CM-P00058463

Ref.TH.1431-CERN

Archives

THE K^- NUCLEON INTERACTION IN K MESIC ATOMS *)

William A. Bardeen +)

CERN - Geneva

and

Institute of Theoretical Physics
Stanford University, Stanford

and

E. Wayne Torigoe ")

Institute of Theoretical Physics
Stanford University, Stanford

A B S T R A C T

A study of the K^- nucleon interaction is made as it pertains to the absorption of K^- mesons in nuclei. Effective interaction strengths are derived which yield good agreement with recent X ray experiments that determine both the strong interaction energy shifts and widths.

*) Research sponsored in part by the Air Force Office of Scientific Research, U.S. Air Force Contract No F44620-71-C-0044.

+) Alfred P. Sloan Foundation Fellow.

") N.S.F. Predoctoral Fellow.

K mesic atoms can be used to probe our understanding of the K^- nucleon interaction as well as the nuclear surface. The theoretical analysis of K meson absorption in nuclei is based upon the construction of a K nucleus pseudopotential that can be related to the free K nucleon scattering amplitudes. However, there are difficulties associated with the simple application of the pseudopotential technique. The effective K^- nucleus interaction is strongly absorptive which can cause distortions of the K meson wave-function making perturbation techniques unreliable. Also the K nucleon interaction is expected to be strongly energy dependent in the range of energies reached during the absorption process. In this paper, we present a pseudopotential method which can incorporate these effects.

Pseudopotential calculations by Ericson and Scheck ¹⁾, by Bloom, Teller and Johnson ²⁾, by the present authors (referred to as I) ³⁾ give results which are in good agreement with the initial studies of X ray yields by Wiegand et al. ⁴⁾. While the experiments confirm our general picture of K meson absorption, such as absorption from predominantly circular orbits, they are not sufficiently precise as to distinguish the different treatments of the absorption process or to indicate possible modifications of nuclear surface densities.

The recent experiments of Backenstoss et al. ^{5),6)} which determine the strong interaction energy shifts and line broadenings may make such a distinction possible. Indeed, on the basis of the initial studies on sulphur ⁵⁾, Krell ⁷⁾ has argued that there must be strong distortion effects occurring in the lower electronic orbits due to the strongly absorptive potential obtained from the free K meson scattering lengths. Hence perturbation treatments of the lower levels are not reliable. Krell suggests that the distortion effects may be incorporated by solving the eigenvalue problem for the K meson with strong interaction pseudopotential included as a part of the potential in the Klein-Gordon equation describing the dynamics of the K meson. Using this approach, Krell finds that agreement with the sulphur data can be attained with a pseudopotential whose absorptive part is in agreement with that expected from the free scattering lengths but whose real part is attractive rather than repulsive as expected from the free scattering lengths. Similar results have been obtained by Seki ⁸⁾ in fitting the sulphur data.

In our previous paper (I), we have argued that the presence of the $\gamma_0^*(1405)$ resonance in the K^- proton channel can strongly influence the absorption process as the energies available to the K^- proton system during absorption are well below the free particle threshold and near the resonance energy. One qualitative result of the resonance is an enhancement of the absorption rates on protons relative to neutrons as can be seen in our estimates of the absorptive parts of the pseudopotential strengths in I. Another qualitative effect that is possible if the γ_0^* resonance dominates in the K^- proton channel is a change in sign of the real part of the scattering amplitude near the resonance energy. To see whether this effect can account for the results of Krell's analysis of the sulphur data, we determine energy averaged pseudopotential parameters by the methods used in I.

We give a brief review of the pseudopotential methods used in I. We construct a K^- nucleon pseudopotential which, in Born approximation, gives the correct K^- nucleon s wave scattering amplitudes. As the K^- nucleon interaction is short range, we use a potential of the form

$$V_{KN}(\vec{x}-\vec{y}) = (U-iW) \delta(\vec{x}-\vec{y}) \quad (1)$$

where

$$-U+iW = 4\pi (k^2/2\mu) f_{KN} \quad (2)$$

f_{KN} is the free scattering amplitude and μ is the reduced mass, $\mu = m_K m_N / (m_K + m_N)$. The K^- nucleus pseudopotential is constructed as a sum of two-body potentials

$$V_{KA} = \sum_j (U_j - iW_j) \delta(\vec{x}-\vec{y}_j) \quad (3)$$

where the sum (j) ranges over all nucleons in the nucleus. The effective strong interaction potential seen by the K meson is then

$$V_{KA}(\vec{x}) = (U_p - iW_p) \rho_p(\vec{x}) + (U_n - iW_n) \rho_n(\vec{x}) \quad (4)$$

where $\rho_p(\vec{x})$ and $\rho_n(\vec{x})$ are the respective proton and neutron densities.

