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SELECTION RULES AND RATIOS FOR AXIAL COUPLINGS

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ABSTRACT

We study the predictions for the axial couplings following from the use of the mixing operator $U(Z)$, previously introduced to tilt the axial charges of $SU(6)$ in the physical ones.

The quantum number $(-1)^{L+L_3}$, where L and L_3 are the $O(3)$ angular momentum and its third component, is shown to be conserved.

From the properties of Z further predictions can be achieved as the $(D/F) = (3/2)$ for the $\frac{1}{2}^+$ baryon octet in general agreement with experiment.

1. INTRODUCTION

The presence in nature of $SU(6) \otimes O(3)$ hadron supermultiplets ¹⁾ has been connected to the $SU(3) \otimes SU(3)$ algebra ²⁾ described by the vector and axial charges. In fact, the identification of these charges, in the $P_z = \infty$ frame of reference, with the $SU(6)$ generators

$$Q_i = A(\lambda_i) \quad (1a)$$

$$Q_{5i} = A(\sigma_z \lambda_i) \quad (1b)$$

gives the right commutation rules, since

$$\begin{aligned} [A(\lambda_i), A(\lambda_j)] &= i f_{ijk} A(\lambda_k) \\ [A(\lambda_i), A(\sigma_z \lambda_j)] &= i f_{ijk} A(\sigma_z \lambda_k) \\ [A(\sigma_z \lambda_i), A(\sigma_z \lambda_j)] &= i f_{ijk} A(\lambda_k) \end{aligned} \quad (2)$$

Moreover, the axial charge defined in (1b) transforms like 1^+ under $\tilde{J}^P$ ^{*)}, as it is strictly required for the diagonal matrix elements; in general, such a behaviour is consistent with the negative normality of the axial charge.

The couplings computed by Eq. (1b) are larger than the experimental ones inside the $SU(6) \otimes O(3)$ supermultiplets and vanish elsewhere; this is in qualitative agreement with the fact

*) $\tilde{J}$ and $\tilde{P}$ are the angular momentum and parity defined in the " $P_z = \infty$ " frame ³⁾; in the following the $\sim$ will be omitted for simplicity.

that the largest contributions in the Adler-Weisberger ⁴⁾ sum rule come from the nucleon and the $N^*(1236)$ resonance. However, since an important role is played by higher states, it is reasonable to conceive Eq. (1b) as a zero order approximation, while the physical axial charge is obtained through a suitable mixing of the different $SU(6) \otimes O(3)$ multiplets.

In this direction, some attempts ⁵⁾ have been made within a finite set of states with the reasonable hope to get right predictions at least for the matrix elements of the $L = 0$ multiplet. To obey the $\Delta J = 1$ rule for the diagonal matrix elements the mixing should concern only states with equal angular momentum (not necessarily same parity) and should be independent of helicity; this leads to vanishing couplings between states with J different by more than one unit, for instance, the nucleon and the $\frac{5}{2}^{-(+)}$. To overcome this difficulty people looked ⁶⁾ for a suitable mixing procedure concerning an infinite set of states, namely baryons, classified, according to experiment, in the 56 even L and 70 odd L representations.

Two conditions were imposed on the mixing at first order:

- i) it should involve only multiplets contiguous in L to recover the fast convergence of A-W relations;
- ii) only s and d waves should arise as a threshold condition.

The last property was achieved with a mixing operator transforming under the angular momentum as the z component of a vector ⁷⁾. Following these ideas one computed ⁸⁾ the axial couplings concerning the 15 $L = 0$ and $L = 1$ $SU(4) \otimes O(3)$ supermultiplets, where the lowest mesons of both parities are classified: one gets predictions in rather good agreement with experiment. However, a large arbitrariness is present in the mixing procedure at higher values of L for mesons as well as for baryons.

