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AMPLITUDE ANALYSIS FOR HYPERCHARGE EXCHANGE REACTIONS *)

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A B S T R A C T

The s channel helicity non-flip amplitudes for the line reversed reactions $\pi N \rightarrow K[\Sigma, \Lambda]$ and $\bar{K}N \rightarrow \pi[\Sigma, \Lambda]$ are determined directly from cross-section and polarization data at 4 GeV/c. Rigorous bounds are obtained on the magnitudes of the flip amplitudes, whose phases are assumed to be given by an exchange degenerate K^*-K^{**} Regge trajectory. Our solution for the non-flip amplitude is unique and shows the following characteristics: i) $\text{Im } K^*(\Delta\lambda=0)$ has a "cross-over" zero at $t \approx -0.15$ in both Σ and Λ reactions, ii) $\text{Im } K^{**}(\Delta\lambda=0)$ has an approximate double zero near $t \approx -0.6$ in Σ reactions and a positive minimum there in Λ reactions, iii) $\text{Re } K^*(\Delta\lambda=0)$ and $\text{Re } K^{**}(\Delta\lambda=0)$ are less peripheral in character than their imaginary counterpart and have similar behaviour at $\alpha=0$ to simple Regge poles.

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Hypercharge exchange reactions are of especial importance in the study of absorptive corrections to Regge poles because of the simplicity of duality diagram phase predictions ¹⁾ for these amplitudes. For the processes

$$\pi N \rightarrow K [\Sigma, \Lambda] \quad (1a)$$

the Regge exchange amplitudes ^{*)}

$$A_+(s, t) = Q + K^* = -\gamma(t) e^{-i\pi\alpha(t)} s^{\alpha(t)} \quad (1b)$$

have a rotating phase determined by the exchange-degenerate trajectory $\alpha(t)$. The amplitudes for the processes

$$\bar{K} N \rightarrow \pi [\Sigma, \Lambda] \quad (2a)$$

are purely real

$$A_-(s, t) = Q - K^* = -\gamma(t) s^{\alpha(t)} \quad (2b)$$

In a dual pole model the differential cross-sections $\sigma (\equiv d\sigma/dt)$ for the line reversed reactions (1a) and (2a) are equal

$$\sigma_+ = \sigma_- \quad (3)$$

and the polarizations are zero

$$P_+ = P_- = 0 \quad (4)$$

Substantial absorptive corrections to the amplitudes of Eqs. (1b) and (2b) are clearly necessary to account for the experimental data which exhibit

^{*)} For convenience of notation we use the symbol Q to denote the K^{**} exchange amplitude.

large polarizations of opposite sign for the line reversed pairs of reactions (1a) and (2a) and a substantial splitting of the forward differential cross-sections with

$$\sigma_+ < \sigma_- \quad (5)$$

Although conventional absorptive corrections to the non-flip amplitudes from a Pomeranchuk trajectory with slope $\alpha'_P \sim 0.5$ can reproduce the features of the experimental polarizations, they lead to $\sigma_- < \sigma_+$ in disagreement with data [Eq. (5)]²⁾.

In order to determine the nature of absorptive corrections to the pole amplitudes implied by the data on hypercharge exchange reactions, model independent deductions of the amplitudes from experiment are needed, such as those which have been carried out for $\pi^- p \rightarrow \pi^0 n$ ^{3),4)}. A $\pi^- p \rightarrow \pi^0 n$ amplitude analysis supplies information about absorption of vector exchange (ρ). The hypercharge exchange processes are probably the best source of information on absorbed tensor exchange (K^{**}), as well as for additional information about absorptive corrections to vector exchange (K^*). However, since spin-correlation measurements have not yet been made for reactions (1a, 2a), some theoretical input is required for an amplitude analysis. In this Letter we show that with plausible assumptions for the helicity-flip amplitudes, the non-flip amplitudes are determined by the existing cross-section and polarization data.

For our analysis we assume that the s channel helicity flip amplitudes are essentially uncorrected by initial and final state absorption. This assumption is routinely accepted as a working hypothesis in most current phenomenological analyses. The results from $\pi^- p \rightarrow \pi^0 n$ amplitude analysis^{3),4)} and the outstanding success of SU(3) sum rules for the flip dominated charge exchange processes⁵⁾ provide supporting evidence for the uncorrected Regge pole approximation for flip amplitudes.