As argued in I, the pseudopotential in Eq. (4) cannot be used due to the strong energy dependence of free scattering amplitudes which determine the pseudopotential parameters through Eq. (2). The energy dependence of fits to the scattering data by Kim⁹⁾ and by Martin and Sakitt¹⁰⁾ is shown in Fig. 1. However, as shown in I, effective pseudopotential parameters could be derived as a suitable energy averaging of the scattering amplitudes in Fig. 1. Hence the scattering amplitudes in Eq. (2) must be replaced by the energy averaged scattering amplitudes, $\bar{f}_{KN}$,

$$f_{KN} \rightarrow \bar{f}_{KN} = \int dW f_{KN}(W) P(W) \quad (5)$$

The function $P(W)$ is the probability of finding the K^- absorbed nucleon system with total centre-of-mass energy W .

If we neglect interactions between the K^- absorbed nucleon system and the residual nucleus, then the centre-of-mass energy is the initial energy less the kinetic energy of the motion of the centre-of-mass,

$$W_i = m_K - \epsilon_K + m_i - \epsilon_i - P_{cm}^2 / 2W_i \quad (6)$$

where ϵ_K and ϵ_i are the K meson and absorbed nucleon binding energies respectively and P_{cm} is the momentum of the centre-of-mass. The energy distributions are then simply related to the centre-of-mass momentum distributions. These are given by the Fourier transform of the product of the K meson and nucleon wave-functions under the assumption of a single particle description of the nucleus and the short range nature of the force in Eq. (1). For nucleon states of definite angular momentum we can derive the explicit formula for $P(W)$, analogous to Eq. (26) in I

$$P(W) \propto \sum_i \sum_J (2J+1) \left| \begin{pmatrix} J & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right|^2 \times \int_0^\infty dp \, p^2 |Z_{ij}(p)|^2 \delta(W - W_i(p)) \quad (7)$$

where $Z_{ij}(p)$ is the overlap of the radial wave-functions

$$Z_{ij}(p) = \int_0^\infty dr \, r^2 j_J(pr) U_{iL}(r) R_{n\ell}(r) \quad (8)$$

We note that $j_J(\rho r)$ is the spherical Bessel function, $U_{iL}(r)$ the nucleon wave-function of orbital angular momentum L , and $Rn_l(r)$ the radial wave-function for the K meson ; the sum over i is over all nucleon states.

Using the shell model description of the nucleus as given in I and undistorted K meson wave-functions, the energy distributions were calculated using Eqs. (7) and (8). The results of these calculations for selected nuclei and indicated circular orbital angular momentum states for the K meson are shown in Fig. 2 for $^{16}\text{O}(\ell=2)$, $^{40}\text{Ca}(\ell=3)$, $^{96}\text{Mo}(\ell=4)$. In comparison with Fig. 1, the energy distributions show a strong peak in the region of the zero of the real part of the K^- proton amplitude with a tail extending to lower energies or more attractive scattering amplitudes.

Using the energy distributions of Fig. 2 and the two sets of scattering amplitudes shown in Fig. 1, we can compute the energy averaged scattering amplitudes defined in Eq. (5). The results of these calculations are presented in Table I along with the threshold values of the scattering amplitudes or the scattering lengths.

We see that the conjecture of Krell is verified for the resonance dominated amplitudes of Fig. 1. The energies available during the absorption process are sufficiently below threshold as to reflect the change in sign of the real part of the scattering amplitudes. By comparing the absorptive parts of the averaged amplitudes, we see the large enhancement of the absorption strength on protons. As suggested in I, this enhancement implies that X ray studies in K mesic atoms are not sensitive to the neutron distributions.

