Archives

CRUCIAL TESTS OF THE VENEZIANO MODEL
FROM DIP BEHAVIOUR AT LOW ENERGY

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A B S T R A C T

A phenomenological exploration of the low energy behaviour of dips in some hadronic two-body processes (πp , Kp , $\bar{p}p$ scatterings and Λ production) is presented. It is found that such behaviour is in most cases unexpectedly simple, and that in several cases dips and resonances describe simple patterns as the bare Veneziano formula suggests. Phase shifts are systematically used, and prove to be of critical importance to get an understanding of the low energy dip structure. The results of the study support physical significance of the Veneziano formula and contribute to clarify its phenomenological role. The view that the formula already contains important contributions from unitarity appears to be encouraged. On the other hand, the apparent limits of applicability of the formula seem to deflate the problem of avoiding parity doublets in the theory.

1. INTRODUCTION

A characteristic feature of the Veneziano formula ¹⁾

$$\frac{\Gamma(1-\alpha(t)) \Gamma(1-\alpha(s))}{\Gamma(2-\alpha(s)-\alpha(t))} \quad (1)$$

is to exhibit a family of real zeros at fixed values of the variable u , with positions corresponding to the intersections of s and t channel resonances (Fig. 1). Conventional phenomenological applications of the formula are usually concerned with obtaining quantitative fits to data ²⁾. Since several beta function terms are commonly employed in this aim, the role of these zeros remains marginal in such applications. A relevant, but isolated, exception is the interpretation just in terms of these zeros of the hole structure exhibited by $\bar{p}n \rightarrow \pi^- \pi^- \pi^+$ Dalitz plot ^{3),4)} *).

One has to realize that the existence and the behaviour of the zeros are not an accidental feature of the formula but that they are at a considerable extent implied by the main assumptions of a Regge recurrent structure for the resonances and of narrow width limit. In fact, a zero must always exist near the intersection of two resonances, in order to avoid double poles and consequent infinite values for the angular momenta of the two resonances. The amplitude being exactly real, because of the assumptions, these zeros cannot propagate into the complex plane but are constrained to move in the real plane of s , t variables. That makes intersections of zeros with resonances extremely probable. When that occurs the zeros correspondingly induced in the resonance residue cannot be arbitrary but have to be consistent with the angular momentum of the resonance. All these restrictions, though stringent, are not

*) The original proposal of Lovelace ⁵⁾ to understand that structure in terms of the Veneziano formula must be somewhat changed, however, especially in view of the features exhibited by the recently appeared data in flight ⁴⁾. Another structure for the zeros, still of the Veneziano type, seems to be at work [see Ref. 6) and Section 4].

sufficient to determine completely the behaviour of the zeros (think for instance of the possible addition of Veneziano satellites). What the Veneziano formula specifically adds to all that is to assume straight-line behaviour for the zeros.

The existence of links between duality ⁷⁾ and straight-line behaviour of zeros can be realized in a rather direct way. The zero at the intersection of two resonances has an orientation in the Mandelstam plane related to the ratio of the resonance residues at that point. Near the intersection one can write for the amplitude $A(s, t)$

$$\begin{aligned} (s - m_a^2)(t - m_b^2) A(s, t) = \\ = b(s - m_a^2) + a(t - m_b^2) + O(s - m_a^2, t - m_b^2) \end{aligned} \quad (2)$$

a , b being the residues at the intersection. Locally, then, the zero runs along the line $b(s - m_a^2) + a(t - m_b^2) = 0$. Now, a way to interpret local duality is to require that a and b have to stay in some fixed relationship. If, to fix the ideas, one requires this relationship to be just equality, then all the above local zeros must be oriented systematically at fixed u . It is then natural to expect that, at least in the $s, t \geq 0$ region where resonance intersections occur, all these zeros be embedded in a global system of straight-line behaved zeros at fixed u .

Some empirical observations seem to encourage the hope that, experimentally, dips corresponding to amplitude zeros could have a simple behaviour in the s, t plane regardless of the conventional separation between high energy, low energy and unphysical regions. We recall the remark by Dolen et al. ⁷⁾ about the approximate coincidence, in $\pi^- p \rightarrow \pi^0 n$, of the positions in t of high energy zeros and of low energy resonance minima. Particularly relevant is Booth ⁸⁾ observation concerning the presence in the 180° energy distribution of $\pi^\pm p$ elastic scattering of a pronounced minimum where the $t \simeq -2.8(\text{GeV}/c)^2$ dip leaves the physical region.

This fact could indeed suggest survival of the dip in the unphysical region. A similar empirical observation for $\bar{p}p \rightarrow \bar{p}p$ dips has been made by Yoh et al.⁹⁾.

Besides the intrinsic interest of possibly finding simple dip regularities at low energy, exploring in this direction opens a way to provide some badly needed experimental tests for the Veneziano model. Because of the limitations of the model, indeed, extra assumptions are usually necessary in order to have quantitative comparison with data. This difficulty can be circumvented by considering its qualitative predictions, as for instance the behaviour of amplitude zeros and their interplay with resonances.

This article deals with a phenomenological study of the low energy behaviour of dips in some hadronic two-body processes (πp , Kp , $\bar{p}p$ scatterings and Λ production)^{*)}. The choice of the processes has been mainly dictated by the relative abundance of data. Phase shifts have been extensively used, this source of information proving to be decisive to put in evidence certain Veneziano features of low energy amplitudes. The results of this study provide evidence supporting phenomenological relevance for the Veneziano formula, and contribute to clarify its role in the phenomenology of two-body processes. It appears that the formula should be interpreted as describing a particular possible pattern of real and straight-line behaved zeros for physical amplitudes not containing diffraction, alternative patterns being also possible and physically realized (e.g., in Λ production). Concerning the unitarization program, this implies in particular that the formula already contains large contributions from unitarity¹¹⁾. The formula seems to have little to say about helicity non-flip amplitudes, which fact reflects negatively on the physical relevance of constructing Veneziano-like models avoiding parity doublets.

