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ϵ MESON COUPLING CONSTANTS AND SCALE INVARIANCE BREAKING

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A B S T R A C T

A general method for obtaining ratios of coupling constants (defined by pole residues) in a way which is completely free of resonance/background separation troubles is devised and applied to the ϵ meson. Huge discrepancies between previous determinations are shown to arise (1) from inherent ambiguities in the methods used, (2) from lack of knowledge about the ϵ pole position and (3) from the well-known up-down ambiguity in the isospin-0 s wave $\pi\pi$ phase-shift δ_0^0 . Taking as input πN phase shifts, available information on δ_0^0 and including all possible uncertainties we find for down-up or up-up δ_0^0 : $g_{\epsilon NN}/g_{\epsilon\pi\pi} = (6 \pm 3)\mu^{-1}$ and for down-down or up-down δ_0^0 : $g_{\epsilon NN}/g_{\epsilon\pi\pi} = (1.8 \pm 0.5)\mu^{-1}$. The precise validity of the scale invariance breaking prediction $(g_{\epsilon NN}/g_{\epsilon\pi\pi}) \cdot m_\epsilon^2/2M = 1$ is fulfilled in some theoretical $\pi\pi$ models having $\Gamma_\epsilon \approx m_\epsilon \approx 500$ MeV and δ_0^0 of up-down type, but is excluded if δ_0^0 is down-up or up-up.

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1. INTRODUCTION

We define $\mathcal{E}$ meson couplings to pions and nucleons by the Lagrangian (units $\hbar = c = 1$ are used throughout)

$$\mathcal{L} = g_{\mathcal{E}\pi\pi} \mathcal{E} \vec{\pi} \cdot \vec{\pi} + g_{\mathcal{E}NN} \mathcal{E} \bar{N}N. \quad (1)$$

Then we have in the narrow width approximation (nwa)

$$\Gamma_{\mathcal{E} \rightarrow 2\pi} = 3 g_{\mathcal{E}}^2 \frac{g_{\mathcal{E}\pi\pi}^2}{4\pi} \frac{1}{m_{\mathcal{E}}^2} \quad \left. \vphantom{\Gamma_{\mathcal{E} \rightarrow 2\pi}} \right\} \quad (2)$$

where

$$g_{\mathcal{E}} \equiv \frac{1}{2} \sqrt{m_{\mathcal{E}}^2 - 4\mu^2}$$

μ is the pion mass and $m_{\mathcal{E}}$ is the $\mathcal{E}$ meson mass [the $\mathcal{E}$ object which we have in mind is the one denoted by η_{o+} (700 - 1000) or $\mathcal{E}$, in the Particle Data Group Tables ¹⁾].

If further $f_{\pm}^J(b)$ are the usual ²⁾ amplitudes describing the process

$$\pi\pi \rightarrow \bar{N}N$$

in a state of angular momentum J , with isospin $0(1)$ for J even (odd) and with nucleon-antinucleon helicity configuration described by the label, $\pm$, then Eq. (1) gives in the nwa

$$\text{Im} [f_{\pm}^0(t)/(M^2 - t/4)] = -2 \frac{g_{\mathcal{E}\pi\pi} g_{\mathcal{E}NN}}{4\pi} (-\pi \delta(t - m_{\mathcal{E}}^2)) \quad (3)$$

where M is the nucleon mass.

Equations (2) and (3) suggest to obtain the $\mathcal{E}$ couplings $g_{\mathcal{E}\pi\pi}$ and $g_{\mathcal{E}NN}$ by measuring the combinations

$$\frac{g_{\epsilon\pi\pi}^2}{4\pi} \quad \text{and} \quad \frac{g_{\epsilon\pi\pi} g_{\epsilon NN}}{4\pi} \quad (4)$$

respectively from the ϵ width and mass and from $I = 0$ exchange in πN scattering.

