarrow \bar{K}_0 p)$$

to the regeneration amplitude, one can set bounds, using (4), on the regeneration cross-section in terms of the $K^\pm n$ ones. These bounds can be useful, in the absence of actual regeneration measurements, in the study ²⁸⁾ of possible Pomeranchuk theorem violation. The bounds on the regeneration cross-section deduced from our $K^\pm n$ bounds at 12 GeV/c are not very useful. Also, of course, if the bounds for $K^\pm n$ ($K^\pm n$) scattering, obtained by using regeneration and $K^\pm n$ ($K^\pm n$) data (when available) are combined with the independently valid bounds obtained by using $K^+ p \rightarrow K^+ p$ ($K^- p \rightarrow K^- p$) and $K^+ n \rightarrow K^0 p$ ($K^- p \rightarrow \bar{K}^0 n$) data, one may make the bounds on $K^\pm n$ ($K^\pm n$) scattering even more restrictive.

D. VECTOR-MESON PRODUCTION

As simple illustrations of the bounds for resonance production, we consider the vector-meson production processes $\pi N \rightarrow \rho N$ (Figs. 5, 6 and 7) and $KN \rightarrow K_{890}^* N$ (Figs. 8, 9 and 10). The bounds (9) for the density matrix elements ^{*}) (all in the Gottfried-Jackson frame) and (10) for the differential cross-sections for $\pi^- p \rightarrow \rho^0 n$ at 4 and 8 GeV/c are compared in Fig. 5 with the available data which are found to be generally quite consistent with isospin invariance; the points (Fig. 5f) where the upper bound is lower than the lower bound may be traced to the behaviour of the corresponding differential cross-sections. This apparent disagreement with charge-independence may well be due to our data interpolations; also, we have treated all errors in an uncorrelated way. The 4 GeV/c, $\pi^- p \rightarrow \rho^0 n$ data are from Ref. 29) and the $\pi^\pm p \rightarrow \rho^\pm p$ data from Ref. 30). For 8 GeV/c, the ρ^0 production data are from Ref. 31) and the $\rho^\pm$ production data from Ref. 32). The S wave background in the case of $\pi^- p \rightarrow \rho^0 n$ data was accounted for, as in Ref. 33).

The effect of rotation of the reference axes on the 15 GeV/c, $\pi^- p \rightarrow \rho^0 n$ data ³⁴⁾ is illustrated in Fig. 6 which shows the value of α , the angle of rotation from the s channel helicity frame to the DH2 frame.

^{*}) All the density matrix data (except those of Fig. 6) used in this paper were given in the Gottfried-Jackson frame.

It is remarkable to note that for all but the three smallest scattering angles shown, the quantity ρ_{00} in the DH2 frame becomes consistent with zero (within one standard deviation). This property of data $[\rho_{00}]_{\text{DH2}} \rightarrow 0$, typical of the majority of high energy K^* and ρ production data, implies a saturation of the positivity condition on the observed part of the density matrix. Such a saturation could follow in some simple models³³⁾. The vanishing of ρ_{00}^θ leads to the relation (11) and to the bounds (12b) for the density matrix. For the same reason, rotation around the normal to the production plane provides a simple visual check on the well-known positivity property⁶⁾ of the original density matrix, as is apparent from the figures below, especially Figs. 7 and 9.

The behaviour of ρ_{00}^θ of Eq. (13) is shown as a function of the angle of rotation θ , starting from the Gottfried-Jackson frame, for the 4 GeV/c, $\pi^- N \rightarrow \rho^- N$ data^{29),30)} at $t = -0.12 \text{ GeV}^2$ in Fig. 7a. Here again, ρ_{00} becomes consistent with zero in the DH2 frame; the errors have not been shown for clarity.