Such an ambiguity is resolved assuming that the mixing operator transforms as a 35 under $SU(6)$ and as a vector under $O(3)$ ⁹⁾; this is consistent with having matrix elements between

56 and 70 for baryons and two 35 for mesons, both of contiguous orbital momentum L . Since in the 35 the only $SU(3)$ singlet has spin one, the mixing operator should transform as a vector $\vec{W}$ under the $SU(6)$ spin; if we call $\vec{M}$ the vector under $O(3)$ the only object with positive normality one can build is just the vector product

$$Z = (\vec{W} \wedge \vec{M})_z \quad (3)$$

where the z component is chosen to obey $\Delta J_z = 0$.

The 70 is contained only once in the product 56 $\otimes$ 35, while charge conjugation selects the symmetric 35 $\otimes$ 35 $\otimes$ 35 coupling for mesons; so all the matrix elements of Z are given in terms of one parameter for each pair of contiguous multiplets.

Through Eq. (3) one is able to connect different parameters introduced in the problems previously studied: for the baryons of 56 $L = 0$ and 70 $L = 1$ the predicted values for the ratios of the mixing angles are near to the fit values ⁶⁾, while for the mesons of 35 $L = 0$ and $L = 1$ just the ratio of the two parameters ⁸⁾ involved is recovered. In order to relate the mixing between different pairs of multiplets we need the reduced matrix elements of M . Indeed, these have been studied for mesons starting from the Weinberg equation ¹⁰⁾

$$[Q_{5a}, [Q_{5b}, m^2]] \propto S_{ab} \quad (4)$$

In the following we are showing the exact relations for the axial couplings, which hold irrespectively of the particular form of the operator $\vec{M}$, once the charge is given by tilting the right-hand side of (1b) with an arbitrary unitary function $U(Z)$ of the

of the operator defined in (3)

$$Q_{5i} = U(Z) A(\sigma_z \lambda_i) U^+(Z) \quad (5)$$

First, we derive a selection rule from (3) and (5), which applies to mesons and baryons; then we shall derive further consequences of these equations.

2. A SELECTION RULE FOR THE AXIAL CHARGE

If one writes Eq. (3) in the form

$$Z = i (W_+ M_- - W_- M_+) \quad (3')$$

it is easy to see that $U(Z)$ has the property

$$\Delta(L + L_3) = \text{even} \quad (6)$$

since both L and L_3 are changed by one unit by $M_{\pm}$; moreover, $A(\sigma_z \lambda_i)$ does not act on $O(3)$, so, from Eqs. (5) and (6), one deduces

$$\langle 2', L', L'_3, S', S'_3 | Q_{5i} | 2, L, L_3, S, S_3 \rangle = 0 \quad (7)$$

whenever $L' + L'_3 - L - L_3 = \text{odd}$. It means in the physical base that

$$\sum_{J'J} C_{L'_3 h - L'_3 h}^{L' S' J'} C_{L_3 h - L_3 h}^{L S J} \langle \alpha', L', S', J', h | Q_{5i} | \alpha, L, S, J, h \rangle = 0 \quad (8)$$

where α' and α label the internal quantum numbers. When $L = 0$ or $S = S' = 0$ Eq. (8)

$$\sum_{L'_3 h - L'_3 h} C_{L'_3 h - L'_3 h}^{L' S' J'} \langle \alpha', L', S', J', h | Q_{5i} | \alpha, L=0, S, J=S, h \rangle = 0 \quad (8a)$$

if $L' + L'_3 = \text{odd}$, or

$$\langle \alpha', L', S'=0, J=L', h | Q_{5i} | \alpha, L, S=0, J=L, h \rangle = 0 \quad (8b)$$

if $L' - L = \text{odd}$. We write down the content of Eqs. (8a) and (8b) more explicitly where a comparison with experiment is possible. In the boson case we obtain from (8a) the following results