Apart from their general interest these couplings are important for the theory of scale invariance breaking (SIB) or ϵ meson dominance of the trace of the energy momentum tensor,

$$\langle 0 | \Theta_{\mu}^{\mu} | \epsilon \rangle = - \frac{m_{\epsilon}^2}{b} \quad (5)$$

which yields the following predictions ³⁾

$$\begin{aligned} g_{\epsilon NN} &= -b M \\ g_{\epsilon\pi\pi} &= -\frac{b}{2} (m_{\epsilon}^2 + (d-2)\mu^2) \end{aligned} \quad (6)$$

where d is the "dimension" of the SIB part of the interaction - possibly integer between 1 and 3 - and b is a fundamental universal constant (ϵ graviton coupling).

Provided the two ϵ couplings could be obtained, the theory could be tested on the rule

$$\frac{g_{\epsilon NN}}{g_{\epsilon\pi\pi}} \frac{m_{\epsilon}^2}{2M} = \frac{m_{\epsilon}^2}{m_{\epsilon}^2 + (d-2)\mu^2} \approx 1 \quad (7)$$

and if the test is positive, the universal constant, b , could be obtained from either of Eqs. (6).

In practice, it turns out that the two parameters (4) cannot be obtained from Eqs. (2) and (3) with a very useful accuracy [for a recent compilation of wildly varying results, see Genz and Steiner, Ref. 3].

This is due in part to the fact that the ϵ mass and ϵ width are extremely poorly known [even accepting that the state exists - by this we mean that the $J = I = 0$ partial wave amplitude (pwa) has a second sheet pole somewhere above threshold and below 1 GeV]. That, in turn, is due to lack of knowledge on the $J = I = 0$ $\pi\pi$ phase shift

δ_0^0 in this region [for reviews with several references, see Refs. 4]. We shall base the discussion in this paper mainly on the dispersion relation fits of Morgan and Shaw classified according to the up, down and between terminology [U, D, B, Fig. 1, Ref. 5]. We shall for the most part simplify the discussion by grouping together the various possible δ_0^0 's into two classes: (1) up-above- ρ (i.e., DUI, DUUI, UU, BU) and (2) down-above- ρ (i.e., BD, DD, UD). Then, in the up-above- ρ group, ϵ parameters are relatively well defined and something like

$$m_\epsilon = (750 \pm 100) \text{ MeV}, \quad T_\epsilon = (300 \pm 200) \text{ MeV} \quad (9)$$

is suggested, corresponding to

$$\left(\frac{g_{\epsilon\pi\pi}^2}{4\pi} \right)_{\text{up}} = \left(8_{-5}^{+6} \right) \mu^2 \quad (10)$$

For the down-above- ρ group on the other hand, a very wide object is indicated with mass and width values of the same magnitude, but both very poorly known. Thus

$$m_\epsilon = (700 \pm 200) \text{ MeV}, \quad T_\epsilon = (500 \pm 300) \text{ MeV} \quad (11)$$

gives

$$\left(\frac{g_{\epsilon\pi\pi}^2}{4\pi} \right)_{\text{down}} = \left(12_{-7}^{+10} \right) \mu^2 \quad (12)$$

The situation regarding the second parameter in (4) is not better. Two recent estimates exist based on an analysis of backward pion nucleon scattering. Using a real axis ϵ pole at 700 MeV and a pole at the f position (which may be considered a phenomenological way of dealing with all far away contributions to the low energy region) both with free residues, Engels⁶⁾ obtained a result that may be quoted as [using Eq. (3)]

$$\int_{\epsilon\text{-peak}} dt \text{ Im } f_t^0(t) = (3400 \pm 200) \mu \quad (13)$$

or

$$\frac{g_{\epsilon\pi\pi} g_{\epsilon NN}}{4\pi} = (13.8 \pm 1.0) \mu.$$

Engels further took $\Gamma_{\mathcal{E}} = 400 \text{ MeV}$ giving

$$\frac{g_{\mathcal{E}\pi\pi}^2}{4\pi} = 10.7 \mu^2$$

so that the test [Eq. (7)] reads

$$\left(\frac{g_{\mathcal{E}NN} m_{\mathcal{E}}^2}{g_{\mathcal{E}\pi\pi} 2M} \right)_{\text{Engels}} = 2.42 \pm 0.14 \quad (14)$$

which is 10 standard deviations away from 1.0. If allowance for uncertainties in $\mathcal{E}$ mass and $\mathcal{E}$ width is made like in Eqs. (9) or (11) the errors increase so that 1 is possible in Eq. (14), however, little useful information is left then.