Isospin invariance and this vanishing of ρ_{00} in the DH2 frame (determined by the angle α) for one-charge mode (say, $\pi^- p \rightarrow \rho^0 n$) implies the equality, (11), of the combination $[\rho_{00}(d\sigma/dt)]$ for the other two modes ($\pi^\pm p \rightarrow \rho^\pm p$), ρ_{00} being a known combination of ρ_{00} , ρ_{11} , $\text{Re } \rho_{10}$ and ρ_{1-1} of the original frame. This prediction has been compared with relevant data (wherever available for all the modes) and found to be satisfied within the (unfortunately large) errors. For example, this equality due to a vanishing of ρ_{00} at $\alpha \simeq 113^\circ$ (starting with the Gottfried-Jackson frame) for the 8 GeV/c, $\pi^+ p \rightarrow \rho^+ p$ data³²⁾ at $t = -0.25 \text{ GeV}^2$ reads as

$$(0.04 \pm 0.04) \text{ mb/GeV}^2 = (0.025 \pm 0.04) \text{ mb/GeV}^2$$

To illustrate the effect of rotation on the relations (9), the 4 GeV/c, $\rho_{00}(\pi^- p \rightarrow \rho^0 n)$ bounds, at $t = -0.12 \text{ GeV}^2$, are compared with data²⁹⁾ in Fig. 7b as a function of the angle θ of rotation (starting from the Gottfried-Jackson frame). The input data for Fig. 7b are the same as for Fig. 5b wherein the $\theta = 0$ point is shown. As examples of the use of the bounds (9) in a rotated frame, we have evaluated bounds (14) on the quantity $(2\text{Re } \rho_{10} \pm \rho_{1-1})$; unfortunately, the data are such that the bounds

obtained were rather poor, even though the corresponding errors on the ρ_{00}^θ bounds are quite reasonable.

For $KN \rightarrow K_{890}^* N$, we illustrate the effect of the rotation of the reference axes in Figs. 8, 9 and 10. The angle α of rotation to the DH2 frame (starting from the Gottfried-Jackson frame) is shown in Fig. 8 for the 2.64 GeV/c, $K^- p \rightarrow K^{*-} p$ data³⁵⁾; here also, the quantity ρ_{00} in the DH2 frame does become zero (within one standard deviation) for a number of t values. The prediction (11) for the equality of the remaining two modes when ρ_{00} for the first mode vanishes is shown as a function of t in Fig. 9 for the 4.5 GeV/c, $K^- p \rightarrow K^{*-} p$ and $K^- n \rightarrow K^{*-} n$ data³⁶⁾ and for the 4.57 GeV/c, $K^- p \rightarrow K^{*0} n$ data³⁷⁾. In fact, as for a lot of the $\pi N \rightarrow \rho N$ data (see Fig. 7a), the behaviour of ρ_{00}^θ as a function of the angle θ of rotation (starting from the Gottfried-Jackson frame) is qualitatively the same for all the three charge modes: the angle α of rotation to the DH2 frame is the same (within errors), and $\rho_{00} \rightarrow 0$ for all these modes. For clarity, ρ_{00} is shown in Fig. 9 for the three charge modes at only some "average" value (104°) of this angle of rotation; the equality (11) is fulfilled, both sides being consistent with zero. Figure 10 shows the isospin bounds (9) on $\rho_{00}^\theta d\sigma/dt(K^- p \rightarrow \bar{K}^{*0} n)$ at 4.5 GeV/c, $t = -0.15 \text{ GeV}^2$ as a function of the angle θ of rotation, starting from the Gottfried-Jackson frame. The "data" for comparison come from the density matrix elements of the 4.57 GeV/c, $K^- p \rightarrow \bar{K}^{*0} n$ experiment³⁷⁾ and the $t = -0.15 \text{ GeV}^2$ differential cross-section interpolated between the 4.1 and 5.5 GeV/c experiments³⁸⁾. The input 4.5 GeV/c, $K^- p, n \rightarrow K^{*-} p, n$ data are from Ref. 36). The corresponding ρ_{11} bounds are not restrictive.