i) $\alpha_{L=0} = \pi$

$$\frac{G_{\pi f_{L+1}}^2}{G_{\pi \sigma_{L-1}}^2} \stackrel{L \text{ odd}}{=} \frac{L}{L+1} \stackrel{L \text{ even}}{=} \frac{G_{\pi \rho_{L-1}}^2}{G_{\pi \rho_{L+1}}^2} \quad (9)$$

ii) $\alpha_{L=0} = \rho \ (h = \pm 1)$

$$G_{\rho \pi_L}(h = \pm 1) = 0 \quad L \text{ even} \quad (10)$$

$$\begin{aligned} G_{\rho A_{L+1}}^2(h = \pm 1) : G_{\rho A_L}^2(h = \pm 1) : G_{\rho A_{L-1}}^2(h = \pm 1) = \\ L \text{ odd} \\ = L^2(L+2) : (2L+1) : (L+1)^2(L-1) \end{aligned} \quad (11)$$

iii) $\alpha_{L=0} = \rho \ (h = 0) \text{ or } w \ (h = 0)$

$$G_{\rho H_L}(h=0) = G_{w B_L}(h=0) \stackrel{L \text{ odd}}{=} 0 \quad (12)$$

Equation (8b) brings to the selection rule

$$G_{\pi_L B_{L'}}(L) = 0 \quad L \text{ even, } L' \text{ odd} \quad (13)$$

For baryons Eq. (8a) implies the following equations

$$\text{iv)} \quad 2_{L=0} = 8_{\frac{1}{2}+}$$

$$\frac{G_{8_{\frac{1}{2}} + \beta_{L+\frac{1}{2}}}^2}{G_{8_{\frac{1}{2}} + \beta_{L-\frac{1}{2}}}^2} \stackrel{\text{odd } L}{=} \frac{L}{L+1} \stackrel{\text{even } L}{=} \frac{G_{8_{\frac{1}{2}} + 8_{L-\frac{1}{2}}}^2}{G_{8_{\frac{1}{2}} + 8_{L+\frac{1}{2}}}^2} \quad (14)$$

where $\beta_{\frac{1}{2}}$ can be the $SU(3)$ decuplet, octet or singlet of the 70 odd L with $S = \frac{1}{2}$.

3. MESON SYSTEM

In a slightly different way from what has been done in Ref. 9), where it was defined only on $I = 1$ states, here we assume that the $\vec{W}$ vector is just the W spin operator ¹¹⁾ which applies in the same way to $I = 0$ and $I = 1$ states; this is in agreement with $SU(3)$ and shows up to be necessary in the study of Eq. (4) ¹²⁾. The $S = 1$, $S_3 = 0$ state is a W singlet and is, therefore, annihilated by the Z operator; so

$$U^+(Z) | I, S=1, S_3=0, L, L_3 \rangle = | I, S=1, S_3=0, L, L_3 \rangle \quad (15)$$

The behaviour under $A(\sigma_z \frac{\tau_a}{2})$ of the different isospins is

$$A(\sigma_z \frac{\tau_a}{2}) | I=0, S=1, S_3=0, L, L_3 \rangle = 0$$

$$A(\sigma_z \frac{\tau_3}{2}) | I=1, I_3, S=1, S_3=0, L, L_3 \rangle = I_3 | I=1, I_3, S=0, L, L_3 \rangle \quad (16)$$

From (15) and (16) one derives

$$\begin{aligned} & \langle I=1, S'=1, S'_3=0, L', L'_3 | Q_{5a} | I=1, S=0, L, L_3 \rangle = \\ & = \langle I=1, S'=0, L', L'_3 | Q_{5a} | I=1, S=1, S_3=0, L, L_3 \rangle \end{aligned}$$

which together with (8) implies

$$\frac{\langle I=1, L', S'=1, J', h | Q_{5a} | I=1, L, S=0, J=L, h \rangle}{\langle I=1, L', S'=0, J'=L', h | Q_{5a} | I=1, L, S=1, J, h \rangle} = \frac{C_{L'1J'}^{L'1J'}}{C_{L1J}^{L'1J}} \quad (17)$$