Nielsen et al. ⁷⁾ analyzed the data using the modern prescriptions for doing analytic extrapolations. Their result was

$$\int_{\mu^2}^{50\mu^2} dt \text{Im} f_+^0(t) = (1100 \pm 600) \mu \quad (15)$$

which is independent of the $\mathcal{E}$ parameters (or even existence of a resonance) and also does not imply any specific assumptions on the form of the far away contributions (they turn out to be large in the low energy region). Using Eq. (3) in a blind fashion, this gives a value for the product in (4) a factor 3 smaller than Engel's value (13) and tends to improve agreement with SIB [Eq. (7)].

The value of Eq. (13) is likely to be too large since in the two-pole fit, opposite signs of the residues have to be used and to simulate the effect of the neglected low energy $\pi\pi$ interaction, both will tend to be numerically too large. The value of Eq. (15) on the other hand is likely to be too small since in the analysis one effectively asks how smooth and slowly varying an $\text{Im} f_+^0(t)$ can fit the data, so that genuine $\mathcal{E}$ peak contributions may be interpreted as far away ones. However, even if an accurate estimate of the integrals (13) and (15) was possible, to obtain the product

$$\frac{g_{\mathcal{E}\pi\pi} g_{\mathcal{E}NN}}{4\pi}$$

one would have to make specific assumptions on which part of the integral is due to a second sheet pole and how much comes from the background. Secondly, having decided that, the pole residue would depend heavily on the assumed pole position.

We conclude that existing data are insufficient to yield accurate estimates of the two parameters (4) proceeding as outlined above. What we show in this paper, however, is that important correlations between the two may be obtained from the data and, in fact, fairly accurate values of the ratio of $\mathcal{E}$ couplings

$$g_{\pi\pi} / g_{\pi NN}$$

may be obtained.

In Section 2, we discuss a general method for finding ratios of resonance pole residues in different interactions. The method is based on the simple observation that in an n channel two-body to two-body scattering the resonance pole occurs in all amplitudes (having appropriate quantum numbers). The pole cancels in ratios of amplitudes enabling one thus to obtain by extrapolation reliable results about the ratio of coupling constants even in cases where data are insufficient to make extrapolations of individual amplitudes. In practical applications, it may be more advantageous to extrapolate the amplitude multiplied by a known function which kills the pole and construct the ratio of amplitudes afterwards. We show that in the present two-channel problem ($\pi\pi \rightarrow \pi\pi$ and $\pi\pi \rightarrow \bar{N}N$) this can be done in a particularly attractive way. In Sections 3 and 4, we give computational details and in Section 5 we present our numerical results. In Section 6, we apply our method to the model of Le Guillou et al ⁸⁾ (GMN) and attempt conclusions with respect to SIB.

2. OBTAINING RATIOS OF COUPLING CONSTANTS

To avoid as much as possible the inherent nwa associated with defining coupling constants in terms of effective Lagrangians like Eq. (1), we shall be mainly concerned with the residues of second sheet resonance poles. These are also the dynamically significant quantities since they factorize properly in different reactions. For definiteness, we consider the case of $\mathcal{E}$ contributions to $\pi\pi \rightarrow \bar{N}N$ and $\pi\pi \rightarrow \pi\pi$ but the scheme can obviously be generalized.

Suppose that the $I = J = 0$ $\pi\pi$ phase shift δ_0^0 is given from the 2π threshold to the $K\bar{K}$ threshold, $t_1 = 995$ MeV. Then, we define an Omnès function

$$F_{\delta}(t) = \exp \left\{ \frac{t}{\pi} \int_{\frac{4\mu^2}{9}}^{t_1} \frac{\delta_0^0(t')}{t'(t'-t)} dt' + \int_{t_1}^{\infty} \frac{\delta(t')}{t'(t'-t)} dt' \right\} \quad (16)$$