4. CONCLUDING REMARKS

Strong interactions are believed to be isospin invariant to a very good approximation. Isospin invariant models would fail to fit data on the different charge modes if the data do not fulfil the simple triangular inequalities. On the other hand, isospin invariant models with certain specific dynamical assumptions may misfit a particular piece of data rather strongly even though they fit the remaining data reasonably well, and all the relevant data satisfy charge independence requirements quite well; for an example, see Ref. 5). Our $np \rightarrow np$ and $K^\pm n \rightarrow K^\pm n$ bounds seem to point out another use of the isospin inequalities: that charge independence alone can teach one something about dynamics in certain situations.

For resonance production, rotation around the normal to the production plane can provide a simple way of checking the well-known positivity requirements ⁶⁾ on the density matrix of the produced resonance; this is illustrated for high energy $\pi N \rightarrow \rho N$ and $KN \rightarrow K_{890}^* N$ data for a majority of which the density matrix element ρ_{00} in one of the Donohue-Högaasen frames becomes consistent with zero. This rotation allows ones to calculate isospin bounds on certain density matrix elements even in situations where only a partial knowledge of the density matrix for the other two modes is available. Also, isospin invariance predicts relations [like (11) and (12)] between the density matrices in the rotated frame for the other two modes, if the density matrix data on the first mode satisfy certain conditions. The apparent inconsistencies with positivity and with isospin invariance that came up at some points cannot be taken literally because of our inadequate treatment of errors, especially for the density matrix.

Since data on all the three charge modes are available in πN elastic scattering, it would be interesting to get dynamical information from a study of the relative positioning (especially its energy dependence) of the data within the isospin bounds, similar to that in Fig. 1. We intend to study this, along with a study of the bounds on A_1 and A_2 production (also the density matrix bounds).

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R E F E R E N C E S

- 1) B.A. Jacobsohn, Phys.Rev. 89, 881 (1953);
D. Feldman, Phys.Rev. 89, 1159 (1953).
- 2) L. Michel, Nuovo Cimento 22, 203 (1961); Brandeis University Lectures
[Axiomatic Field Theory, Gordon and Breach Science Publishers, p. 247
(1965)];
R.J.N. Phillips, Nuovo Cimento 26, 103 (1962).
- 3) O. Kamei and S. Sasaki, Nuovo Cimento 69A, 535 (1969).
- 4) S.M. Roy, Nuclear Phys. B34, 528 (1971);
M. Korkea-aho and N. Törnqvist, Nuclear Phys. B33, 239 (1971).
- 5) G.V. Dass, J. Froyland, F. Halzen, A. Martin, C. Michael and S.M. Roy,
Phys.Letters 36B, 339 (1971).
- 6) R.H. Dalitz, Nuclear Phys. 87, 89 (1966);
P. Minneart, Phys.Rev.Letters 16, 672 (1966).
- 7) J.T. Donohue and H. Högaasen, Phys.Letters 25B, 554 (1967).
- 8) J.T. Donohue, Phys.Rev. D1, 1979 (1970).
- 9) U. Amaldi et al., CERN preprint "Study of Elastic Proton-Neutron
Scattering by Means of Proton-Deuteron Scattering at 24 GeV/c".
- 10) B.G. Gibbard et al., Nuclear Phys. B30, 77 (1971).
- 11) J. Engler et al., Phys.Letters 29B, 321 (1969);
Private communication from Dr. K. Runge.
- 12) J.V. Allaby et al., Phys.Letters 34B, 431 (1971).
- 13) J. Engler et al., CERN preprint (Feb. 1971).
- 14) E.L. Miller et al., Phys.Rev.Letters 26, 984 (1971);
J. Engler et al., Phys.Letters 34B, 528 (1971).
- 15) W. Galbraith et al., Phys.Rev. 138, B913 (1965);
M. Kreisler et al., Phys.Rev.Letters 20, 468 (1968).
- 16) G. Belletini et al., Phys.Letters 19, 341 (1965).
- 17) M. Holder et al., Phys.Letters 36B, 400 (1971);
R.A. Carrigan Jr. et al., Phys.Rev.Letters 24, 168 (1970).