($L'-L = \text{even}$)

A particular situation shows up at $h = 0$ where

$$\begin{aligned} Z | I, S=1, L, J=L\pm 1, h=0 \rangle &= 0 \\ Z A(\sigma_z \frac{\tau_a}{2}) Z | I, S, L, J, h=0 \rangle &= 0 \end{aligned}$$

so that the matrix elements of the axial charge are given in terms of

$$\langle S'=1, J'=L', h=0 | U(Z) | S=0, J=L, h=0 \rangle$$

and

$$\langle S'=0, J'=L', h=0 | U(Z) | S=1, J=L, h=0 \rangle$$

if $L'-L = \text{odd}$

or

$$\langle S'=1, J'=L', h=0 | U(Z) | S=1, J=L, h=0 \rangle$$

and

$$\langle S'=0, J'=L', h=0 | U(Z) | S=0, J=L, h=0 \rangle$$

if $L-L' = \text{even}$

When $L = 0$ only one parameter is needed. Then one finds

$$\frac{G_{\pi f_{L+1}}^2 (h=0)}{G_{\rho A_L}^2 (h=0)} \stackrel{L \text{ odd}}{=} \frac{L}{2L+1} \quad (18)$$

4. BARYON SYSTEM

We recall what are the matrix elements of $W_{\pm}^9$, we need

$$\begin{aligned} W_+ |8_+ S=S_3=\frac{1}{2}\rangle &= - |8_- S=S_3=\frac{3}{2}\rangle \\ (W_+ (6,3)_8 &= - (8,1)) \\ W_- |8_+ S=S_3=\frac{1}{2}\rangle &= -\frac{1}{\sqrt{3}} |8_- S=\frac{3}{2}, S_3=-\frac{1}{2}\rangle \\ &\quad - \frac{2}{\sqrt{3}} |8_- S=-S_3=\frac{1}{2}\rangle \\ (W_- (6,3)_8 &= \frac{1}{\sqrt{6}} [(3,6)_8 - 3(3,\bar{3})_8]) \\ W_{\pm} |1_- S=-S_3=\frac{1}{2}\rangle &= 0 \\ (-W_{\pm} (3,\bar{3})_7 &= 0) \end{aligned} \quad (19)$$

The $SU(3) \otimes SU(3)$ behaviour of the two sides is written in the brackets. From (3'), (19) and

$$\langle L \pm 1, L_3 = \mp 1 | M_+ | L, L_3 = 0 \rangle = \langle L \pm 1, L_3 = \mp 1 | M_- | L, L_3 = 0 \rangle$$

one gets

$$U^+(Z)|8_+, L=0, J=\frac{1}{2}\rangle = \cos\varphi (6,3)_8^I + \sqrt{\frac{3}{8}} \sin\varphi \left[(8,1) + \frac{1}{\sqrt{6}} (3,6)_8 - \sqrt{\frac{3}{2}} (3,\bar{3})_8 \right]^I \quad (20)$$

The chiral content of the baryon octet depends only on φ , while the $\frac{D}{F}$ ratio for the diagonal matrix elements is φ independent and predicted to be $\frac{3}{2}$, the actual value of $\frac{G_A}{G_V} = \frac{5}{4}$ fixes

$$\varphi = \frac{\pi}{4} \quad (21)$$

The baryon octet chiral wave function given by (20) and (21) is slightly different from what one finds in a model independent way⁹⁾.

Content in	Exp	Th	Th (with 4% of $(3,6)_8$ added)
$(6,3)_8 + (3,6)_8$	0.67	0.53	0.67
$(3,\bar{3})_8 + (\bar{3},3)_8$	0.18	0.28	0.22
$(8,1) + (1,8)$	0.15	0.19	0.11

In the third row the correction was quoted due to the presence of a 56 $L=1$ representation, which would not spoil the result found for the $\frac{D}{F}$ ratio; however, since no such supermultiplet has been found in nature we shall not consider it in the following and just restrict to 56 even L and 70 odd L representations.