where δ in principle is an arbitrary function for which the last integral is reasonably well behaved. We shall dispose of this arbitrariness and choose a function that joins onto δ_0^0 at t_1 and goes to 0 at $t = \infty$ to make $F_{\delta}(t)$ exhibit features characteristic of δ_0^0 mainly. In fact, we replace Eq. (16) by

$$F(t) = \exp \left\{ \frac{t}{\pi} \int_{\mu^2}^{t_1} \frac{\delta_0^0(t')}{t'(t'-t)} dt' \right\} e^{\delta_0^0(t_1)/\pi} \left(\frac{t_1 - t}{t_1} \right)^{\frac{\delta_0^0(t_1) \cdot t_1}{\pi t}}$$

We then define functions $N(t)$ and $J(t)$ like

$$N(t) \equiv f_0^0(t)/F(t)$$

$$J(t) \equiv \frac{f_+^0(t)/(M^2 - t/4)}{F(t)} \quad (17)$$

where f_0^0 is the $\pi\pi$ $J = I = 0$ pwa normalized in the elastic range as

$$\left. \begin{aligned} f_0^0(t) &= \frac{\sqrt{t}}{q} e^{i\delta_0^0(t)} \sin \delta_0^0(t) \\ q &\equiv \frac{1}{2} \sqrt{t - 4\mu^2} \end{aligned} \right\} \quad (18)$$

We assume throughout that this pwa is purely elastic below the $K\bar{K}$ threshold. Above that, both $K\bar{K}$ and 4π inelasticity is abundant, but below, essentially no 4π events have been observed (in high energy peripheral $\pi N \rightarrow 4\pi N$)⁷⁾.

Then N is a function analytic in the t plane cut along $(-\infty, 0)$ and (t_1, ∞) and $J(t)$ is analytic in the t plane cut along $(-\infty, 4\mu^2(1 - \mu^4/4M^2))$ and (t_1, ∞) . Thus, unlike $f_0^0(t)$ and $f_+^0(t)$, N and J have no second sheet ε pole and will be correspondingly more slowly varying functions on $(4\mu^2, t_1)$.

Next, we define the functions $R(t)$ and $T(t)$ by

$$R(t) \equiv \frac{f_+^0(t)/(M^2 - t/4)}{f_0^0(t)} \frac{3}{2} \frac{\sqrt{t}}{\text{Re}\sqrt{t}} = \frac{\sqrt{t}}{\text{Re}\sqrt{t}} \frac{J(t)}{N(t)} \frac{3}{2}$$

$$T(t) \equiv \frac{f_+^0(t)/(M^2 - t/4)}{f_0^0(t)} \sqrt{t} \text{Re}\sqrt{t} \frac{3}{4M} \quad (19)$$

These contain no ε poles either, but at the complex pole position, their values are nevertheless given by the ratio of pole residues. Defining $m_\varepsilon, T_\varepsilon$ by

$$\sqrt{t_p} = m_\varepsilon - i \frac{T_\varepsilon}{2} \quad (20)$$

where t_p is the pole position,

$$R(t_p) = g_{\epsilon NN} / g_{\epsilon \pi \pi} \quad , \quad T(t_p) = \frac{g_{\epsilon NN}}{g_{\epsilon \pi \pi}} \frac{m_\epsilon^2}{2M} \quad (21)$$

which contains a definition of $g_{\epsilon NN}/g_{\epsilon \pi \pi}$ based on analogy with the nwa forms (2) and (3). $T(t_p)$ should be close to 1 provided the SIB prediction of Eq. (7) is valid. Note that with this definition, there is no a priori reason for $g_{\epsilon NN}/g_{\epsilon \pi \pi}$ to be real.

The idea now is to attempt an evaluation of the functions R and T in the complex region of the t plane where the ϵ pole is supposed to lie (11). The factor $\text{Re} \sqrt{t}$ introduced for convenience in Eqs. (19) make $T(t)$ and $R(t)$ non-analytic. This, however, does not matter since in evaluating $R(t)$ and $T(t)$ in the complex t plane we extrapolate the analytic functions $J(t)$ and $N(t)$. Clearly R and T cannot both be very slowly varying since

$$T = \frac{1}{2M} (\text{Re} \sqrt{t})^2 R \quad (22)$$

but from the arguments concerning N and J we expect one to be more slowly varying than f_0^0 and f_+^0 individually so that ignorance on the ϵ pole position does not show up in large errors on the corresponding ratio (21). Also the connection between R or T and the appropriate combination of coupling constants is completely free of resonance vs. background separation troubles.