*) Preliminary accounts on part of the study are contained in Ref. 10).

Sections 2 to 5 contain a phenomenological inspection of the low energy region of πp , Kp , $\bar{p}p$ scatterings and Λ production. In order to have only an idea of the main results one can skip these sections looking only at the figures and the captions. Sections 6 and 7 contain the discussion and the conclusions.

2. K^-p SCATTERING

K^-p scattering (elastic and charge exchange) is a particularly significant process for the Veneziano model. The absence (or at least the smallness) of resonance bumps in K^+N channel, makes indeed the resonance structure of this process to look very similar to that described by the bare Veneziano formula. One would correspondingly expect the appearance of dips at fixed u in the backward region of K^-p scattering (Fig. 2).

In $K^-p \rightarrow K^-p$, one observes at high energy some clear structure at fixed t . This structure persists smoothly when going to low energy, until the boundary of the physical region is met (Fig. 3). Of course, such a structure has nothing to do with that suggested by the Veneziano formula. Rather, a dominant role in it seems to be played by diffraction. Since this mechanism appears to conserve nucleon helicity at high energy [see Gilman et al. ¹²⁾ and references therein, and Ref. 13) for further evidence], one can hope to circumvent its disturbing effects looking to helicity flip. The invariant amplitude $A(s,t)$ represents s channel helicity flip at high energy. If high and low energy are connected by the analyticity properties of invariant amplitudes, one may expect that this amplitude remains free of diffraction also at low energy. In Fig. 4 one can look at the low energy behaviour of the amplitude A , calculated from phase shifts ¹⁴⁾. One can remark the absence there of any structure at fixed t , and the presence of some structure at fixed u . In particular a dip at $u \simeq -0.3(\text{GeV}/c)^2$ ($u' = u - u_{\min} \simeq -0.4$ in the considered momentum interval) is visible until it leaves the physical region through the forward direction.

At higher energies (1.4-2.4 GeV/c) structure at this value of u is also suggested by the data of Daum et al.¹⁵⁾ (differential cross-section and polarization).

A more impressive evidence is provided by $K^-p \rightarrow \bar{K}^0 n$ data. In the interval 0.45-1.8 GeV/c this process is rather well known experimentally. Plotting angular distributions against t and u (Fig. 5) one easily recognizes that $K^-p \rightarrow \bar{K}^0 n$ differential cross-section exhibits three dips at remarkably constant values of $u \simeq -0.1$, $u \simeq -0.7$ and $u \simeq -1.7(\text{GeV}/c)^2$. One also notices at ~ 0.850 and ~ 1.400 GeV/c, i.e., where $u \simeq -0.7$ and $u \simeq -1.7$ respectively leave the physical region, a hook structure to be present near the forward direction, which fact can be interpreted as indicating survival of the dips in the unphysical region.

A quantitative association of dip positions to resonance intersections is not much significant in K^-p scattering, since besides ($\Lambda_\alpha, \Lambda_\gamma$) resonances also ($\Sigma_\delta, \Sigma_\beta$) resonances are important¹⁶⁾, and the two sets of resonances are somewhat shifted in M^2 ($\sim 0.2-0.4 \text{ GeV}^2$). On the other hand it is worth stressing that the prediction of having $u \leq 0$ dips with $\sim 1 (\text{GeV}/c)^2$ spacing is characteristic of the Veneziano model in this process, since conventional high energy mechanisms have little to say about the angular structure of exotic exchanges.

3. πp SCATTERING

At high energy $\pi^\pm p$ elastic scattering exhibits prominent structure at $t \simeq -0.6$, $t \simeq -2.8 (\text{GeV}/c)^2$, and at $u \simeq -0.2 (\text{GeV}/c)^2$ in $\pi^+ p$ channel. From the compilation of data¹⁷⁾ in Fig. 6 one can follow such dips down to momenta $\sim 2.5 \text{ GeV}/c$.

The resonance spectrum appearing in $\pi^+ p \rightarrow \pi^+ p$ at low energy is particularly simple (at least as far as real resonance bumps are concerned) with a regular spacing $\sim 2 \text{ GeV}^2$ in M^2 (Fig. 7e).

As one can see from Fig. 7, also the low energy dip structure is a rather simple one. Like what was observed by Booth ⁸⁾ concerning the $t \simeq -2.8$ dip, also the $t \simeq -0.6$ dip travels down (Fig. 7b) until it meets the backward direction there producing a marked dip in the 180° energy distribution (Fig. 7d). The dip at $u \simeq -0.2$, instead, persists down to ~ 1 GeV/c, where it meets the $t \simeq -0.6$ dip (some effect in this structure near 2 GeV/c, corresponding to the crossing of the $t \simeq -2.8$ dip, is also observed) (Fig. 7c). That is similar to the disappearance at low energy of the $u \simeq -0.3$ dip in $K^-p \rightarrow K^-p$ differential cross-section. Like there, we can look at the invariant amplitude A , where diffraction is expected to be absent, and see that the $u \simeq -0.2$ dip, like the $t \simeq -0.6$ dip, persists in this amplitude until leaving the physical region ^{*)} (Fig. 8). For a more quantitative inspection, we have also plotted the real and the imaginary parts of this amplitude. The plots are at fixed t and u , rather than at fixed energy, so as to single out u and t structure respectively ^{**) .}

In $\pi^-p \rightarrow \pi^-p$ scattering, as it is known, the resonance spectrum is somewhat more complicated. Besides the prominent $N(1680)$ bump, which is $\sim 2 \text{ GeV}^2$ away from the nucleon, one observes other bumps corresponding to $\Delta(1236)$ and $N(1520)$ (Fig. 9a). The low energy dip structure also is not a very simple one. The $t \simeq -0.6$ and $t \simeq -2.8$ dips behave more or less regularly like in π^+p scattering, though what occurs to the $t \simeq -0.6$ dip when it approaches the backward direction (Figs. 9b and 9c) is not clear. In the amplitude A the behaviour of this dip near 180° is clean, but some spoiling near 1 GeV/c is present (Fig. 9d). As to the u structure it has no simple relation here with that observed at high energy.