One may write the analogous of (20) for the singlet and octet states of the 70 $L = 1$ which couple to the $h = \frac{1}{2}$ baryon octet; from (8a) it follows

$$S_3 = \frac{1}{2} \pm 1$$

so we consider

$$U^+(\Sigma) | 8, S=S_3=\frac{3}{2}, L=L_3=1 \rangle = (8, 1)^{L=1} + \psi(6, 3)_8^{\Pi} \\ + \sqrt{\frac{3}{8}} \chi \left[(8, 1) + \frac{1}{\sqrt{6}} (3, 6)_8 - \sqrt{\frac{3}{2}} (3, \bar{3})_8 \right]^{\Pi}$$

$$U^+(\Sigma) | 8, S=\frac{3}{2}, S_3=-\frac{1}{2}, L=L_3=1 \rangle = \frac{1}{\sqrt{2}} [(3, 6)_8 + (3, \bar{3})_8]^{L=1} \\ - \frac{1}{\sqrt{3}} \psi(6, 3)_8^{\Pi} - \frac{1}{2\sqrt{2}} \chi \left[(8, 1) + \frac{1}{\sqrt{6}} (3, 6)_8 - \sqrt{\frac{3}{2}} (3, \bar{3})_8 \right]^{\Pi}$$

$$U^+(\Sigma) | 8, S=-\frac{1}{2}, S_3=\frac{1}{2}, L=L_3=1 \rangle = \frac{1}{\sqrt{2}} [-(3, 6)_8 + (3, \bar{3})_8]^{L=1} \\ - \frac{2}{\sqrt{3}} \psi(6, 3)_8^{\Pi} - \frac{1}{\sqrt{2}} \chi \left[(8, 1) + \frac{1}{\sqrt{6}} (3, 6)_8 - \sqrt{\frac{3}{2}} (3, \bar{3})_8 \right]^{\Pi}$$

$$U^+(\Sigma) | 1, S=-\frac{1}{2}, S_3=\frac{1}{2}, L=L_3=1 \rangle = -(3, \bar{3})_7^{L=1}$$

From Eqs. (20) and (22) the couplings of the $\frac{1}{2}^+$ octet to the singlets and octets of the 70 $L = 1$ are given in Table I.

TABLE I

State of the <u>70</u> $L = 1$ (SU(3) S J)			Coupling to the baryon octet
8	$\frac{3}{2}$	$\frac{5}{2}$	$- U \sqrt{\frac{2}{5}} (D - \frac{1}{3}F)$
8	$\frac{3}{2}$	$\frac{3}{2}$	$\tau \frac{\sqrt{10}}{3} (D + \frac{2}{3}F) + U \sqrt{\frac{2}{5}} (\frac{11}{9}F + \frac{4}{3}D)$
8	$\frac{3}{2}$	$\frac{1}{2}$	$\tau \frac{\sqrt{2}}{3} (D + \frac{2}{3}F) + U \frac{\sqrt{2}}{3} (\frac{4}{3}F - D)$
8	$\frac{1}{2}$	$\frac{3}{2}$	$- \tau \frac{2}{3} (D + \frac{2}{3}F) - U \frac{1}{3} (D - \frac{1}{3}F)$
8	$\frac{1}{2}$	$\frac{1}{2}$	$- \tau \frac{2\sqrt{2}}{3} (D + \frac{2}{3}F) - U \frac{\sqrt{2}}{3} (D - \frac{1}{3}F)$
1	$\frac{1}{2}$	$\frac{3}{2}$	$\sqrt{\frac{2}{3}} U$
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{\sqrt{3}} U$

where

$$\begin{aligned} \tau &= \psi \cos \varphi \langle (6,3)_8^I | (6,3)_8^{\pi} \rangle + \frac{1}{2} \chi \sin \varphi \langle (8,1)^I | (8,1)^{\pi} \rangle \\ U &= \sqrt{\frac{3}{8}} \sin \varphi \langle (8,1)^I | (8,1)^{L=1} \rangle \end{aligned} \quad (23)$$