According to the preceding argument, the exchange degenerate flip amplitudes of reactions (1), (2) can be parameterized at any given energy by

$$A_+(\Delta\lambda=1) = -\rho_f(s) e^{-i\pi\alpha(s)} \quad (6)$$

$$A_{-}(\Delta\lambda=1) = -\rho_f(t) \quad (7)$$

where $\Delta\lambda$ is the net s channel helicity flip. To facilitate comparison with the exchange degenerate Regge forms [cf. Eqs. (6) and (7)] we write the s channel non-flip amplitudes as

$$A_{+}(\Delta\lambda=0) = -\rho_{+} e^{i[\phi_{+}-\pi\alpha]} \quad (8)$$

$$A_{-}(\Delta\lambda=0) = -\rho_{-} e^{i\phi_{-}} \quad (9)$$

where the magnitudes, $\rho_{\pm}(t)$, and the phases, $\phi_{\pm}(t)$, are to be determined directly from the data. In terms of Eqs. (6) through (9) the experimental observables ^{*)} can be expressed as

$$\sigma_{\pm} = |\rho_{\pm}|^2 + |\rho_f|^2 \quad (10)$$

$$P_{\pm}\sigma_{\pm} = 2\rho_f\rho_{\pm} \sin\phi_{\pm} \quad (11)$$

$$T_{\pm}\sigma_{\pm} = 2\rho_f\rho_{\pm} \cos\phi_{\pm} \quad (12)$$

$$S_{\pm}\sigma_{\pm} = |\rho_{\pm}|^2 - |\rho_f|^2 \quad (13)$$

Given the modulus of the flip amplitude $|\rho_f(t)|$ as input, the non-flip quantities $\rho_{\pm}$ and $\phi_{\pm}$ can be calculated directly from $\sigma_{\pm}$ and $P_{\pm}$ data.

Fortunately, the data impose fairly restrictive bounds on the modulus of the flip amplitude $|\rho_f|$. From Eq. (11) we have

*) The helicity correlation parameters T and S are defined in terms of the s channel helicity amplitudes $A(\Delta\lambda)$ as

$$T\sigma = 2 \operatorname{Re} [A(\Delta\lambda=0)A(\Delta\lambda=1)^*] \quad \text{and} \quad S\sigma = |A(\Delta\lambda=0)|^2 - |A(\Delta\lambda=1)|^2$$

$$-2|p_f|p_{\pm} \leq p_{\pm} \sigma_{\pm} \leq 2|p_f|p_{\pm} \quad (14)$$

When combined with Eq. (10), Eq. (14) leads to the bounds

$$\frac{1}{2} \sigma_{\pm} (1 - \sqrt{1 - p_{\pm}^2}) \leq |p_f|^2 \leq \frac{1}{2} \sigma_{\pm} (1 + \sqrt{1 - p_{\pm}^2}) \quad (15)$$

The calculations of these bounds from the data ^{*}) at 4 GeV/c are shown by the dashed curves in Fig. 1. The results are particularly restrictive for the Λ production amplitude.

Guided by the success for SU(3) in relating the flip amplitudes for charge exchange reactions, we expect the $\pi^- p \rightarrow K^0 \Lambda$ and $\pi^+ p \rightarrow K^+ \Sigma^+$ flip amplitudes to be in the ratio ⁶⁾

$$\frac{(p_f)_{\Sigma}}{(p_f)_{\Lambda}} = \sqrt{6} \left(\frac{2F-1}{2F+1} \right) \quad (16)$$

where F is the usual SU(3) parameter ($0.23 \lesssim F \lesssim 0.32$). Thus, given the bounds on $(p_f)_{\Lambda}$ in Fig. 1a, $(p_f)_{\Sigma}$ is similarly constrained up to the scale factor given in Eq. (16).

For our representation of the helicity flip contribution $(p_f)_{\Lambda}$ we take the logarithmic mean of the Λ bounds in Fig. 1 for $|t| \geq 0.4$ and match to this mean the form $|p_f|^2 = |t| 700 e^{7t} \mu b / (\text{GeV}/c)^2$ for the region $0.025 \leq |t| \leq 0.4$. We then calculate $(p_f)_{\Sigma}$ from $(p_f)_{\Lambda}$ using $F=0.25$ in Eq. (16). Our convention is to take p_f positive in Λ reactions which means p_f is negative for Σ reactions. This representation is shown by the solid curves in Fig. 1.