Of course, it would be extremely interesting to see what happens to π^+p and π^-p fixed u dips in the low energy region of $\bar{p}p \rightarrow \pi^- \pi^+$. Unfortunately, presently available data do not allow to say much about that.

^{*)} The phase shifts of Ref. 18) are used (CERN Theoretical solution).

^{**) .} Actually, the $u \simeq -0.2$ dip shifts to $u \simeq -0.4, -0.5$. That is, however, not much relevant at the level of these considerations.

In Fig. 10 the dips previously considered are traced in the Mandelstam plane, together with the prominent resonance bumps observed in π^+p [$\Delta(1236)$, $\Delta(1950)$, $\Delta(2420)$] and in π^-p [$N(938)$, $N(1680)$] scatterings (see Figs. 7e and 9a). Also traced are the $\rho(765)$ and $g(1670)$ mesons in t channel. One can see that dips have positions corresponding to resonance intersections, just as the Veneziano formula suggests. We stress again that this fact is directly connected to (approximate) straight-line behaviour of dips in the real Mandelstam plane. Consider for instance the $t \simeq -0.6$ dip in the invariant amplitude $B(s,t)$ (though, as we shall see now, this amplitude does not have a complete Veneziano structure). When this dip, supposed to be propagating in the unphysical region, crosses the nucleon pole, it should produce a zero in the corresponding residue. This is not allowed, since the nucleon, having $J^P = \frac{1}{2}^+$, must have a constant residue in this amplitude. The $\Delta(1236)$, passing through the intersection removes the unwanted zero, thus allowing a regular propagation of the dip ^{*)}.

It is important, in order to understand the content of Fig. 10, to recall that while the ρ meson has a negligible coupling to s channel helicity non-flip [associated to the combination $A + (s-u)/(4M^2)B$], the f^0 meson almost decouples from s channel helicity flip (associated to the amplitude A) ^{13),19)}. The dips traced in Fig. 11 can be quite naturally associated to baryon-baryon intersections and to intersections of baryons with ρ . But there is no trace of backward dips associated to intersections of baryons with f^0 . Therefore, while the evidence presented in Fig. 11 suggests a Veneziano structure for the amplitude A , such a structure does not seem to be shared by the amplitude B . This does not exclude that some episodic Veneziano features be present also in B [as we have seen in the case of nucleon- $\Delta(1236)$ intersection] ^{**))}.

^{*)} In the amplitude A' of $K^-p \rightarrow K^-p$ nothing of this sort can happen when the $t \simeq -0.5$ dip crosses the $\Lambda(1115)$ pole. The dip has then likely to shift into the complex plane.

^{**))} One can remark that in the amplitude $A + (s-u)/(4M^2)B$ (where the ρ meson is negligible) f^0 , N_α and Δ_8 cannot combine into a Virasoro ²⁰⁾ structure since the product of their signature is odd. The presence of the π^+p $u \simeq -0.2$ dip also in the amplitude B is discussed in Section 6.

If fixed t dips really propagate onto the physical region and cross there baryon-baryon intersections, then the corresponding resonance residues should be equal at the intersection points (see Section 1). Using the information extracted from phase shifts one can evaluate and compare the relevant quantities. These are proportional to $M_R \Gamma_T x_R = M_R \Gamma_{\pi N}$, where M_R is the mass of the resonance, Γ_T its total width, x_R its elasticity, $\Gamma_{\pi N}$ its partial width for πN channel. Apart from the $\Delta(1236)$ resonance, typical errors on x_R are $\sim 20\%$, and on Γ_T $\sim 50\%$ or more (see for instance the compilation of the Particle Data Group ²¹). In Fig. 11 one can find the comparison for the amplitude A and B. For A the agreement is reasonably good, while for B qualitative violations are observed, as one would expect. Rather surprising in this context is the exceptionally good agreement at the nucleon $\Delta(1236)$ intersection in B.

4. $K^- N \rightarrow \pi \Lambda$ and $\pi N \rightarrow K \Lambda$

The low energy behaviour of $\bar{K}N \rightarrow \pi \Lambda$ differential cross-section (Fig. 12) looks considerably different from that observed in Kp and πp scatterings. In particular its dip structure cannot be simply described in terms of fixed t and u minima. To understand what is happening in this process we pick up the suggestion of duality diagrams ²²), according to which direct channel and t channel resonances are here exchange degenerate, so as to give an imaginary part averaging to zero. Actually, hyperon resonance parameters, as determined from phase-shift analysis, seem to fit approximately into the pattern of two exchange degenerate families of (Σ_S, Σ_P) and $(\Sigma_\alpha, \Sigma_\gamma)$ resonances ^{16), 23}) *).

*) If such exchange degeneracy were exact, a vanishing polarization would be predicted at high energy, in contrast with data ²⁴).