3. EVALUATION OF $N(t)$

For given $\delta_0^0(t)$ in $4\mu^2 \leq t \leq t_1$, $f_0^0(t)$ and $F(t)$ and hence $N(t)$ may be directly obtained on that same interval. To calculate $N(t)$ in the complex plane, we use the conformal mapping techniques ¹⁰⁾ and fit N to polynomials in the variable z defined by

$$\begin{aligned} t \rightarrow z &= \varphi(t) \\ \varphi(4\mu^2) &= -1 \\ \varphi(t_1') &= 1 \\ \varphi(0) &= -\varphi(t_1) \equiv -a \end{aligned}$$

$\varphi(t)$ plane cut along $(-\infty, 0)$, (t_1, ∞) = ellipse with foci ± 1 and semi-major axis = a . (23)

We choose $t_1 = t_1 - 5\mu^2$ but nothing depends very much on that choice. Since we only need evaluate N in the part of the holomorphy region which is relatively close (in a conformally invariant sense) to the physical region $-1 \leq z \leq 1$ (i.e., $4\mu^2 \leq t \leq t_1$), extrapolation errors are not a great problem and are very much smaller than in the application to backward πN scattering ⁷⁾ [for a review, see Pišút ¹¹⁾]. We shall use the value of N associated with a second order fit in z and quote as the "error" the difference between that and the third order fit result.

4. EVALUATION OF $J(t)$

We use the values of $f_+^0(t)$ calculated by Nielsen et al. ⁷⁾ on $-26\mu^2 \leq t \leq 0$. Also $\text{Im} f_+^0(t)$ is known on $0 \leq t \leq 4\mu^2 - \mu^4/M^2$ from the πNN coupling constant. Then for given δ_0^0 , $J(t)$ may be directly evaluated on $-26\mu^2 \leq t \leq 0$ using Eq. (17) [note that $J(t)$ has no pole at $t = 4M^2$ ²⁾]. That, however, is on part of the cut in J and to perform the extrapolation to the $\mathcal{E}$ region of the complex plane we use the discrepancy method ⁷⁾ and write

$$J(t) = \frac{1}{\pi} \int_{-26\mu^2}^{4\mu^2 - \mu^4/M^2} \frac{\text{Im} J(t')}{t' - t} dt' + \frac{t_0 \mu^2}{t} \text{Im} f_+^0(t_0) / (M^2 - t_0/4) / F(t_0) \cdot \frac{1}{\pi} \log \left| \frac{t - t_0}{t_0} \right| + \Delta(t) \quad (24)$$

where $t_0 = -26\mu^2$ and where $\Delta(t)$ may be directly evaluated on $-26\mu^2 \leq t \leq 0$ by

$$\Delta(t) = \text{Re} J(t) - \frac{P}{\pi} \int_{-26\mu^2}^{4\mu^2 - \mu^4/M^2} \frac{\text{Im} J(t')}{t' - t} dt' - \frac{t_0 \text{Im} f_+^0(t_0)}{t(M^2 - t_0/4) F(t_0)} \frac{1}{\pi} \log \left| \frac{t - t_0}{t_0} \right| \quad (25)$$

The last term prevents $\Delta(t)$ from getting a strong logarithmic singularity at $t = t_0 = -26\mu^2$. Now $\Delta(t)$ is analytic in the t plane cut along $(-\infty, -26\mu^2)$ and (t_1, ∞) and is known in part of its holomorphy domain so that it may be continued onto the complex $\mathcal{E}$ region just as $N(t)$. The two first terms in Eq. (24) may be directly evaluated in that same region so that finally $J(t)$ is calculated for t arbitrary, complex.