^{*}) For our analyses we have used interpolations through the available data in the region of 4 GeV/c shown in Ref. 6).

With these deduced forms for ρ_f , we solve for the non-flip amplitudes using Eqs. (10) and (11). In general there are four sets of solutions corresponding to phase ambiguities. Since absorptive corrections at $t=0$ do not lead to over-all sign changes of the unabsorbed amplitudes, we select the solution with non-flip amplitude phases such that

$$\frac{\text{Re } K^*}{\text{Im } K^*} > 0, \quad \frac{\text{Re } Q}{\text{Im } Q} < 0 \quad (17)$$

at $t=0$ in correspondence with the signs for the Regge pole amplitude. The Λ and Σ solutions obtained for $\rho_{\pm}$ and $\phi_{\pm}$ are shown in Fig. 2. The phases $\phi_{\pm}^{\Sigma}$ are qualitatively similar to $\phi_{\pm}^{\Lambda}$, but the moduli are not. The cross-over of ρ_{-}^{Σ} and ρ_{+}^{Σ} at $t \approx -0.5$ reflects a change in sign of the differential cross-section difference ($\sigma_{-}^{\Sigma} - \sigma_{+}^{\Sigma}$) at that t value.

The physical interpretation of the results is more transparent in terms of the K^* and Q exchange contributions

$$\begin{aligned} K^*(\Delta\lambda=0) &= -\frac{1}{2} [\rho_{+} e^{i(\phi_{+}-\pi\alpha)} - \rho_{-} e^{i\phi_{-}}] \\ Q(\Delta\lambda=0) &= -\frac{1}{2} [\rho_{+} e^{i(\phi_{+}-\pi\alpha)} + \rho_{-} e^{i\phi_{-}}] \end{aligned} \quad (18)$$

These combinations are plotted in Fig. 3, based on the trajectory $\alpha(t) = 0.4 + t$. Our results are not qualitatively dependent on the particular choice of the flip contributions. We tried other choices of the flip amplitude closer to the allowed bounds and found that the non-flip amplitudes were of the same structure. We note the following interesting characteristics of the solution for the non-flip contributions.

- i) $\text{Im } K^*$ has a "cross-over" zero at $t = -0.15$ in both Σ and Λ reactions, as is the case for $\text{Im } \rho(\Delta\lambda=0)$ in πN scattering^{3),4)}.
- ii) $\text{Im } Q$ has an approximate double zero for Σ reactions near $t = -0.6$ and a positive minimum at this t value for Λ reactions. The structure of $(\text{Im } Q)_{\Sigma}$ is similar to the tensor exchange contribution

- $\text{Im } P'(\Delta\lambda=0)$ in πN scattering in which the double zero of $\pi^\pm_p$ polarization at $t \approx -0.6$ requires a double zero of $\text{Im } P'$ ⁷⁾.
- iii) $(\text{Re } K^*)_\Sigma$ has a double zero near $t = -0.4$, corresponding to $\alpha = 0$, in perfect correspondence to Regge pole behaviour with $\text{Re } K^* \sim \alpha \tan \frac{\pi\alpha}{2}$. This result agrees with the $\text{Re } P(\Delta\lambda=0)$ results from πN FESR analysis ^{4),8)}. $(\text{Re } K^*)_\Lambda$ has a slightly positive minimum near $t = -0.4$.
- iv) $(\text{Re } Q)_{\Lambda,\Sigma}$ have no zero in the vicinity of $\alpha = 0$, again in correspondence with Regge pole behaviour for which $\text{Re } Q \sim \alpha \cot \frac{\pi\alpha}{2}$. A zero in $\text{Re } Q$ is observed at larger $|t| \approx 0.8$.
- v) Near $t = 0$ we find $\text{Re } K^* \approx -\text{Re } Q$ which is in accord with the expectations from exchange degenerate poles. The exchange degeneracy prediction of equal imaginary parts is violated by about a factor of 3 near $t = 0$.