According to this hypothesis s channel resonances should have residues of alternate signs in the $t > 0$ region, and similarly for t channel resonances at $s > 0$. Therefore, when resonances intersect, zeros cannot be always at constant u , as the Veneziano formula prescribes. It is still possible, however, to have a regular structure with straight-line behaved dips if one allows some of them to move at $s - t = \text{const.}$, which gives $a = -b$ in Eq. (2) (see the Introduction). Figure 13 represents a structure of resonances and zeros of this type ^{*}). Such a structure has also been proven ⁶⁾ to be useful in understanding the Dalitz plot of $\bar{p}n \rightarrow \pi^- \pi^- \pi^+$. It implies in particular the existence of u channel $(\pi N \rightarrow K \Lambda)$ resonances spaced by $\sim 2 \text{ GeV}^2$ (if $\sim 1 \text{ GeV}^2$ is the resonance spacing in s and t channel), and this is in agreement with what is known about the resonance spectrum in $\pi N \rightarrow K \Lambda$ ²⁵⁾. One can see that within the given assumptions (straight-line behaved zeros and duality diagrams), the structure of Fig. 13 turns out to be fairly unique. From it one predicts the existence of fixed u dips spaced by $\sim 2 \text{ GeV}^2$ and, what is most characteristic of such a structure, the presence of $s - t = \text{constant}$ dips spaced by 2 GeV^2 . These "slanting" dips, however, should not be observable at high energy, since, when taking the large width limit for higher resonances, one ends with an high energy average exhibiting fixed t and u zeros for the imaginary part, with also the real part having zeros in the same positions (because of dispersion relations) apart from exceptional values of the effective trajectory α .

Figure 14 shows evidence for the existence of fixed $s-t$ dips with spacing $\sim 2 \text{ GeV}^2$ in the intermediate energy region of $\bar{K}N \rightarrow \pi \Lambda$. Since $\bar{K}N \rightarrow \pi \Lambda$ data are rather dense in energy (about eighty energies between 0.44 and 1.9 GeV/c), we can profit of this fact plotting differential cross-sections at fixed u rather than

^{*}) It can be analytically represented by

$$A(s, t) = \frac{\Gamma(1 - \alpha_{\bar{b}os}^{(t)}) \Gamma(3/2 - \alpha_{hyf}^{(s)})}{\Gamma(5/2 - \alpha_{\bar{b}os}^{(t)} - \alpha_{hyf}^{(s)})} \frac{\sin \frac{\pi}{2} (\alpha_{\bar{b}os}^{(t)} - \alpha_{hyf}^{(s)} + 3/2)}{\sin \frac{\pi}{2} (\alpha_{\bar{b}os}^{(t)} + \alpha_{hyf}^{(s)} + 1/2)}$$

at fixed momentum. In this way, confusion due to overlapping with fixed u structure is definitely avoided. Of course, the shape of the corresponding distributions is then affected by normalization errors. However, this kind of uncertainty is under control in our case, since in almost all energy regions data from different sources are available. One can observe in this way the existence of two dips at approximately constant $s-t \simeq 3$ and 5 GeV^2 (Fig. 14). The dip at $s-t \simeq 5$ falls always in the physical region, in the considered energy range. Its constancy in $s-t$, though affected by some minor fluctuations, can be compared to its sizable and systematic variation in t . The dip at $s-t \simeq 3$, instead, can be observed to leave the physical region. It appears to be in part spoiled when approaching $t=0$ ($u \simeq -0.65$). When it crosses this point, however, one observes a pronounced depression in the forward direction, and, immediately after its leaving the physical region, the formation of a hook structure near $t=0$.

According to the scheme of Fig. 13 also in $\pi^- p \rightarrow K^0 \Lambda$ one should observe dips at fixed $s-t$. This variable is $u-t$ in conventional notations for this channel, and it will be denoted throughout by s^*-t . In Fig. 15 $\pi^- p \rightarrow K^0 \Lambda$ differential cross-section is plotted at fixed momenta versus s^*-t . A dip at constant $s^*-t \simeq 1 \text{ GeV}^2$ is well visible. Its position satisfies $\sim 2 \text{ GeV}^2$ spacing with respect to $\bar{K}N \rightarrow \pi \Lambda$ dips. One should expect a dip also at $s^*-t \simeq -1$. Something which might be interpreted as an effect near this point is observable. One gets, however, the impression that vicinity to the forward direction definitely disturbs these dips, as in part observed for $\bar{K}N \rightarrow \pi \Lambda$.

In $\bar{K}N \rightarrow \pi \Lambda$ one can plot data versus u at fixed $s-t$ in order to see fixed u dips (Fig. 16). As one can see a dip at approximately constant $u \simeq -0.2 (\text{GeV}/c)^2$ is present. This dip should be related to the analogous structure observed at higher energies ²⁶⁾.

From a glance at Fig. 17, representing the amplitude $A(s,t)$ of $\bar{K}N \rightarrow \pi \Lambda$ in the range 1.2-1.8 GeV/c^{*}), one gets the impression that in this amplitude dips have a behaviour more regular than in the differential cross-section^{**}).

According to the scheme of Fig. 13, one should also be able to relate dip positions to resonance intersections. The non-negligible presence of (Z_α, Z_γ) resonances¹⁶⁾ beside the dominant (Z_δ, Z_β) is expected, however, to perturb such relations.

5. $\bar{p}p$ ELASTIC SCATTERING

We consider $\bar{p}p \rightarrow \bar{p}p$ not to put in evidence some experimental pieces of Veneziano structure, but in order to discuss some peculiar effects suggesting that also the low energy behaviour of "diffractive" dips and discontinuities is not completely independent from the resonance structure.