The energies available during absorption will of course vary with the nucleus and also with the initial state of the K mesic atom. However, from Table I, we observe the effective neutron parameters are essentially constant and approximately the same for the two fits to the scattering data. The absorptive part of the proton amplitudes are also essentially constant but differ considerably between the two fits reflecting the much broader resonance peak for the Kim fit. The real part of the proton amplitude is the most sensitive to the energy averaging although the sign will clearly remain opposite to the threshold value for the resonance dominated amplitudes in Fig. 1.

The energy averaged amplitudes of Table I can be used to make a detailed comparison with the recent experiments of Backenstoss et al.⁶⁾ which observe with some precision the strong interaction energy shifts and line broadenings of the K mesic X rays for a number of light nuclei. We have used energy averaged amplitudes for $^{16}\text{O}(\ell=2)$ given in Table I to construct the effective pseudopotential determined through Eqs. (4) and (2). The strong interaction energy shifts and widths were computed using a program developed by Krell¹¹⁾ which solves the eigenvalue problem for the Klein-Gordon equation with the potential including the strong interaction pseudopotential. The results of these calculations for the isotopes ^{10}B , ^{11}B , ^{12}C , ^{31}P , ^{32}S , ^{35}Cl are given in Table II and compared with the experimental data⁶⁾. The pseudopotential shapes for the lighter isotopes were harmonic wells with root mean square radii determined from electron scattering¹²⁾. Two parameter Fermi wells were used for the heavier isotopes with root mean square radii determined from the muonic atom studies of Backenstoss et al.¹³⁾.

From Table II, we see that the theoretical predictions based on the resonance dominated amplitudes are generally in quite good agreement with the data. Calculations based on the threshold scattering lengths (shown in brackets) generally give widths which are a factor of two to three too small. We note that in this range of pseudopotential parameters the energy shift varies most strongly with the imaginary part of the pseudopotential, while the width depends most strongly on the real part, contrary to perturbation theory. We observe that the Martin-Sakitt fit parameters seem to give a better fit to the energy shifts in the heavier nuclei than the Kim fit parameters. This may indicate that the Kim fit gives too large an imaginary part for the proton amplitude in the resonance region. Definite conclusions concerning this point must await a more complete analysis of data particularly a study of the radius and amplitude dependences of the measured quantities.

There are of course many ways these calculations could be improved. For example, distorted K meson wave-functions should be used in computing the energy averaging functions. In principle, the energy averaging will differ depending upon the K meson orbit as well as the isotope involved. These effects are being studied and will be reported elsewhere.

There are also basic limitations to the pseudopotential approach. The use of the free scattering amplitudes involves the neglect of interactions of the K meson-nucleon system with the residual nucleus. We expect this approximation to be reasonably good as absorption occurs in the nucleon periphery, but this assumption should be studied in realistic atom situations.

We conclude that the pseudopotential method works quite well when suitably modified to the kinematical situation found in K^- mesic atoms. The fits to the Backenstoss data in Table II imply that the K^- proton scattering amplitude must be resonance dominated without a large repulsive background ¹⁴⁾. As our understanding of the absorption process improves, we may be able to use K^- mesic atoms as a probe of the K^- proton scattering amplitudes in the resonance region as well as a probe of the nuclear surface.

ACKNOWLEDGEMENTS

One of us (W.A.B.) would like to thank the members of the Backenstoss group at CERN for many useful discussions concerning the data and the theory. W.A.B. would also like to thank T.E.O. Ericson, M. Krell and F. Scheck for stimulating discussions of the theory.

	$\bar{f}_{K^-p} - \text{Kim} - \bar{f}_{K^-n}$ fm	$\bar{f}_{K^-p} - \text{M.S.} - \bar{f}_{K^-n}$ fm
$^{16}\text{O} (\ell = 2)$	$0.93 + i2.27$ $0.10 + i0.47$	$1.12 + i1.48$ $0.14 + i0.40$
$^{40}\text{Ca} (\ell = 3)$	$0.73 + i2.24$ $0.09 + i0.46$	$0.94 + i1.60$ $0.13 + i0.39$
$^{96}\text{Mo} (\ell = 4)$	$0.79 + i2.32$ $0.10 + i0.48$	$1.01 + i1.62$ $0.13 + i0.42$
THRESHOLD	$-0.89 + i0.62$ $-0.13 + i0.51$	$-0.88 + i0.62$ $-0.09 + i0.54$

