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BOUNDS ON PION MULTIPLICITIES AND FLUXES
FROM ISOSPIN INVARIANCE

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A B S T R A C T

From isospin invariance we derive lower and upper bounds for π^0 multiplicities (fluxes) in terms of $\pi^\pm$ multiplicities (fluxes) for various processes like NN^- , $N\bar{N}^-$, KN^- and πN scattering.

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During the last years, increasing attention has been paid to high multiplicity processes ^{1),2)} of the kind

$$A+B \rightarrow n_+ \pi^+ + n_- \pi^- + n_0 \pi^0 + \text{(anything without pions)} \quad (1)$$

where n_+ , n_- and n_0 denote the number of produced π^+ , π^- and π^0 , respectively. A first step to study these processes is to look at the inclusive cross-sections, where only one pion with momentum p_1 and isospin $I_1^{(3)}$ is measured

$$\frac{d^3 \sigma(I_1^{(3)}, I_A^{(3)}, I_B^{(3)})}{d^3 \vec{p}_1} = \sum_n \frac{d^3 \sigma^{(n)}(I_1^{(3)}, I_A^{(3)}, I_B^{(3)})}{d^3 \vec{p}_1} \quad (2)$$

$$2 p_{10} (2\pi)^4 \frac{d^3 \sigma^{(n)}(I_1^{(3)}, I_A^{(3)}, I_B^{(3)})}{d^3 \vec{p}_1} = \frac{1}{(n-1)!} \frac{1}{2} \frac{(2\pi)^4}{\lambda(m_A^2, m_B^2, s)} \sum_{I_2^{(3)} \dots I_n^{(3)}} \sum_i \int d p_2 \dots d p_n \dots \quad (3)$$

$$\cdot \delta(p_1 + \dots + p_n + \dots - p_A - p_B) |\langle p_1 I_1^{(3)} \dots p_n I_n^{(3)} \dots | T | p_A I_A^{(3)} p_B I_B^{(3)} \rangle|^2$$

By Eq. (3) we define the inclusive cross-section $A + B \rightarrow \pi^{I_1}(p_1) + \text{(anything with } (n-1) \text{ pions)}$. Furthermore, we have used in Eq. (3) the abbreviations

$$d p_i = \frac{d^3 p_i}{(2\pi)^4 2 p_{i0}}$$

and

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$$

The $I_1^{(3)}$, $I_A^{(3)}$, $I_B^{(3)}$ are the three components of the isospins of the outgoing pions and incoming particles A and B. The dots stand for the momenta and quantum numbers of all outgoing particles, except for the pions. We sum over all these momenta and quantum numbers, which is indicated by the summation symbol in Eq. (3). The average π^+ , π^- and π^0 multiplicities $\langle n_{I_1^{(3)}}^{(3)} \rangle_{AB}$ ($I_1^{(3)} = 1, -1, 0$) are related to the fluxes :

$$N(p_1 I_1^{(3)}; I_A^{(3)}, I_B^{(3)}) = \frac{2 p_{10} (2\pi)^4}{\sigma_{\text{tot}}(AB)} \frac{d^3 \sigma(I_1^{(3)}, I_A^{(3)}, I_B^{(3)})}{d^3 \vec{p}_1} \quad (4)$$

by

$$\langle n_{I_1^{(3)}} \rangle_{AB} = \int d p_1 N(p_1 I_1^{(3)}, I_A^{(3)} I_B^{(3)}) \quad (5)$$

There exist some theoretical predictions for multiplicities ; e.g., in the framework of the multiperipheral model ³⁾ the multiplicities increase slowly with increasing energy. In particular, all pion multiplicities $\langle n_+ \rangle$, $\langle n_- \rangle$ and $\langle n_0 \rangle$ become equal if the total energy tends to infinity. However, as long as we have not yet reached this asymptotic region, there may be differences between the pion multiplicities, which have been really observed ^{4),5)}, even for large incident energies. Especially, it might be interesting to have bounds on the π^0 multiplicities (fluxes) in terms of the $\pi^\pm$ multiplicities (fluxes). These bounds may be useful for the analysis of a recent ISR experiment ⁶⁾ which has not yet been finished, where the inclusive cross-section

$$p + p \rightarrow \gamma + (\text{anything})$$

is measured. The photons with high energy $E_\gamma \gtrsim 100$ MeV are predominantly produced via π^0 decays. Thus, it is possible to reconstruct the π^0 spectrum from the γ spectrum.