It has been remarked by Yoh et al.,⁹⁾ that the high energy structures showing up in this process at $t \simeq -0.8$ and -2.5 (GeV/c)² continue regularly down to very low energies exhibiting a pronounced bending towards lower $|t|$ values. From the data represented in Figs. 18a and 18b one can realize that these dips, when leaving the physical region, have values $t \simeq 0$ and $t \simeq -0.7$ respectively. This phenomenon does not occur in πp

*) The phase shifts of Litchfield²⁷⁾ are used.

**) Phase-shifts covering also the lower energy region of $\bar{K}N \rightarrow \pi \Lambda$ are in progress and some preliminary results from them have already appeared²⁸⁾ but the numbers were not available to us. As to $\pi^- p \rightarrow K \Lambda$, because of the scarcity of experimental data, the existing phase-shifts²⁵⁾ are not reliable enough for the present considerations.

and K^-p scattering. It is tempting to interpret this fact as due to matching of the dips with the zeros generated at the intersection of meson resonances (Fig. 18d). If it is really so, then here dips would approximately describe branches of hyperbola in the Mandelstam plane. It is relevant to observe that, actually, $pp \rightarrow pp$ discontinuities do describe branches of hyperbola in such a plane. As remarked by Krisch ²⁹⁾, indeed, such discontinuities occur at approximately constant $\beta^2 p_{\perp}^2 \equiv (u \cdot t)/s$ (Fig. 18c), which means that they describe paths satisfying $u \cdot t + a(u + t - 4M^2) = 0$, where a is a real constant and M is the nucleon mass ^{*}).

In $np \rightarrow np$, on the contrary, the discontinuity at $t \simeq -0.8$ which one observes in the data (Fig. 19a), does not seem to move appreciably with the energy. According to what we have seen in the other processes, it appears natural to associate with this structure the prominent effect ³¹⁾ observed in the 180° energy distribution near 1 GeV/c (Fig. 19b) ^{**)}. We recall that in $np \rightarrow np$, differently from $pp \rightarrow pp$, besides the pion only ρ and A_2 are expected to be exchanged near the backward direction, and that these two resonances seem to be rather weakly coupled to the nucleon-nucleon vertex, as the small difference between np and pp total and differential cross-section (near forward direction) indicates ³²⁾.

6. DISCUSSION

a) From the evidence presented in the preceding sections one can make the general remark that the behaviour in the Mandelstam plane of dips and discontinuities may happen to be extremely simple, regardless of distinctions between high energy, low energy and unphysical region. This fact, when occurring, appears to be strictly

*) Practically the same conclusions are reached considering constancy in the Orear parameter ³⁰⁾ $p_{\perp}$, since in the region of interest β approaches 1.

**) I thank J. Froyland for pointing that to me.

associated to the existence of a simple resonance spectrum, with a regular spacing in mass squared. The regularity of the dip behaviour is observed to be spoiled in the measure in which such condition is not fulfilled. This can be realized, for instance, comparing $\pi^-p \rightarrow \pi^0 n$ to π^+p scattering, the first process receiving sizable contributions from N_α , Δ_8 , N_δ resonances, whilst in the second only Δ_8 prominent resonances are observed. As one can see from Fig. 20, $\pi^-p \rightarrow \pi^0 n$ exhibits at low energy a complicated dip structure contrasted to the regular behaviour of dips in low energy π^+p scattering (Fig. 7a). The general statement which can be made is, therefore, that a simple and regularly spaced resonance spectrum seems generally to induce a simple and regular dip structure.

b) Coming more specifically to Veneziano-like connections between dips and resonance intersections, one is able to observe some unambiguous evidence for them. We recall

- i) the fixed u dips with $\sim 1 \text{ GeV}^2$ spacing in K^-p scattering;
- ii) the characteristic $s-t = \text{constant}$ dips with $\sim 2 \text{ GeV}^2$ spacing in Λ production;
- iii) the quantitative coincidence in πp scattering of $t \simeq -0.6$, -2.8 and (in π^+p) $u \simeq -0.2$ dips with positions of resonance intersections;
- iv) still in πp scattering, the quantitative relations between residues of intersecting baryon resonances.

To all that one could also add the peculiar low-energy bending of $\bar{p}p \rightarrow \bar{p}p$ dips ⁹⁾.

Using the available phase-shifts one is able to realize, in πp and K^-p scattering, that only the amplitude $A(s,t)$ (associated to s channel helicity flip) has systematically a Veneziano structure of resonances and straight-line behaved dips. In the helicity non-flip part of K^-p elastic scattering one observes fixed t structure not associable to resonance intersections. In πp scattering structure corresponding to intersections of the f^0 meson (mainly coupled to helicity non-flip ^{13), 19)) with baryon resonances is not observed. Even subtracting diffraction in some way, it is unlikely that what remains in the helicity non-flip}

amplitude might have a Veneziano structure of resonances and straight-line behaved dips. There is, for instance, no place in the Veneziano formula for cross-over zeros. Also the right signature zeros required for f^0 exchange ³³⁾ in πp scattering are extraneous to Veneziano. Still in πp scattering f^0 , N_α and Δ_δ cannot combine into a crossing-invariant dual resonance structure, since the product of their signatures is odd. We notice that, because of $SU(3)$, one expects that the perturbation caused by diffraction in helicity non-flip amplitudes of elastic processes should propagate also to the similar amplitudes of inelastic processes.