TABLE I : The energy averaged scattering amplitudes

Atom Transition	$r_{r.m.s.}$ (fm)	$r_{0.5}$ (fm)	t (fm)	Kim		M.S.		Experimental	
				ϵ_L (KeV)	Γ_L (KeV)	ϵ_L (KeV)	Γ_L (KeV)	ϵ_L (KeV)	Γ_L (KeV)
$^{10}\text{B} \ ((3,2) \rightarrow (2,1))$	2.45	-	-	-0.30 (-0.23)	0.65 (0.24)	-0.25 (-0.22)	0.72 (0.25)	-0.208 $\pm$ 0.035	0.81 $\pm$ 0.10
$^{11}\text{B} \ ((3,2) \rightarrow (2,1))$	2.42	-	-	-0.30 (-0.23)	0.64 (0.24)	-0.26 (-0.23)	0.70 (0.26)	-0.167 $\pm$ 0.035	0.70 $\pm$ 0.08
$^{12}\text{C} \ ((3,2) \rightarrow (2,1))$	2.42	-	-	-0.80 (-0.59)	1.44 (0.55)	-0.67 (-0.58)	1.58 (0.58)	-0.59 $\pm$ 0.08	1.73 $\pm$ 0.15
$^{31}\text{P} \ ((4,3) \rightarrow (3,2))$	3.188	3.34	2.45	-0.52 (-0.49)	1.65 (0.58)	-0.36 (-0.48)	1.68 (0.60)	-0.33 $\pm$ 0.08	1.44 $\pm$ 0.12
$^{32}\text{S} \ ((4,3) \rightarrow (3,2))$	3.244	3.33	2.60	-0.88 (-0.82)	2.73 (0.94)	-0.61 (-0.80)	2.78 (0.98)	-0.55 $\pm$ 0.06	2.33 $\pm$ 0.06
$^{35}\text{Cl} \ ((4,3) \rightarrow (3,2))$	3.335	3.42	2.60	-1.44 (-1.29)	4.06 (1.45)	-1.05 (-1.27)	4.28 (1.51)	-0.77 $\pm$ 0.40	3.8 $\pm$ 1.0

TABLE II : Pseudopotential calculation of the energy shifts and widths.
Threshold parameter calculations in brackets.

REFERENCES

- 1) T.E.O. Ericson and F. Scheck - Nuclear Phys. B19, 450 (1970).
- 2) S.D. Bloom, M.H. Johnson and E. Teller - Phys.Rev.Letters 23, 28 (1969) ;
UCRL-72807 (1970).
- 3) W.A. Bardeen and E.W. Torigoe - Phys.Rev. C3, 1785 (1971).
- 4) C.E. Wiegand - Phys.Rev.Letters 22, 1235 (1969) ;
C.E. Wiegand and D.A. Mack - Phys.Rev.Letters 18, 685 (1967).
- 5) G. Backenstoss et al. - Phys.Letters 32B, 399 (1970).
- 6) G. Backenstoss et al. - Submitted to Phys.Letters.
- 7) M. Krell - Phys.Rev.Letters 26, 584 (1971).
- 8) R. Seki - Submitted to Phys.Rev.
- 9) J.K. Kim - Phys.Rev.Letters 19, 1074 (1967) ;
F. Von Hippel and J.K. Kim - Phys.Rev.Letters 20, 1303 (1968).
- 10) B.R. Martin and M. Sakitt - Phys.Rev. 183, 1345 (1969) ; 183, 1352 (1969).
- 11) M. Krell - Submitted to "Computer Physics Communications".
- 12) R. Hofstadter and H.R. Collard - In Landolt-Börnstein, edited by
K.H. Hellwege, Springer Verlag, New York (1967), group I,
Vol. 2, p. 32.
- 13) G. Backenstoss et al. - Phys.Letters 25B, 547 (1967).
- 14) D. Cline, R. Laumann and J. Mapp - Phys.Rev.Letters 26, 1194 (1971).

FIGURE CAPTIONS

- Figure 1 The energy dependence of the free K^- nucleon scattering amplitudes for the fits of Kim and of Martin and Sakitt.
- Figure 2 The probability function for K^- nucleon system to have an energy, W , in the centre-of-mass.

FIG.1

FIG.2