It is the purpose of this paper to derive bounds on the π^0 multiplicities (fluxes) in terms of the $\pi^\pm$ multiplicities (fluxes) which follow from isospin invariance alone. Of course, these bounds are model independent and hold for any energy.

For the derivation of the multiplicity bounds, we make an isospin analysis of the T matrix elements, which occur in the definition of the cross-sections (3)

$$\begin{aligned} [I_1^{(3)}; I_A^{(3)} I_B^{(3)}]^{(n)} &= 2 p_{10} (2\pi)^4 \frac{d^3 \sigma^{(n)}(I_1^{(3)}, I_A^{(3)} I_B^{(3)})}{d^3 \vec{p}_1} = \\ &= \frac{1}{2(n-1)!} \frac{(2\pi)^4}{\lambda(m_A^2, m_B^2, s)} \sum_{\tilde{I}, \tilde{\eta}} \int d p_2 \dots d p_n \dots \delta(p_1 + \dots + p_n + \dots - p_A - p_B) \cdot \\ &\quad \left| \sum_{I=|I_A-I_B|}^{I_A+I_B} (1 I_1^{(3)}, \tilde{I} I_A^{(3)} + I_B^{(3)} - I_1^{(3)}) (I I_A^{(3)} + I_B^{(3)}) (I_A I_A^{(3)}, I_B I_B^{(3)}) (I I_A^{(3)} + I_B^{(3)}) \right|. \end{aligned} \quad (6)$$

$$\langle p_1 \dots p_n \dots I \tilde{I} \tilde{\eta} | T | p_A p_B, I I_A I_B \rangle|^2$$

Here, we have introduced a new basis $|\tilde{I}, \tilde{I}^{(3)}, \tilde{\eta}\rangle$ for the isospins of all outgoing particles except for the first pion. This basis is characterized by a total isotopic spin $\tilde{I}$, its three component $\tilde{I}^{(3)}$ and some degeneration parameters $\tilde{\eta}$. Next, we have coupled the isospins I_A and I_B of the incoming particles to a total isospin I . The same was done with the isospin of pion 1 and the total isospin $\tilde{I}$ of the rest of the final state. The point is that we can sum over the total isospin $\tilde{I}$ of the rest in an incoherent way. Isospin invariance of the T matrix only allows three values of $\tilde{I}$:

$$\tilde{I} = |I-1|, I, I+1$$

The $3(2I_A + 1) \cdot (2I_B + 1)$ cross-sections (6) are trivially related by charge symmetry.

$$[I_1^{(3)}; I_A^{(3)} I_B^{(3)}]^{(n)} = [-I_1^{(3)}; -I_A^{(3)} -I_B^{(3)}]^{(n)} \quad (7)$$

First we discuss reaction (1) where A and B are two isospin $1/2$ particles. From Eq. (6) we find that the six cross-sections

$$[I_1^{(3)}; \frac{1}{2} \frac{1}{2}]^{(n)}, [I_1^{(3)}; \frac{1}{2} -\frac{1}{2}]^{(n)} \quad I_1^{(3)} = -1, 0, 1 \quad (8)$$

can be expressed by five independent quantities

$$(101)^{(n)}, (111)^{(n)}, (121)^{(n)}, (010)^{(n)}, (011)^{(n)} \quad (9)$$

where

$$(\bar{I} \tilde{I} I)^{(n)} = \frac{1}{2^{(n-1)}!} \frac{(2\pi)^4}{\lambda(m_A^2, m_B^2, s)} \sum_{\tilde{\eta}} \int d p_2 \dots d p_n \dots \delta(p_1 + \dots + p_n + \dots - p_A - p_B) \quad (10)$$

$$\cdot \text{Re}(\langle p_1 \dots p_n \dots \bar{I} \tilde{I} \tilde{\eta} | T | p_A p_B \bar{I} I_A I_B \rangle \langle p_1 \dots p_n \dots I \tilde{I} \tilde{\eta} | T | p_A p_B I I_A I_B^* \rangle)$$

