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A NEW VALUE FOR THE LAMB SHIFT

A. Peterman
CERN - Geneva

A B S T R A C T

The recent analytic value of the α^2 slope of the Dirac form factor of the free electron, from the crossed ladder diagram, is added to the contributions of the fourth order diagrams already evaluated by different workers. The sum is found to be

$$\begin{aligned} F_1'(0)_{\text{total}} &= \frac{\alpha^2}{\pi^2} \left\{ -\frac{4819}{5184} - \frac{49\pi^2}{432} + \frac{\pi^2}{2} \text{Log } 2 - \frac{3}{4} \zeta(3) \right\} \\ &= \frac{\alpha^2}{\pi^2} \{0.4699\}. \end{aligned}$$

This contributes to the Lamb shift in H :

$$\left(\frac{\alpha^2}{\pi^2}\right) \alpha^2 \frac{m\alpha^2}{2} 0.4699 = 0.444 \text{ MHz.}$$

The total Lamb shift in H is then :

$$S_H = (2S_{\frac{1}{2}} - 2P_{\frac{1}{2}})_H = 1057.911 \pm 0.011 \text{ MHz (s.d.)},$$

and the separation $\Delta E_H - S_H$:

$$(2P_{\frac{3}{2}} - 2S_{\frac{1}{2}})_H = 9911.115 \pm 0.031 \text{ MHz | s.d.).}$$

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It is only very recently that an analytic value for the contribution of diagram (1) of the Figure, to $F_1'(0)$, in very good agreement with numerical estimates, has been reported ¹⁾. Those of diagrams (2)-(5) were already known from previous works that we shall briefly discuss. The most schematic way to analyze them is to give a table compiling the contributions from the various diagrams of the Figure. This is done in Table I.

As one can see, there are at least two calculations for diagrams (3), (4) and (5) that agree together ^{2),3),4),5)}.

For diagram (2), two calculations have been done ^{2),4)}. We discarded Ref. 4), because of its conflicting value with numerical estimates ^{6),7)}.

Finally, diagram (1) has been computed by three different authors ^{1),4),8)}. For the same reason as before, we have discarded Refs. 4) and 8).

The sum of the five terms gives the total slope of the Dirac form factor at $q^2 = 0$ of the free electron :

$$\begin{aligned} m^2 F_1'(0) &= \frac{\alpha^2}{\pi^2} \left[-\frac{4819}{5184} - \frac{49\pi^2}{432} + \frac{\pi^2}{2} \log 2 - \frac{3}{4} \zeta(3) \right] \\ &= \frac{\alpha^2}{\pi^2} [0.4699] \end{aligned}$$

On Table II, we have summarized the various contributions to the $2S_{\frac{1}{2}} - 2P_{\frac{1}{2}}$ Lamb-shift in H. The total amounts to 1057.911 ± 0.011 MHz. The theoretical error for the $2P_{\frac{3}{2}} - 2S_{\frac{1}{2}}$ separation is now of the same order of magnitude as the three measurements, Refs. 10), 11), 12), of this frequency. To give an example on how to compare Q.E.D. theoretical predictions and experiment, the fine structure $2P_{\frac{3}{2}} - 2P_{\frac{1}{2}}$ is combined with the new value of the Lambshift $2S_{\frac{1}{2}} - 2P_{\frac{1}{2}}$:

$$\begin{aligned} (2P_{3/2} - 2S_{1/2})_H^{th} &= 9911.115 \pm 0.011 \text{ MHz (s.d.)} \\ \alpha^{-1} &= 137.03608 \quad . \quad 13) \end{aligned}$$

Then we compare with the value measured by the authors of Ref. 12)

$$(2P_{3/2} - 2S_{1/2})_H^{\text{exp}} = 9911.17 \pm 0.04 \text{ MHz (s.d.)}$$

This provides a 2.0 p.p.m. precision value of α^{-1} ;

$$\alpha^{-1} = 137.03570 (27)$$

which compares fairly well with the ac Josephson effect value

$$\alpha^{-1}_{\text{WQED}} = 137.03608 (26)$$

Finally, higher order Lamb transitions ($n = 3, 4, \dots$) and Lamb shifts for other hydrogenic atoms ($Z = 2, 3, \dots$) are given

H (n=2)	1057.911 $\pm$ 0.011
D (n=2)	1059.271 $\pm$ 0.025
He ⁺ (n=2)	14044.76 $\pm$ 0.61
He ⁺ (n=3)	4184.42 $\pm$ 0.18
He ⁺ (n=4)	1769.097 $\pm$ 0.076
Li ⁺⁺ (n=2)	62763.4 $\pm$ 9.1

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We want to thank Professor M. Veltman for informing us that R. Barbieri, J. Mignaco and E. Remiddi ¹⁴⁾ had confirmed independently, with a different method, the result of Ref. 1). We thank also Professor G.W. Erickson for making available to us the details of his calculations on various terms afflicted by uncertainties ¹⁵⁾.

TABLE I. Compilation of contribution to the slope of the Dirac form factor of the electron at fourth order ^{*}). The graphs refer to those shown in the Figure.