- 18) R. Klanner, private communication.
- 19) P.R. Robrish et al., Phys.Letters 31B, 617 (1970).
- 20) D. Cheng et al., Phys.Rev. 163, 1470 (1967).
- 21) J. Froyland and G.A. Winbow, to be published in Nuclear Phys.
- 22) K. Böckmann et al., Nuovo Cimento 42, 954 (1966);
D. Birnbaum et al., Phys.Rev.Letters 23, 663 (1969).
- 23) P. Astbury et al., Phys.Letters 22, 537 (1966) and 23, 166 (1966).
- 24) K.J. Foley et al., Phys.Rev.Letters 11, 503 (1963) and 15, 45 (1965);
M. Aderholz et al., Phys.Letters 24B, 434 (1967);
D.P. Owen et al., Phys.Rev. 181, 1794 (1969);
P. Laurelli, CERN Report NP 71-3 "Polarization Measurements in Elastic
Scattering of $\pi^\pm$, $K^\pm$, p and $\bar{p}$ on Polarized Protons in the
Range of Energy between 6 and 17.5 GeV/c".
- 25) P. Astbury et al., Phys.Letters 23, 396 (1966).
- 26) K.J. Foley et al., Phys.Rev.Letters 11, 503 (1963).
- 27) A. Firestone et al., Phys.Rev.Letters 25, 958 (1970).
- 28) J. Finkelstein and S.M. Roy, Phys.Letters 34B, 322 (1971).
- 29) I. Derado et al., Phys.Rev.Letters 14, 872 (1965);
R.L. Eisner et al., Phys.Rev. 164, 1699 (1967).
- 30) M. Aderholz et al., Phys.Rev. 138, B897 (1965);
M. Aderholz et al., Nuovo Cimento 34, 495 (1964);
I. Bondar et al., Nuovo Cimento 31, 729 (1964);
R.L. Eisner et al., Phys.Rev. 164, 1699 (1967).
- 31) J.A. Poirier et al., Phys.Rev. 163, 1462 (1967).
- 32) I. Derado et al., Phys.Letters 24B, 112 (1967);
M. Aderholz et al., Nuclear Phys. B24, 509 (1970);
M. Aderholz et al., Nuclear Phys. B8, 45 (1968).
- 33) G.V. Dass and C.D. Froggatt, Nuclear Phys. B8, 661 (1968).
- 34) F. Bulos et al., Phys.Rev.Letters 26, 1453 (1971).
- 35) J.H. Friedman and R.R. Ross, Phys.Rev.Letters 16, 485 (1966).
- 36) D.D. Carmony et al., Nuclear Phys. B12, 9 (1969).
- 37) Y.W. Kang, Phys.Rev. 176, 1587 (1968).
- 38) F. Schweingruber et al., Phys.Rev. 166, 1317 (1968).

FIGURE CAPTIONS

Figure 1 : Isospin bounds on $d\sigma/dt$ [mb/GeV²] for $np \rightarrow np$, shown as a function of t (GeV²), are compared with data. The dotted lines joining the central values of the upper (and similarly, lower) bounds are meant only as a guide to the eye.

Figure 2 : Isospin bounds on $d\sigma/dt$ (mb/GeV²) for $\bar{p}n \rightarrow \bar{p}n$ are shown as a function of t (GeV²). The dotted lines have the same significance as in Fig. 1.

Figure 3a : Isospin bounds on $d\sigma/dt$ (mb/GeV²) for $K^-n \rightarrow K^-n$ are shown as a function of t (GeV²). The dotted lines have the same significance as in Fig. 1.