Thus, we have one relation, which reads for the multiplicities (5) [and fluxes (4)]

$$\frac{(\frac{1}{2} \langle n_+ + n_- \rangle - \langle n_0 \rangle)_{AB}}{(\frac{1}{2} \langle n_+ + n_- \rangle - \langle n_0 \rangle)_{A'B'}} = - \frac{\sigma_{\pi^0\pi^0}(A'B')}{\sigma_{\pi^0\pi^0}(AB)} \quad (11)$$

where

$$\begin{pmatrix} I_A^{(3)} & I_B^{(3)} & I_{A'}^{(3)} & I_{B'}^{(3)} \end{pmatrix} = \pm \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

Equation (11) relates the multiplicities of different processes of the kind (1). It holds especially for

$$(AB A'B') = (pp \, p\bar{n}), (\bar{p}n \, \bar{p}p), (pK^+ pK^0), (p\bar{K}^0 pK^-) \quad (12)$$

From the positivity of the "irreducible" cross-sections [cf. Eq. (10)]

$$(111)^{(n)}, (101)^{(n)}, (010)^{(n)}$$

one derives immediately the inequalities which hold for multiplicities (5) and fluxes (4) :

$$\frac{1}{2} \langle n_- \rangle_{AB} \leq \langle n_0 \rangle_{AB} \leq \langle n_+ \rangle_{AB} + \frac{1}{3} \langle n_- \rangle_{AB} \quad (13a)$$

where

$$\begin{pmatrix} I_A^{(3)} & I_B^{(3)} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad (\text{e.g. } (AB) = (pp), (pK^+))$$

$$\frac{1}{2} \langle n_+ \rangle_{AB} \leq \langle n_0 \rangle_{AB} \leq \langle n_- \rangle_{AB} + \frac{1}{3} \langle n_+ \rangle_{AB} \quad (13b)$$

where

$$\begin{pmatrix} I_A^{(3)} & I_B^{(3)} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \end{pmatrix} \quad (\text{e.g. } (AB) = (\bar{p}n))$$

$$\langle n_0 \rangle_{AB} \leq \langle n_+ + n_- \rangle_{A'B'} \frac{\sigma_{\text{tot}}(A'B')}{\sigma_{\text{tot}}(AB)} \quad (14a)$$

where

$$\begin{pmatrix} I_A^{(3)} & I_B^{(3)} & I_{A'}^{(3)} & I_{B'}^{(3)} \end{pmatrix} = \pm \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

The last configuration of isospins are realized again for the processes listed in Eq. (12). Combining Eq. (11) with the inequalities (13a) or (13b) we obtain upper and lower bounds for $\langle n_0 \rangle_{A'B'}$ in terms of the charged multiplicities (fluxes) :

$$\begin{aligned} \frac{1}{2} \langle n_+ + n_- \rangle_{A'B'} + \left(\frac{1}{6} \langle n_- \rangle - \frac{1}{2} \langle n_+ \rangle \right)_{AB} \frac{\sigma_{tot}(AB)}{\sigma_{tot}(A'B')} &\leq \\ \leq \langle n_0 \rangle_{A'B'} &\leq \frac{1}{2} \langle n_+ + n_- \rangle_{A'B'} + \frac{1}{2} \langle n_+ \rangle_{AB} \frac{\sigma_{tot}(AB)}{\sigma_{tot}(A'B')} \end{aligned} \quad (14b)$$

where

$$(ABA'B') = (ppp\pi), (pK^+pK^0), (p\bar{K}^0pK^-)$$

For $(ABA'B') = (\bar{p}n\bar{p})$ the positive and negative charged multiplicities $\langle n_+ \rangle$ and $\langle n_- \rangle$ have to be interchanged in Eq. (14b). Next we consider the case where A and B are isospin 1 and 1/2 particles respectively. Equation (6) provides for $I_A^{(3)} = 1$ and $I_B^{(3)} = 1/2$ an expansion of the three cross-sections $[I_1^{(3)}, 11/2]^{(n)}$ ($I_1^{(3)} = 0, \pm 1$) into three irreducible cross-sections :

$$\left(\frac{3}{2} \quad \tilde{I} \quad \frac{3}{2} \right)^{(n)} \quad \tilde{I} = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \quad (15)$$