The results for the phases $\phi_\pm$ in our analysis are relatively insensitive to the exact choice of the flip amplitude, within the bounds of Fig. 1. Consequently, we can predict the helicity correlation parameters $T_\pm$ of Eq. (12) with reasonable confidence. These predictions are shown in Fig. 4 along with the interpolation of the polarization data $P_\pm$. Our predictions for T_- are quite different from those resulting ^{2),9)} from conventional absorptive model fits to hypercharge exchange data.

The reaction $K^-p \rightarrow \eta_8 \Lambda$ also involves the K^*_Λ and Q_Λ exchanges which, if we assume $SU(3)$, are in the combination $(Q_\Lambda + 3K^*_\Lambda)$ ¹⁰⁾. From the non-flip amplitudes of Fig. 3 we observe an almost complete zero of both the real and imaginary parts of this combination around $t \approx -0.3$, in agreement with the striking dip observed ¹¹⁾ in the $K^-p \rightarrow \eta_8 \Lambda$ cross-section at 4 GeV/c.

In summary, we have shown that the non-flip K^* and K^{**} exchange amplitudes can be deduced directly from cross-section and polarization data on hypercharge exchange reactions with the very plausible assumption that the flip amplitude is given by exchange degenerate Regge poles. Our results for the $K^*(\Delta\lambda=0)$ amplitude are qualitatively similar to the $P(\Delta\lambda=0)$ amplitude obtained from amplitude and FESR analyses of πN scattering. $\text{Im } K^*(\Delta\lambda=0)$ exhibits a $t = -0.15$ cross-over. $\text{Re } K^*(\Delta\lambda=0)$ has a double zero at $\alpha = 0$, similar to a pure Regge pole. For tensor exchange we have obtained evidence that the

behaviour of $\text{Im } Q$ is similar to $\text{Im } P'$, indicating an approximate double zero in the vicinity of $t \approx -0.6$. For $\text{Re } Q$ we find no zeros near $\alpha = 0$, again in agreement with a pure Regge pole amplitude.

Our results tie in nicely with the over-all empirical picture that is emerging on the nature of absorption of s channel amplitudes:

- 1) helicity flip amplitudes are pure Regge, with very little absorption,
- 2) imaginary parts of helicity non-flip amplitudes are strongly absorbed,
- 3) real parts of helicity non-flip amplitudes show the zero structure of pure Regge poles, indicating little absorption.

The failures of the conventional absorption prescription are associated with over-absorption of the real parts of the non-flip amplitudes. We note that the application ¹²⁾ to hypercharge exchange reactions of the effective empirical absorptive cuts needed in $\pi^- p \rightarrow \pi^0 n$, produces an $\text{Im } Q$ similar to our results, but gives a $\text{Re } Q$ with a zero at $t \approx -0.2$ in conflict with our deductions.

Insufficient data on hypercharge exchange processes exist above 4 GeV/c to carry out a determination of the non-flip amplitudes as we have done here. It will be very interesting to have more data at higher energies, particularly needed for the Δ reactions, to carry out such an analysis.

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FIGURE CAPTIONS

Figure 1 Bounds on $|A(\Delta\lambda=1)|^2$ in hypercharge exchange reactions. Expressions for the bounds in terms of the experimental observables are given in Eq. (15).

Figure 2 Phase representation of the $A(\Delta\lambda=0)$ amplitudes from the data on hypercharge exchange reactions at 4 GeV/c, [cf., Eqs. (8) and (9) of text].

Figure 3 K^* and K^{**} s channel helicity non-flip amplitudes from amplitude analysis of data on hypercharge exchange reactions at 4 GeV/c.

Figure 4 Polarization data interpolations at 4 GeV/c and predictions of the helicity correlation parameter T from our amplitude analysis. The definition of T is $T d\sigma/dt = 2\text{Re} [A(\Delta\lambda=0)A(\Delta\lambda=1)^*]$.

BOUNDS ON |FLIP AMPLITUDE|²

FIG. 1

FIG. 2

NONFLIP AMPLITUDES AT 4 GeV/c

FIG. 3

INTERPOLATIONS OF POLARIZATION DATA

PREDICTIONS OF HELICITY CORRELATIONS

FIG. 4