Graphs	Analytic results	Numerical results
cross (Diagram 1)	$-\frac{13}{36}\log\lambda^{-2} + \frac{1181}{1728} + \frac{\pi^2}{72} + \frac{7}{12}\pi^2\log^2 - \frac{7}{8}\zeta(3) = 2.3925 \quad [\text{Ref. 1}]$	$-\frac{13}{36}\log\lambda^{-2} + 2.37 \pm 0.02 \quad [\text{Ref. 6}]$ $-\frac{13}{36}\log\lambda^{-2} + 2.3924 \pm 0.0002 \quad [\text{Ref. 1}]$
corner (Diagram 2)	$\frac{1}{12}\log^2\lambda^{-2} - \frac{1}{72}\log\lambda^{-2} + \frac{1511}{1728} + \frac{209}{864}\pi^2 - \frac{\pi^2}{12}\log^2 + \frac{1}{8}\zeta(3) = -1.9328 \quad [\text{Ref. 2}]$	$\frac{1}{12}\log^2\lambda^{-2} - \frac{1}{72}\log\lambda^{-2} - 1.91 \pm 0.02 \quad [\text{Ref. 6}]$ $\frac{1}{12}\log^2\lambda^{-2} - \frac{1}{72}\log\lambda^{-2} - 1.95 \pm 0.05 \quad [\text{Ref. 7}]$
vac.pol. (Diagram 3)	$-\frac{1099}{1296} + \frac{77}{864}\pi^2 = 0.0316 \quad [\text{Refs. 3), 4), 5}]$	$0.0316 \pm 0.0002 \quad [\text{Ref. 6}]$
self-energy (Diagram 4)	$-\frac{1}{12}\log^2\lambda^{-2} + \frac{1}{72}\log\lambda^{-2} + \frac{1109}{1728} + \frac{17}{72}\pi^2 = 1.688 \quad [\text{Refs. 2, 3, 4}]$	$-\frac{1}{12}\log^2\lambda^{-2} + \frac{1}{72}\log\lambda^{-2} + 1.68 \pm 0.01 \quad [\text{Ref. 6}]$
ladder (Diagram 5)	$\frac{13}{36}\log\lambda^{-2} + \frac{319}{864} - \frac{91}{432}\pi^2 = -1.710 \quad [\text{Refs. 3, 4}]$	$\frac{13}{36}\log\lambda^{-2} - 1.69 \pm 0.02 \quad [\text{Ref. 6}]$

^{*}) All the diagrams have been evaluated in the Feynman gauge. For the analytic results of the vacuum polarization, self-energy and ladder diagrams, we have corrected the over-all sign error. All the results are given in units $(\alpha/\pi)^2$.

Table II. Compilation of contributions to the Lamb-shift: $2S_{1/2} - 2P_{1/2}$ in hydrogen *).

ORDER	DESCRIPTION AND REFERENCES	CORRECTION, UNITS $\frac{\alpha}{\pi} (Z\alpha)^4 \frac{m}{6}$	NUMERICAL VALUE in MHz
$\alpha(Z\alpha)^4_m$	2nd order self-energy	$(-2\log(Z\alpha) + \frac{11}{24} \log \frac{K_0(2,0)}{K_0(2,1)}) (1 - \frac{3}{M})$	1009.920
$\alpha(Z\alpha)^4_m$	2nd order magnetic moment	$\frac{1}{2}(1 - 2.75 \frac{m}{M})$	67.720
$\alpha(Z\alpha)^4_m$	2nd order vac. polarization	$-\frac{1}{5}(1 - \frac{3}{M})$	-27.084
$\alpha(Z\alpha)^5_m$	2nd order binding	$(Z\alpha)(3\pi)(1 + \frac{11}{128} \log 2 + \frac{5}{192})(1 - \frac{3}{M})$	7.140
$\alpha(Z\alpha)^6_m$	4th order binding + higher orders	$(Z\alpha)^2(a+b \log(Z\alpha)^2 + c \log^2(Z\alpha)^2) + (Z\alpha)^3 \pi \cdot 9.56$	-0.372
$\alpha^2(Z\alpha)^4_m$	4th order self-energy	$3 \frac{\alpha}{\pi} 0.470$	0.444
$\alpha^2(Z\alpha)^4_m$	4th order magnetic moment	$\frac{\alpha}{\pi} (-0.328)$	-0.102
$\alpha^2(Z\alpha)^4_m$	4th order vac. polarization	$\frac{\alpha}{\pi} (\frac{41}{54})$	-0.239
$\alpha(Z\alpha)^4 \frac{Zm}{M}$	Recoil corrections	$\frac{Zm}{M} (a_1 + b_1 \log(Z\alpha)^2)$	0.359
$\alpha(Z\alpha)^4 (\frac{m}{e})^2_m$	Proton size	$\frac{\pi^2}{2} (\frac{m}{e})^2$	0.125
TOTAL = 1057.911 ± 0.011			

*) This table is an updated version of the compilation made by A. Peterman. The value of the Bethe logarithm $\log(K_0(2,1)/K_0(2,0))$ is that evaluated by Schwartz and Tiemann⁹⁾. The constants a, b, c, a_1 and b_1 are the following: $a = -\frac{4\pi^2}{3} - 4 \log^2 2 - 0.28 \pm 0.5$; $b = \frac{55}{48} 4 \log^2 2$; $c = -3/4$; $b_1 = -1/4$;
 $a_1 = 2 \log \frac{K_0(2,1)}{K_0(2,0)} + \frac{97}{12}$.

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FIGURE CAPTION

The five diagrams contributing to $F_1^+(0)$ and their denomination.

Crossed ladder

1)

+

Corner

2)

Vacuum polarization

3)

+

Self - energy

4)

Direct ladder

5)

FIG. 1