The positivity of the irreducible cross-sections leads again to bounds for the multiplicities and fluxes which read in the π^+p case

$$\frac{2}{5} \langle n_- \rangle_{\pi^+p} \leq \langle n_0 \rangle_{\pi^+p} \leq 1.5 \langle n_+ \rangle_{\pi^+p} + 0.25 \langle n_- \rangle_{\pi^+p} \quad (16)$$

The expansion (6) of the cross-sections $[I_1^{(3)}, -11/2]^{(n)}$ contains again the three irreducible cross-sections (15) and in addition :

$$\left(\frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \right)^{(n)}, \left(\frac{1}{2} \quad \frac{3}{2} \quad \frac{1}{2} \right)^{(n)}, \left(\frac{3}{2} \quad \frac{1}{2} \quad \frac{1}{2} \right)^{(n)}, \left(\frac{3}{2} \quad \frac{3}{2} \quad \frac{1}{2} \right)^{(n)} \quad (17)$$

In order to evaluate this expansion, we consider the combination $(\alpha_1, \alpha_2$ arbitrary real numbers)

$$\begin{aligned}
 & [-1; -1 \frac{1}{2}]^{(n)} + \alpha_1 [0; -1 \frac{1}{2}]^{(n)} + \alpha_2 [1; -1 \frac{1}{2}]^{(n)} = \\
 & = C_1(\alpha_1) [1; 1 \frac{1}{2}]^{(n)} + C_0(\alpha_1, \alpha_2) [0; 1 \frac{1}{2}]^{(n)} + C_{-1}(\alpha_1, \alpha_2) [-1; 1 \frac{1}{2}]^{(n)} \\
 & + \frac{2}{9} (2 + \alpha_1) \left\{ \left(\frac{1}{2} \frac{1}{2} \frac{1}{2} \right)^{(n)} + 2a(\alpha_1) \left(\frac{3}{2} \frac{1}{2} \frac{1}{2} \right)^{(n)} + a(\alpha_1)^2 \left(\frac{3}{2} \frac{1}{2} \frac{3}{2} \right)^{(n)} \right\} \quad (18) \\
 & + \frac{1}{9} (1 + 2\alpha_1 + 3\alpha_2) \left\{ \left(\frac{1}{2} \frac{3}{2} \frac{1}{2} \right)^{(n)} + 2b(\alpha_1, \alpha_2) \left(\frac{1}{2} \frac{3}{2} \frac{3}{2} \right)^{(n)} + b(\alpha_1, \alpha_2)^2 \left(\frac{3}{2} \frac{3}{2} \frac{3}{2} \right)^{(n)} \right\}
 \end{aligned}$$

where

$$C_1(\alpha_1) = \frac{1}{9} \frac{11\alpha_1 + 4}{2 + \alpha_1} \quad (19)$$

$$C_0(\alpha_1, \alpha_2) = -\frac{2}{27} \frac{11\alpha_1 + 4}{2 + \alpha_1} + \frac{1}{3} \frac{\alpha_1 + 6\alpha_2 + 3\alpha_1\alpha_2}{1 + 2\alpha_1 + 3\alpha_2}$$

$$C_{-1}(\alpha_1, \alpha_2) = \frac{1}{10} (1 + 2\alpha_1 + 2\alpha_2) - \frac{2}{15} \frac{\alpha_1 + 6\alpha_2 + 3\alpha_1\alpha_2}{1 + 2\alpha_1 + 3\alpha_2} + \frac{1}{54} \frac{11\alpha_1 + 4}{2 + \alpha_1}$$

$$a(\alpha_1) = -\frac{1 - \alpha_1}{2 + \alpha_1} \quad (20)$$

$$b(\alpha_1, \alpha_2) = \frac{2}{\sqrt{10}} \frac{2 + \alpha_1 - 3\alpha_2}{1 + 2\alpha_1 + 3\alpha_2}$$