There is sometimes a puzzling resemblance between the dip structure of helicity non-flip and flip amplitudes (for instance the $u \simeq -0.2$ dip in $\pi^+ p$ scattering is present in both). This circumstance allows part of the dual dip structure of the helicity flip amplitude to show up directly in the differential cross-section. When that occurs (πp scattering and $\mathcal{A}$ production) one observes, at the same time, that the dominant direct channel resonances ^{*)} have a definite natural parity, which fact is understandable ^{**)}. In $K^- p$ scattering, where the dip structures of helicity non-flip and helicity flip amplitudes are rather different, resonances of opposite J^P are comparable [$\Delta_\alpha, \Delta_\gamma$] and [$\Sigma_\delta, \Sigma_\beta$].

One may look for theoretical arguments accounting for the non-Veneziano structure of helicity non-flip amplitudes. It is worth recalling that the reality of zeros in the Veneziano formula is strictly connected to the reality of the amplitude, which is a consequence of the exclusion of non-pole singularities and of the narrow width limit. The presence of an imaginary part due to diffraction clearly destroys such conditions. Even subtracting in some way diffraction, one must still have at $t=0$ positive

^{*)} Δ_δ in $\pi^+ p$, N_α in $\pi^- p$, ($\Sigma_\delta, \Sigma_\beta$) in $\mathcal{A}$ production.

^{**)} Differences between helicity non-flip and flip amplitudes require the interference of partial waves of opposite normality.

resonance residues. In a self-consistent approach involving $SU(3)$ and Veneziano duality, this is a further constraint which probably prevents from having a solution ^{*)}.

Concerning the problem of avoiding parity doublets in Veneziano theory, the observed non-Veneziano structure of dips in helicity non-flip amplitudes (contrasted to the simple Veneziano dip structure of amplitudes involving helicity flip) does not seem to encourage efforts in the search of simple Veneziano-like models avoiding parity doublets, at least as far as phenomenological applications are concerned.

c) The observation in some amplitudes of a simple interplay of dips and resonances, like that suggested by the Veneziano formula, seems to emphasize the role of the formula as describing a particular pattern of resonances and zeros for the physical amplitude. In the frame of the unitarization program of Veneziano theory this indicates that the simple Veneziano formula does already contain important contributions from unitarity ¹¹⁾. Since, however, other regular structures of resonances and straight-line behaved dips are possible and experimentally realized (as we have seen in Λ production), what seems to be of real phenomenological relevance is the approximate reality and straight-line behaviour of amplitude zeros rather than the details of the Veneziano formula.

d) In processes where all the three channels (s , t and u) are populated by resonances, the presence of a Veneziano structure is associated to restrictions on the relevant mass spectrum. The structure existing in Λ production implies having satisfied the supplementary condition $\alpha_s(s) + \alpha_t(t) + \alpha_u(u) = 2$ where α_s , α_t , α_u are the trajectory functions corresponding to s , t , u channels respectively ¹⁾. The same condition is satisfied in πp scattering in order to have zeros corresponding to wrong signature values of α_p trajectory ²⁰⁾. The supplementary condition establishes a quantitative relation between external masses and

*) Actually, Auvil et al. ³⁴⁾ are able to find such a solution in meson-baryon scattering only for the amplitude A , and not for B .

"internal" resonance masses. While this condition is approximately satisfied in the two considered processes, it is difficult to make a statement about its holding or not in general. Just to make an example, in $\pi^+ p \rightarrow \pi^0 \Delta^{++}$ the dip observed ³⁵⁾ at $t \simeq -0.6$, which one associates to the wrong signature point $\alpha_p = 0$, cannot be made to correspond to $N-\Delta$ intersection like in πp scattering, because now the external masses are different ^{*)}.

This fact could inspire the attitude of considering Veneziano correlations between dips and resonances as episodic and not of fundamental interest. The existence of processes (like $\pi^- p \rightarrow \pi^0 n$), where the presence of a complicated resonance spectrum makes the existence of neat Veneziano structures "a priori" impossible, seems to encourage this view.

On the other hand, one may also consider the possibility that the above constraints (at least sometimes) suggest something approximately true. For instance in $\pi^+ p \rightarrow \pi^0 \Delta^{++}$ since $t \simeq -0.6$ is just the position of $\Delta-\Delta$ intersection, one could expect $\pi\Delta\Delta$ coupling to be much stronger than $\pi N\Delta$ coupling. Lack of knowledge of the relevant vertex couplings makes it impossible to be conclusive about this point. Some encouragement, anyway, comes from the fact that combining systematically PCAC and Veneziano duality, one is led to have analogous mass constraints, a number of which, actually, do not compare unfavourably with data ³⁶⁾. Perhaps, also of relevance in this context is the empirical observation by Freund ³⁷⁾ of the existence of a certain amount of quantization in the hadronic mass spectrum.

*) $t_{\text{intersection}} = -s_{\text{res}} - u_{\text{res}} + (\sum_i m_i^2)_{\text{ext}}$.

7. CONCLUSIONS

From the examination of πp , Kp , $\bar{p}p$ scatterings and Λ production at low energy one can draw the following conclusions.

The existence of a simple and regular (constant M^2 spacing) resonance spectrum seems generally to induce a correspondingly simple and regular low energy dip structure.

Straight-line behaviour of dips and Veneziano correlations between dip positions and resonance intersections seem to be systematically present in inelastic processes and s channel helicity flip amplitudes of elastic processes. In other words, when diffraction is absent. In amplitudes dominated by diffraction some Veneziano-like features of this kind are episodically observed, but the evidence seems to exclude that, even after subtracting diffraction in some way, these amplitudes could have a full Veneziano structure of straight-line behaved dips and resonances. This fact, apparently, casts shadows on the physical relevance of building simple-minded Veneziano-like models avoiding parity doublets.