5. NUMERICAL RESULTS

The inputs used are

- (1) $\text{Re } f_+^0(t)$ on $-26\mu^2 \leq t \leq 0$ and
 $\text{Im } f_+^0(t)$ on $-26\mu^2 \leq t \leq 4\mu^2 - \mu^4/M^2$ 7)
- (2) $\delta_0^0(t)$ on $4\mu^2 \leq t \leq t_1$

Calculations were performed for all the curves shown on Fig. 1 (and for several ones not shown).

To test the method (and the computer program) we constructed simple explicit analytic model functions $f_0^0(t)$ having cuts $(-\infty, 0)$ and $(4\mu^2, \infty)$, a pair of second sheet resonance poles and satisfying elastic unitarity. Then we used as numerical input to the program :

- (1) values substituting those of $f_+^0(t)$ on the above regions defined by

$$f_+^0(t) \equiv f_0^0(t) \cdot (M^2 - t/4)$$

- (2) $\delta_0^0(t) \equiv \arg f_0^0(t)$ on $4\mu^2 \leq t \leq t_1$.

With these the calculated values of R and T trivially should reduce to

$$R(t) \rightarrow \frac{3}{2} \frac{\sqrt{t}}{\text{Re} \sqrt{t}}$$

$$T(t) \rightarrow \sqrt{t} \text{Re} \sqrt{t} \frac{3}{4M}$$

The program correctly reproduced these in the complex region within the error estimate (difference between second and third order extrapolations $\approx 1\% - 30\%$ depending on the complex value of t).

For the actual curves of Fig. 1 we find that $R(t)$ is always very much more slowly varying with t than $T(t)$. This means that the quantity $g_{\epsilon NN}/g_{\epsilon \pi \pi}$ can be determined with useful accuracy without almost any knowledge of (m_ϵ, T_ϵ) . The SIB prediction of Eq. (7) on the other hand can then not be accurately tested without assuming something for (m_ϵ, T_ϵ) .

For the solutions BU, DUI, DUII, UU we summarize the findings by the results

$$\int_{\mu^2}^{50\mu^2} \text{Im} f_+^0(t) dt = (3500 \pm 500) \mu \quad (26)$$

$$\frac{g_{ENN}}{g_{E\pi\pi}} = (6 \pm 3) e^{i(-5^\circ \pm 25^\circ)} \mu^{-1} \quad (27)$$

$$\frac{g_{ENN}}{g_{E\pi\pi}} \frac{m_\pi^2}{2M} = (13 \pm 8) e^{i(-5^\circ \pm 25^\circ)} \quad (28)$$

The error estimates here have several sources :

- (1) they summarize the range of results obtained by all of the four phase-shifts above ;
- (2) they allow for uncertainty in the ϵ pole position corresponding to (9) ;
- (3) they include errors from uncertainties in $(\text{Re}, \text{Im}) f_+^0(t)$;
- (4) they include extrapolation errors ;
- (5) they are totally free of resonance vs. background separation troubles.

Although the estimate (28) is very uncertain, the SIB prediction seems to be ruled out if any of these phase-shifts represent the correct form for δ_0^0 . However, being "up" above the ρ , they are not among the now preferred forms.

For the solutions BD, UB, DD we get

$$\int_{\mu^2}^{50\mu^2} \text{Im} f_+^0(t) dt = (2500 \pm 500) \mu \quad (29)$$

$$\frac{g_{ENN}}{g_{E\pi\pi}} = (1.8 \pm 0.5) e^{i(15^\circ \pm 15^\circ)} \mu^{-1} \quad (30)$$

$$\frac{g_{ENN}}{g_{E\pi\pi}} \frac{m_\pi^2}{2M} = 4 \pm 3 \quad (31)$$

Errors include effects similar to those listed above. For down-above- ρ phase shifts uncertainties in m_ϵ and Γ_ϵ are much larger than the similar uncertainties for the up-above- ρ case. Correspondingly, the fraction of errors due to uncertainties in $m_\epsilon, \Gamma_\epsilon$ is much larger in Eq. (31) than in Eq. (28), whereas no such trouble is present in $g_{ENN}/g_{E\pi\pi}$ since the function $R(t)$ is numerically nearly constant throughout the complex plane.