In order to derive Eq. (18) we have substituted the irreducible cross-sections $(3/2 \tilde{\Gamma} 3/2)^{(n)}$ by the π^+ cross-sections $[\bar{1}, 1 1/2]^{(n)}$ [cf. Eq. (6)]. The quantities $(1/2 \tilde{\Gamma} \bar{1})^{(n)}$ which cannot be expressed uniquely in terms of the cross-sections (6) are arranged in the last two expressions of Eq. (18), in such a way that they build non-negative forms of the kind

$$(\bar{1} \tilde{\Gamma} \bar{1})^{(n)} + 2a(I \tilde{\Gamma} \bar{1})^{(n)} + a^2(I \bar{1} I)^{(n)} \geq 0 \quad (21)$$

[cf. Eq. (10)]. Again, this positivity property enables us to derive inequalities for the pion multiplicities and fluxes in the π^-p scattering

$$\left(\langle n_- \rangle + \alpha_1 \langle n_0 \rangle + \alpha_2 \langle n_+ \rangle \right) \frac{\sigma_{tot}(\pi^-p)}{\sigma_{tot}(\pi^+p)} \geq \quad (22)$$

$$\left(C_1(\alpha_1) \langle n_+ \rangle + C_0(\alpha_1, \alpha_2) \langle n_0 \rangle + C_{-1}(\alpha_1, \alpha_2) \langle n_- \rangle \right)_{\pi^+p}$$

where the $>$ and $<$ signs stand for the cases

$$2 + \alpha_1 \geq 0 \quad 1 + 2\alpha_1 + 3\alpha_2 \geq 0 \quad (23a)$$

and

$$2 + \alpha_1 \leq 0 \quad 1 + 2\alpha_1 + 3\alpha_2 \leq 0 \quad (23b)$$

respectively.

The bounds in Eq. (16) can be obtained from Eq. (22) performing the limits $2 + \alpha_1 \rightarrow 0$ and $1 + 2\alpha_1 + 3\alpha_2 \rightarrow 0$ ($\alpha_1 + 6\alpha_2 + 3\alpha_1\alpha_2 \neq 0$), respectively. Next we can consider the limits

$$1 + 2\alpha_1 + 3\alpha_2 \rightarrow 0 \quad \alpha_1 + 6\alpha_2 + 3\alpha_1\alpha_2 \rightarrow 0$$

i.e.,

$$\alpha_1 \rightarrow -1 \quad \alpha_2 \rightarrow \frac{1}{3}$$

As long as we perform this limit from all directions in the α_1, α_2 plane except on the curves

$$1 + 2\alpha_1 + 3\alpha_2 = 0 \quad \text{or} \quad \alpha_1 + 6\alpha_2 + 3\alpha_1\alpha_2 = 0$$

we get for the ratio

$$\lim_{\substack{\alpha_1 \rightarrow -1 \\ \alpha_2 \rightarrow \frac{1}{3}}} \frac{1}{3} \frac{\alpha_1 + 6\alpha_2 + 3\alpha_1\alpha_2}{1 + 2\alpha_1 + 3\alpha_2} = \frac{1}{3}$$

The inequality (22) reads in this limit

$$\begin{aligned} & \left(\langle n_- \rangle + \frac{1}{3} \langle n_+ \rangle - \langle n_0 \rangle \right)_{\pi^- p} \frac{\sigma_{tot}(\pi^- p)}{\sigma_{tot}(\pi^+ p)} \\ & \geq \left(-\frac{7}{9} \langle n_+ \rangle - \frac{8}{27} \langle n_- \rangle + \frac{23}{27} \langle n_0 \rangle \right)_{\pi^+ p} \end{aligned} \quad (24)$$