Rather than the detailed prescriptions of the Veneziano formula, what seems to be of phenomenological relevance is the tendency of amplitude zeros to remain approximately real (i.e., not to shift into the complex domain) and to have an approximately straight-line behaviour irrespectively of conventional separations between high energy, low energy and unphysical region. This view is supported in particular by the physical existence (e.g., in Λ production) of alternative patterns of resonances and straight-line behaved real zeros.

Concerning the unitarization program of the Veneziano formula, the observation in some physical amplitudes of neat and clean Veneziano zeros seems to indicate that the bare Veneziano formula already contains some relevant contributions from unitarity¹¹⁾.

The correlation between dip positions and resonance intersections implies stringent relations between the masses of external particles (supplementary conditions ¹⁾). A systematic comparison with data of such relations is difficult (especially for quasi-two-body processes) since relevant information about the relative strengths of resonance couplings is in most cases lacking. In two of the considered processes where such relations appear (π p scattering and Λ production) agreement with data is observed.

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FIGURE CAPTIONS

- Figure 1 : Sketch in the Mandelstam plane of the resonance poles (solid lines) and zeros (dashed lines) of the Veneziano formula [Eq. (1)].
- Figure 2 : Schematic representation in the Mandelstam plane of (A_x, A_y) hyperon resonances and t channel boson resonances appearing in K^-p elastic scattering (solid lines), with the corresponding zeros predicted by the Veneziano formula (dashed lines).
- Figure 3 : Perspective plot of $d\sigma/d\Omega$ for K^-p elastic scattering at low energy ($p_{lab} \leq 1.4$ GeV/c) calculated from phase shifts ¹⁴⁾. In the frame: the 180° energy distribution for the same process with indication of the corresponding values of t maximum [solid line drawn by hand to guide the eye; Ref. 38) for data]. The figure shows the smooth continuation at low energy of the high energy fixed t structure.
- Figure 4 : Perspective plot of the modulus squared of the amplitude $A(s,t)$ for K^-p elastic scattering at low energy ($p_{lab} \leq 1.4$ GeV/c) calculated from phase shifts ¹⁴⁾. There are reasons to believe that this amplitude does not contain diffraction (see the text). The figure shows the existence of a dip at $u' \simeq -0.4(\text{GeV}/c)^2$ corresponding to $u \simeq -0.3$ in the considered momentum range, and absence of fixed t structure.
- Figure 5 : $K^-p \rightarrow \bar{K}^0 n$ angular distributions between 0.45 and 1.8 GeV/c plotted versus t and u (p_{lab} in GeV/c is indicated in each frame) (black circles: Armenteros et al. ³⁹⁾; white circles: Wohl et al. ³⁹⁾; white squares: Dauber ³⁹⁾). The solid lines are hand-drawn to guide the eye. The figure shows the existence, in the considered momentum range, of three dips at remarkably constant $u \simeq -0.1, -0.7$ and $-1.7 (\text{GeV}/c)^2$.

One also observes that when the two last dips leave the physical region (~ 0.850 and ~ 1.400 GeV/c respectively), a hook structure is produced near the forward direction.

Figure 6 : Angular distributions of π^+p and π^-p elastic scattering in the intermediate energy region, showing regular persistence in such region of $t \simeq -0.6$ and $t \simeq -2.8$ dips (π^+p and π^-p), and of $u \simeq -0.2$ π^+p dip (by kind permission of Brabson et al. ⁴⁰).

Figure 7 : Low energy structure of π^+p elastic scattering.
a) Perspective plot of $d\sigma/d\Omega$ below 1.9 GeV/c (calculated from phase shifts ¹⁸).
b) Angular distributions near forward direction showing persistence of $t \simeq -0.6$ dip [Ref. 41) for data].
c) Angular distributions near backward direction, showing persistence of $u \simeq -0.2$ dip until crossing $t \simeq -0.6$ dip, with some effect when crossing the $t \simeq -3.0$ dip [Ref. 42) for data].
d) 180° energy distribution showing minima corresponding to crossing fixed t dips [Ref. 43) for data].
e) Forward energy distribution showing π^+p resonance structure [Refs. 44) and 45) for data, dispersion relations from Ref. 46)].
Apart from e), solid lines are hand-drawn to guide the eye.

Figure 8 : Low energy structure of the invariant amplitude $A(s,t)$ of π^+p elastic scattering, associated to s channel helicity flip. The phase shifts of Ref. 18) are used.
a) Perspective plot of $|A|^2$.
b) Plot of the real and imaginary parts versus t at fixed values of u , showing persistence of the (approximate) zero at $t \simeq -0.6$.
c) Plot of the real and imaginary parts versus u at fixed values of t , showing persistence of the (approximate) zero at $u \simeq -0.2$ (with some inessential shifting).

Figure 9 : Low energy structure of π^-p elastic scattering.

- a) Forward energy distribution showing the resonance structure [Refs. 47) and 48) for data, dispersion relations from Ref. 46)].
- b) Angular distributions near forward direction, showing persistence of $t \simeq -0.6$ dip [solid lines are hand-drawn to guide the eye; Ref. 49) for data].
- c) 180° energy distribution showing a minimum when crossing $t \simeq -3.0$ dip, and some unclear structure corresponding to $t \simeq -1.0$. [The solid line is hand-drawn to guide the eye; Ref. 50) for data].
- d) Plot versus t of the modulus of the amplitude $A(s,t)$ at fixed values of u , showing persistence down to backward direction of the $t \simeq -0.6$ dip, with some spoiling, however, near ~ 1 GeV/c. (Phase shifts from Donnachie et al. ¹⁸⁾, Roper et al. ⁵¹⁾, Davies et al. ⁵¹⁾.)

Figure 10 : Plot in the Mandelstam plane of prominent dips and resonances in πp scattering (see the text), showing coincidence of dip positions with resonance intersections.