In general results based on down-above- ρ phase shifts are much closer to the SIB [Eq. (7)] prediction than results obtained with up-above- ρ phase shifts. That is, within the present uncertainties the down-above- ρ type of δ_0^0 is consistent with SIB, whereas the up-above- ρ type is not.

We make a few comments on the comparison between our results and the earlier determinations, Eqs. (13) and (15).

(a) Engel's value for $\int \text{Im} f_+^0(t) dt$ [Eq. (13)] is consistent with ours when δ_0^0 is up-above- ρ , while the value of Nielsen et al. [Eq. (15)] at most is consistent with ours when δ_0^0 is down-above- ρ . We suggest that this be connected with the fact that a real axis pole-fit [Ref. 6] inherently assumes an extreme down-up type of δ_0^0 ,

$$\delta_0^0(t) = \pi \theta(t - m_\rho^2)$$

whereas, of course, in Ref. 7) the down-above- ρ class was indeed preferred (in agreement with Morgan and Shaw⁵⁾).

(b) In the pole-fit method, it is apparently easy to produce a small value for $g_{\mathcal{E}NN}/g_{\mathcal{E}\pi\pi}$ [to get agreement with SIB, Eq. (7)] from any value of $g_{\mathcal{E}NN} \cdot g_{\mathcal{E}\pi\pi}$, just by assuming a sufficiently large value for $\bar{r}_{\mathcal{E}}$. In our case, on the other hand, $g_{\mathcal{E}NN}/g_{\mathcal{E}\pi\pi}$ [or rather $R(t)$] is an increasing function of $\bar{r}_{\mathcal{E}}$ reflecting the fact that correlations between the $\mathcal{E}$ parameters and f_+^0 exist that are ignored in pole fits.

Figure 2 shows some of our calculated forms for $\text{Im} f_+^0(t)$ *).

6. APPLICATION TO THE GMN MODEL

Le Guillou et al.⁸⁾ have constructed low energy model $\pi\pi$ amplitudes that satisfy the requirements of crossing, analyticity and unitarity to a high degree of approximation and fit the experimental ρ meson shape. Their prediction for δ_0^0 is shown on Fig. 1 (GMN) and is close to the preferred between-down solution of Morgan and Shaw⁵⁾. Only their "central" solution is shown - others exist corresponding to a $\Delta\delta_0^0$ increasing with energy and being about $\pm 15^\circ$ at 900 MeV. A further prediction of the model is the existence of an $\mathcal{E}$ pole corresponding to

*) These agree with results recently obtained by Elvekjaer and Nielsen¹²⁾ using a similar method (however, not being concerned with coupling constants).

$$m_{\mathcal{E}} = 420 \text{ MeV} , \quad T_{\mathcal{E}} = 380 \text{ MeV} \quad (32)$$

for the central solution. Other solutions invariantly give the same (very low) $\mathcal{E}$ mass whereas the width is more unstable :

$$T_{\mathcal{E}} = 270 - 520 \text{ MeV}$$

At this moment, it is not clear to what extent the GMN model exhausts the theoretical possibilities (for a discussion, see Petersen ⁴⁾), but similarly low $\mathcal{E}$ mass values (and similar forms for δ_0^0) have been obtained in recent Padé studies of $\pi\pi$ scattering ¹³⁾. We, therefore, consider it to be an interesting exercise to see what the application of our method to such a model (in which the $\mathcal{E}$ pole position is fixed) will yield.

We have calculated $R(t)$ and $T(t)$ for δ_0^0 (GMN) and at the point (32) obtained the results

$$\frac{g_{\mathcal{E}NN}}{g_{\mathcal{E}\pi\pi}} = (1.42 \pm 0.10) e^{i20^\circ} \mu^{-1} \quad (33)$$

$$\frac{g_{\mathcal{E}NN}}{g_{\mathcal{E}\pi\pi}} \frac{m_{\mathcal{E}}^2}{2M} = 0.92 \pm 0.06 \quad (34)$$

in remarkable agreement with Eq. (7).