The inequality (24) represents an upper bound for $\langle n_0 \rangle_{\pi^- p}$ if we use for $\langle n_0 \rangle_{\pi^+ p}$ the lower bound given in Eq. (16) :

$$\langle n_0 \rangle_{\pi^- p} \leq \left(\langle n_- \rangle + \frac{1}{3} \langle n_+ \rangle \right)_{\pi^- p} + \left(\frac{7}{9} \langle n_+ \rangle - \frac{1}{15} \langle n_- \rangle \right)_{\pi^+ p} \frac{\sigma_{tot}(\pi^+ p)}{\sigma_{tot}(\pi^- p)} \quad (25)$$

A lower bound on $\langle n_0 \rangle_{\pi^- p}$ follows immediately from the inequality (22), if we take the limit $\alpha_1 \rightarrow \infty$:

$$\langle n_0 \rangle_{\pi^- p} \geq \frac{1}{5} \langle n_- \rangle_{\pi^+ p} \frac{\sigma_{tot}(\pi^+ p)}{\sigma_{tot}(\pi^- p)} \quad (26)$$

The limit $\alpha_2 \rightarrow \infty$ provides

$$\langle n_+ \rangle_{\pi^- p} \geq \frac{1}{5} \langle n_- \rangle_{\pi^+ p} \frac{\sigma_{tot}(\pi^+ p)}{\sigma_{tot}(\pi^- p)} \quad (27)$$

In general, the inequality (22) can be understood as a lower and an upper bound for $\langle n_0 \rangle_{\pi^- p}$:

$$L(\alpha_1, \alpha_2 \langle n_{\pm} \rangle_{\pi^- p}, \langle n_0 \rangle_{\pi^+ p}) \leq \langle n_0 \rangle_{\pi^- p} \leq U(\alpha'_1, \alpha'_2 \langle n_{\pm} \rangle_{\pi^- p}, \langle n_0 \rangle_{\pi^+ p}) \quad (28)$$

if the multiplicities (fluxes) $\langle n_{\pm} \rangle_{\pi^- p}$, $\langle n_{\pm} \rangle_{\pi^+ p}$ are known. In order to find the best lower and upper bounds, one has to look for the largest value of L and the lowest value of U in the corresponding domains (23a) and (23b) of the α_1, α_2 plane. Indeed, it turns out that (24) and (25) represent the best bounds one can derive from isospin invariance alone, if the five average multiplicities $\langle n_{\pm} \rangle_{\pi^- p}$, $\langle n_{\pm} \rangle_{\pi^+ p}$ are equal, which is at least plausible for large energies. Finally, we would like to mention the cases $(I_A, I_B) = (0, 1/2)$ and $(I_A, I_B) = (0, 0)$ which, of course, cannot be realized experimentally. But they are interesting for some models. If one describes the inelastic proton proton scattering in the diffraction dissociation

region ⁷⁾ by the diagram in Fig. 1 and in the central region ⁸⁾ by the diagram in Fig. 2, the processes

$$Pp, PP \rightarrow n \text{ pions} + (\text{anything without pions})$$

can be analyzed as described above. Equation (6) yields now for the multiplicities and fluxes

$$\langle n_+ + n_- \rangle_{PP} = 2 \langle n_0 \rangle_{PP} \quad (29)$$

$$\langle n_+ \rangle_{PP} = \langle n_- \rangle_{PP} = \langle n_0 \rangle_{PP} \quad (30)$$

The last equality has been found already by Pais ⁹⁾, assuming that the final states consist of pions alone. This assumption does not enter into our approach.

We would like to emphasize that Eq. (29) [and correspondingly (30)] imply, of course, much stronger restrictions on the π multiplicities in pp scattering than the simple isospin bounds [cf. Eq. (13)]. Measurements of the charged and neutral pion multiplicities and fluxes, therefore, decide about the validity of the diffraction dissociation model with simple Pomeron exchange. The experimental values ⁵⁾ for the pion fluxes in pp scattering are compatible with Eq. (29).

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FIGURE CAPTIONS

Figure 1 : Diagram for diffraction dissociation.

Figure 2 : Diagram for diffractive formation in the central region.

FIG. 1

FIG. 2