Figure 11 : Comparison, in πp elastic scattering, of baryon resonance residues in the amplitudes A and B at resonance intersections (see the text).

Figure 12 : Perspective plot of $d\sigma/d\Omega$ for $\bar{K}N \rightarrow \pi\Lambda$ in the low energy region ($0.440 \leq p_{\text{lab}} \leq 1.72$ GeV/c) calculated from a fit to published Legendre coefficients ⁵²⁾.

Figure 13 : Schematic representation in the Mandelstam plane of $\bar{K}N \rightarrow \pi\Lambda$ and $\pi N \rightarrow K\Lambda$ of the dominant Regge families of resonance poles (Z_α, Z_γ resonances though non-negligible are omitted for simplicity), and of Veneziano-like zeros arising from duality diagram prescriptions (see the text).

Figure 14 : World data for $\bar{K}N \rightarrow \pi \Lambda$ between 0.44 and 1.9 GeV/c, plotted at fixed values of u versus $s-t$ ($K^-n \rightarrow \pi^-p$ data are divided by two). See the text for reasons to prefer, in our case, this kind of plotting to the conventional one at fixed momentum. The figure shows the existence of structure at $s-t \simeq 3$ and 5 GeV². Variation in t can be read in the upper scale. Black circles are obtained by linear interpolation from nearby data points at the same energy. White squares are calculated from published Legendre coefficients; their errors are only indicative. The solid line is calculated from a fit to Legendre coefficients. [Reference 52) for data].

Figure 15 : $\pi^-p \rightarrow K^0 \Lambda$ angular distributions at fixed momentum (indicated in each frame) in the intermediate energy region. The variable s^*-t is $s-t$ in the notations of Fig. 13, and $u-t$ in conventional notations for $\pi^-p \rightarrow K^0 \Lambda$. The figure shows the existence of a dip at constant $s^*-t \simeq 1$ GeV². [Reference 53) for data].

Figure 16 : As in Fig. 14 with u and $s-t$ interchanged. The figure shows the existence of a dip at $u \simeq -0.2$ (GeV/c)².

Figure 17 : Perspective plot of $|A|^2$ for $\bar{K}N \rightarrow \pi \Lambda$ in the low energy region ($1.2 \lesssim p_{\text{lab}} \lesssim 1.8$ GeV/c) calculated from the phase shifts of Litchfield²⁷⁾.

Figure 18 : Dips and discontinuities in $\bar{p}p$ and pp elastic scatterings.

a) Schematic compilation of low-energy $\bar{p}p \rightarrow \bar{p}p$ angular distributions, showing regular propagation at low energy of high energy dips at $t \simeq -0.8$ and $t \simeq -2.5$ (GeV/c)² (by kind permission of Yoh et al.⁹⁾).

b) 180° energy distribution of $\bar{p}p \rightarrow \bar{p}p$ showing a minimum ($t \simeq -0.7$) corresponding to leaving the physical region of the (asymptotically) $t \simeq -2.5$ dip [data from Refs. 9) and 54)].

c) Plot of $(d\sigma/d\Omega)^+$ for $pp \rightarrow pp$ versus $\beta_{\perp}^2 \equiv (u \cdot t)/s$, showing (in particular) constancy of

discontinuity positions in this parameter (by kind permission of Krisch ²⁹).

d) Schematic plot in the Mandelstam plane of $\bar{p}p \rightarrow \bar{p}p$ and $pp \rightarrow pp$ of prominent resonances, dips and discontinuities (with a guess for the unphysical region).

Figure 19 : Structure in $np \rightarrow np$ scattering.

a) Angular distributions near forward direction, showing in particular the presence of a discontinuity at approximately constant $t \simeq -0.8 \text{ (GeV/c)}^2$ [data from Ref. 55].

b) 180° energy distribution and energy dependence of the slope of the backward peak, showing structure at $p_{\text{lab}} \simeq 1 \text{ GeV/c}$ [corresponding to $-t \equiv s - 4M^2 \simeq -0.8 \text{ (GeV}^2\text{)}]$ (by kind permission of Mischke et al. ³¹).

Figure 20 : Perspective plot of $d\sigma/d\Omega$ for $\pi^-p \rightarrow \pi^0n$ at low energy, showing that no simple interplay of resonances and dips is present in this process (calculated from phase shifts ¹⁸).

FIG.1

FIG.2

FIG. 3

FIG. 4

$$K^- p \rightarrow \bar{K}^0 n$$

$$\frac{d\sigma}{dt} \left[\frac{\text{mb}}{(\text{GeV}/c)^2} \right]$$

FIG.5

FIG. 6

FIG. 7

$$\pi^+ p \rightarrow \pi^+ p$$

$$|A(s,t)|^2$$

FIG. 8

$$\pi^- p \rightarrow \pi^- p$$

FIG.9

FIG.10

Parameters used (ref. 18)

	M	Γ	x
D ₁₅ (1670)	1680	173	0.391
F ₁₅ (1688)	1690	132	0.68
P ₃₃ (1236)	1236	120	1.0
F ₃₅ (1890)	1910	350	0.16
F ₃₇ (1950)	1950	221	0.39

FIG. 11

$\bar{K}N \rightarrow \pi\Lambda$
 $d\sigma/d\Omega$

FIG. 12

FIG.13

FIG.14

FIG.15

$\bar{K}N \rightarrow \pi\Lambda$
 $d\sigma/d\Omega$ (mb/sr)

FIG. 16

$$\bar{K}N \rightarrow \pi\Lambda$$

$$|A(s,t)|^2$$

FIG.17

FIG.18

a)

b)

FIG.19

FIG. 20