In the GMN model, $g_{\mathcal{E}\pi\pi}$ is further directly obtainable from the pole residue of f_0^0 normalized as in Eq. (18) ¹⁴⁾

$$\text{residue}(m_{\mathcal{E}} = 420 \text{ MeV}, T_{\mathcal{E}} = 380 \text{ MeV}) = 14.7 e^{i56^\circ} \mu^2 \quad (35)$$

That is to be compared with the expected value for the pole residue in the narrow width approximation (always $m_{\mathcal{E}} = 420 \text{ MeV}$, $T_{\mathcal{E}} = 380 \text{ MeV}$)

$$- 2 T_{\mathcal{E}} m_{\mathcal{E}} / \sqrt{m_{\mathcal{E}}^2 - 4\mu^2} = -22.0 \mu^2 \quad (36)$$

which is grossly different from Eq. (35) and expresses some other perils associated with narrow width $\mathcal{E}$ phenomenology.

Since there is no universal consensus on how to define coupling constants from complex pole residues, we just make the following choice which seems reasonable to us and which reduces to the narrow width limit as Γ_ϵ goes to zero. If r is the residue (35) and

$$t_p \equiv (m_\epsilon - i \frac{\Gamma_\epsilon}{2})^2$$

we define

$$\frac{g_{\epsilon\pi\pi}^2}{4\pi} = - \frac{m_\epsilon}{3\sqrt{t_p}} \left[\frac{\text{Re } g_\epsilon - i \frac{\text{Im } t_p}{8g_\epsilon}}{g_\epsilon} \right] r \quad (37)$$

where $g_\epsilon^2 \equiv \frac{1}{4}(t_p - 4\mu^2)$. Other choices will produce other kinematical factors in front of r in Eq. (37) but all reasonable choices appear to give about the same value for $|g_{\epsilon\pi\pi}^2/4\pi|$ [$\arg(g_{\epsilon\pi\pi}^2/4\pi)$ varies from one to the other] and in any case the ratio $g_{\epsilon NN}/g_{\epsilon\pi\pi}$ is free of "kinematical" ambiguities.

From Eqs. (35) and (37) we get

$$\frac{g_{\epsilon\pi\pi}^2}{4\pi} = 3.11 e^{-i32^\circ} \mu^2 \quad (38)$$

compared to the narrow width expectation

$$\left(\frac{g_{\epsilon\pi\pi}^2}{4\pi} \right)_{h.w.} = 7.33 \mu^2 \quad (39)$$

More specific assumptions about the nature of SIB have been used to relate the ϵ width to the ϵ mass ³⁾

$$m_\epsilon^2 = - \frac{b^2 f_\pi^2}{2} d(d-4) (\mu^2 + 2m_K^2) \quad (40)$$

where $f_\pi \approx 95$ MeV. Using Eq. (6) b may be eliminated and assuming d to be integer between 1 and 3, $m_\epsilon = 420$ MeV implies

$$\frac{g_{E\pi\pi}^2}{4\pi} = 0.10 \pm 0.03 \quad (41)$$

which is one to two orders of magnitude smaller than either of the results (38), (39). A much more sensible value for the coupling is predicted by Eq. (40) if $m_E \approx m_\rho$, but then our calculations appear to rule out Eq. (7). Even for the down-above- ρ -solutions we get with $m_E = m_\rho$

$$\frac{g_{ENN}}{g_{E\pi\pi}} \frac{m_E^2}{2M} = 4.0 \pm 1.5 \quad (42)$$

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FIGURE CAPTIONS

Figure 1 :

Various possible forms for δ_0^0 for which the calculations described in the paper were performed. Letters U, D and B refer to the up, down and between classification of the dispersion relation fits by Morgan and Shaw ⁵⁾. The central solution of Le Guillou et al. ⁸⁾ is also shown (GMN).

Figure 2 :

Some of the calculated curves for $\text{Im} f_+^0(t)$. Letters refer to the phase-shift used as input (Fig. 1). The result of Nielsen et al. ⁷⁾ is shown for comparison. Typical expected extrapolation errors are indicated ; they increase drastically towards $t \approx 55 \mu^2$.

FIG.1

FIG. 2