From (21) and (23) $|U| \leq \frac{\sqrt{3}}{4}$; we choose $U = -\frac{\sqrt{3}}{4}$ then $-\frac{\tau}{U}$ is sharply determined by the fit to lie between $\frac{1}{2}$ and $\frac{4}{7}$ *).

*) The number found for $\frac{\tau}{U}$ is rather reasonable since such a ratio computed up to third order in ϑ with $U(Z) = e^{i\sigma Z}$ is $-(1 - \frac{10}{9}\Sigma)/(1 - \frac{4}{9}\Sigma)$, where Σ is of second order in ϑ .

Comparison with experiment is performed for both values of τ in Table II.

TABLE II ^{*)}

N_i	$J^P(S)$	$\tau = \frac{\sqrt{3}}{7}$	$G_{NNi/th}^2$	$\tau = \frac{\sqrt{3}}{8}$	$\tau = \frac{\sqrt{3}}{7}$	$\frac{F}{D}/th$	$\tau = \frac{\sqrt{3}}{8}$	$G_{NNi/exp}^2$	$\frac{F}{D}/exp$
N(1520)	$\frac{3}{2}^- (\frac{3}{2})$	0.070		0.102	$\frac{37}{24}$		$\frac{4}{3}$	0.091	1.2
N(1535)	$\frac{1}{2}^- (\frac{3}{2})$	0.016		0.010	$-\frac{20}{33}$		$-\frac{2}{3}$	0.024	$-\frac{1}{3}$
N(1670)	$\frac{5}{2}^- (\frac{3}{2})$		0.033			$-\frac{1}{3}$		0.070	-0.13
N(1700)	$\frac{1}{2}^- (\frac{1}{2})$	0.064		0.040	$\frac{23}{3}$		pure F	0.063	
N(?)	$\frac{3}{2}^- (\frac{1}{2})$	0.032		0.020	$\frac{23}{3}$		pure F		
Y_{oi}			$G_{\sum Y_{oi}/th}^2$					$G_{\sum Y_{oi}/exp}^2$	
$Y_o(1405)$	$\frac{1}{2}^-$		0.250					0.200	
$Y_o(1520)$	$\frac{3}{2}^-$		0.125					0.020	

*) The calculations of the axial coupling constants performed here compelled us to interchange the assignment early proposed in Ref. 9) of the two $\frac{3}{2}$ states. The quoted experimental values are those of the same reference.

The value $-\frac{1}{3}$ found for the $\frac{F}{D}$ ratio of the $\frac{5}{2}^-$ octet is constant along the trajectory as one can see writing the coupling of the baryon octet to a 70 odd L : the same ratio has been predicted assuming exchange degeneracy of the $\frac{3}{2}^+$ decuplet and $\frac{5}{2}^-$ octet trajectories in meson baryon scattering, Ref. 13).

The $\frac{F}{D}$ ratio is also predicted for the positive parity trajectories; in fact, from

$$U^+(Z) | 8, S=S_3=\frac{1}{2}, L, L_3=0 \rangle = \cos \varphi_L (6, 3)_8^L \\ + \sin \varphi_L \sqrt{\frac{3}{8}} \left[(8, 1) + \frac{1}{\sqrt{6}} (3, 6)_8 - \sqrt{\frac{3}{2}} (3, \bar{3})_8 \right]^L$$

it is easy to deduce that the couplings to the baryon octet have

$$\frac{F}{D} = \frac{2}{3} \quad (24)$$

Such ratio lies in the large range allowed by the different experiments for N ($0.25 \leq \frac{F}{D} \leq 1.1$), Ref. 14).