3b : Isospin bounds on $d\sigma/dt$ (mb/GeV²) for $K^+n \rightarrow K^+n$ are shown as a function of t (GeV²). The dotted lines have the same significance as in Fig. 1.

Figure 4 : Isospin bounds (8b) for polarization P in $K^-n \rightarrow K^-n$ scattering at 10 GeV/c shown as a function of t (GeV²); the dotted lines have the same significance as in Fig. 1.

Figure 5 : Isospin bounds of relation (10) on $d\sigma/dt$ (mb/GeV²) and of relation (9) on ρ_{00} , $\text{Re } \rho_{10}$ and ρ_{1-1} for $\pi^-p \rightarrow \rho^0n$ at 4 GeV/c, shown as a function of t (GeV²) in Figs. a, b, c and d respectively, are compared with data. Figs. e, f, g and h show the corresponding bounds at 8 GeV/c. The dotted lines have the same significance as for Fig. 1.

Figure 6 : The angle α of rotation from the s channel helicity frame to the Donohue-Högaasen frame DH2, for the 15 GeV/c $\pi^-p \rightarrow \rho^0n$ data, shown as a function of t (GeV²). For all but the three lowest $|t|$ values shown by squares, the value of ρ_{00} in the DH2 frame becomes consistent with zero; the dotted line just joins this type of points.

Figure 7a : The density matrix element ρ_{00}^{θ} is shown as a function of the angle θ of rotation (starting from the Gottfried-Jackson frame) for the 4 GeV/c data at $t = -0.12 \text{ GeV}^2$ for the three charge modes $\pi^{\pm} p \rightarrow \rho^{\pm} p$ and $\pi^{\pm} p \rightarrow \rho^0 n$.

7b : Isospin bounds (9) on ρ_{00}^{θ} shown as a function of the rotation angle θ (starting from the Gottfried-Jackson frame) for the mode $\pi^{\pm} p \rightarrow \rho^0 n$ at 4 GeV/c, $t = -0.12 \text{ GeV}^2$ are compared with data. This Figure shows the θ dependence of the $t = -0.12 \text{ GeV}^2$ point of Fig. 5b. The shaded area shows the upper (lower) part of the error on the upper (lower) bound. Some typical error-bars on the data are also shown.

Figure 8 : The angle α of rotation from the Gottfried-Jackson frame to the Donohue-Högaasen frame DH2, for the 2.64 GeV/c, $K^- p \rightarrow K^{*-} p$ data is shown as a function of t (GeV^2). For the points shown by circles (squares), the value of ρ_{00} in the DH2 frame becomes (does not become) consistent with zero.

Figure 9 : The value of ρ_{00}^{α} for one angle ($\alpha = 104^\circ$) of rotation, starting from the Gottfried-Jackson frame. The value 104° is roughly the value where ρ_{00} becomes consistent with zero for the 4.5 GeV/c, $K^- p \rightarrow K^{*-} p$ (circles) and $K^- n \rightarrow K^{*-} n$ (triangles) data. Isospin invariance requires ρ_{00}^{α} for the third mode $K^- p \rightarrow K^{*0} n$ also to vanish; this seems to be borne out by the 4.57 GeV/c data (squares).

Figure 10 : Isospin bounds (9) on $\rho_{00} d\sigma/dt(\text{mb}/\text{GeV}^2)$ for the $K^- p \rightarrow \bar{K}^{*0} n$ mode at 4.5 GeV/c, $t = -0.15 \text{ GeV}^2$ are shown as a function of the rotation angle θ (starting from the Gottfried-Jackson frame). The line with points (.) and typical error-bars shows the data. The shaded area shows the errors on the bounds, as for Fig. 7b.

FIG.1

FIG. 3a

FIG. 3 b

FIG.4

FIG. 5

FIG. 6

FIG.7

FIG. 8

FIG.9

FIG.10