5. COMPARISON WITH EXPERIMENT

We give in Table III a survey of the not yet discussed predictions derived along this paper quoting, for clarity, the equations involved.

TABLE III *)

Ratio (or decay)	EXP.	TH.	Eq.
$\frac{G_{\pi f}^2}{G_{\pi \sigma}^2}$	$\frac{0.11}{\gg 0.10}$	$\frac{1}{2}$	(9)
$\frac{G_{N\Delta(1670)}^2}{G_{N\Delta(1650)}^2}$	$\frac{0.024}{0.021}$	$\frac{1}{2}$	(14)
$\frac{G_{\Sigma Y_0(1520)}^2}{G_{\Sigma Y_0(1405)}^2}$	$\frac{0.020}{0.200}$	$\frac{1}{2}$	(14)
$\frac{G_{NN(1860)}^2}{G_{NN(1688)}^2}$	$\frac{0.047}{0.097}$	$\frac{2}{3}$	(14)
$\frac{G_{\rho_{A_2}(h=1)}^2}{G_{\rho_{A_1}(h=1)}^2}$	$\frac{0.065}{\approx 0.08}$	1	(11)
$\pi_A(1640) \rightarrow B + \pi$	possibly observed $\omega \pi \pi$	forbidden	(13)
$B \rightarrow \omega + \pi$	mainly transverse	purely transverse	(12)
$\frac{G_{\pi \rho_N(1670)}^2}{G_{\rho \pi_A(1640)}^2}$	$\frac{0.11}{0.02}$	$\frac{3}{2}$	(17)

Table III ctd.

Ratio (or decay)	EXP.	TH.	Eq.
$\frac{G_{\pi f}^2}{G_{\rho A_1}^2 (h=0)}$	$\frac{0.11}{0.32}$	$\frac{1}{3}$	(18)
$\frac{G_{NN}^2(1670)}{G_{\Sigma Y_0}^2(1405)}$	$\frac{0.070}{0.200}$	$\frac{2}{15}$	TABLE I
$G_{NN}^2(1535)/G_{NN}^2(1700)$	$\frac{0.024}{0.063}$	$\frac{1}{4}$	TABLE I
$\frac{F}{D} \frac{1}{2}^+ \frac{5}{2}^+$	1.07	$\frac{2}{3}$	(24)

*)

The quoted experimental values are those of Refs. 8) and 9) assuming

$$\left| \frac{G_{\rho A_1}(h=1)}{G_{\rho A_1}(h=0)} \right| = \frac{1}{2}$$

For the predictions coming from Eq. (17) the data of Particle Properties Tables have been inserted in the formula ¹⁰⁾

$$\Gamma_{2 \rightarrow \beta + \pi \alpha} = \frac{(m_2^2 - m_\beta^2)}{16\pi F_\pi^2 m_2^3} \frac{1}{2J_2 + 1} \sum_h |G_{\beta 2 \pi \alpha}(h)|^2$$

The agreement is satisfactory with the exception of the high value predicted for $G_{\Sigma Y_0}^2(1520)$ (the octet singlet mixing explains partly this failure), and the wrong ratio concerning $\rho_N(1670)$ and $\pi_A(1640)$ where anyway, due to their high mass, the situation is still not clear. A lower value for $G_{NN}^2(1670)$ is expected than that found in the present experimental situation. A crucial test of the form proposed for the axial charge would be a comparison with experiment of the selection rules (10), (12) and (13).

6. CONCLUDING REMARK

Since a certain number of predictions derived in this paper are in satisfactory agreement with experiment, the confirmation of this pattern should allow us to think to have found a good model for the axial charge. The form of this operator should be a good starting point to write realistic amplitudes involving two pions and two particles with spin in terms of one-particle intermediate states.

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