THE ASTROPHYSICAL r-PROCESS AND ITS DEPENDENCE ON PROPERTIES OF NUCLEI FAR FROM STABILITY: BETA STRENGTH FUNCTIONS AND NEUTRON CAPTURE RATES

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Abstract

The question of the astrophysical site of the rapid neutron capture (r-) process which is believed to be responsible for the production of the heavy elements in the universe has been a problem in astrophysics for more than two decades. The solution of this problem is not only dependent on the development of realistic astrophysical supernova models, i.e., correct treatment of the hydrodynamics of gravitational collapse and supernova explosion and the equation of state of hot and dense matter, but is shown in this paper to be very sensitive also to "standard" nuclear physics properties of nuclei far from stability such as beta decay properties and neutron capture rates. For both of the latter, strongly oversimplifying assumptions, not applying the development in nuclear physics during the last decades, have been made in almost all r-process calculations performed up to now. A critical discussion of the state of the art of such calculations seems therefore to be indicated. In this paper procedures are described which allow one to obtain 1) beta-decay properties (decay rates, beta-delayed neutron emission and fission rates) 2) neutron capture rates for neutron-rich nuclei considerably improved over what has been used up to now. The beta strength functions are calculated for -6000 nuclei between beta stability line and neutron drip line. By hydrodynamical supernova explosion calculations using realistic stellar models it is shown that as a consequence of the improved beta-rates explosive He burning is a convincing alternative site to the "classical" r-process whose existence still is questionable. The new beta-rates will be important also for the investigation of further astrophysical sites producing heavy elements such as the r(n)-processes in explosive C or Ne burning.

1. Introduction

All elements, except for deuterium and He, which can be produced in big-bang cosmologies, and except for Li, Be, and B, which may be produced by spallation of heavier elements in cosmic rays, are produced in stars. Any theory of nucleosynthesis thus is strongly connected to the theoretical models of star formation and star evolution.

For the formation of the trans-iron-group elements three processes are generally accepted to be responsible: 1) neutron capture by the r- and s-processes and p-process (photodisintegration and proton capture). From these processes in particular the r-process is still far from being well understood, for two reasons: 1) The astrophysical site is rather uncertain. 2) Most of the nuclear physics data needed for r-process calculations cannot be obtained by laboratory experiments. The most important of the nuclear physics input data are the beta-decay rates of nuclei far from stability because they determine the time scale of the synthesis and the relative abundances for different Z.

The beta strength function has been treated, however, in all r-process calculations up to now in extremely oversimplified ways, which are just relics of a time of very limited experimental possibilities.

In a similar way as for the beta-decay properties, the development which occurred in nuclear physics during the last decade has not been applied yet by astrophysicists to the treatment of neutron capture rates.

Because of the strong sensitivity of any r-process calculations on these nuclear physics input data - to be demonstrated in this paper - a critical discussion of the state of the art of the r-process seems to be indicated. In section 2 some basic facts about the r-process are reviewed. In section 3 procedures are described which allow one to obtain sets of beta-decay properties (beta-decay rates, beta-delayed neutron and fission rates) and neutron capture rates considerably improved over what has been used up to now in astrophysical calculations. The beta strength functions are calculated for -6000 nuclei between beta stability line and neutron drip line and are used in section 4 in calculations of heavy element nucleosynthesis by explosive He burning. These calculations are based on realistic star models and a hydrodynamical treatment of the supernova explosion, both for the first time applied for this process. It is shown that as a consequence of the revised beta-decay rates explosive He burning seems at present to be the most convincing site of an r-process leading to a solar r-element distribution. Possible further astrophysical sites producing heavy elements in the universe such as explosive C and Ne burning are also discussed in section 4. Section 5 gives a conclusion.

2. Some basic facts about the r-process

While nuclei \( ^{\alpha}A \) to the "iron group" (\( A \approx 50-60 \)) can be built out of lighter nuclei in successive nuclear charged particle reactions at core temperatures below the Coulomb barrier of A ~ 70 nuclei (which are reached only during the gravitational collapse) also create a Planck distribution of photons to the same temperature. These high energy photons lead to photodisintegration which for not too high temperatures results in abundances of nuclei centered in the mass range with the highest binding energies per nucleon. Heavier elements thus can be produced only via successive neutron captures and beta-decays.

A look at the major abundances of heavy nuclei in the solar system supports this idea and leads to the suggestion of two extreme neutron capture processes: the s-process for \( \tau_n \ll \tau_r \) and the r-process for \( \tau_n \gg \tau_r \), corresponding to "slow" and "rapid" neutron capture. The first of these processes can explain the abundance peaks at nuclei with neutron magic numbers (A = 90, A = 144, A = 208) by the small neutron capture cross sections for these nuclei. Such a process is believed to happen during hydrostatic helium burning in stellar evolution on time scales.

There is another component in the solar abundance distribution with peaks shifted to lower masses by 10-15 mass units on average. Such a component
would be explained by a sudden very high neutron concentration \((n_n > 10^{10} \text{ cm}^{-3})\).

If in addition to \(\epsilon(n_n) < \epsilon_B\) also \(\epsilon(v, n) < \epsilon_B\) is fulfilled, maximum abundances in each isotopic chain are obtained on lines of constant neutron separation energy. These lines come close to the stability line at magic neutron numbers, where then the longer \(\beta\)-decay of half-lives again lead to a pile-up of nuclei. Projecting down these waiting points to the \(\beta\)-stable nuclei by subsequent \(\beta\)-decays leads to the \(\gamma\)-abundance peaks below the magic neutron numbers. In the case of an \((n, \gamma)\) - \((\gamma, n)\) equilibrium the only needed nuclear physics input data are nuclear masses (binding energies), partition functions and \(\beta\)-decay half-lives.

These general ideas have already been discussed in the pioneering work of Burbidge, Burbidge, Fowler and Hoyle\(^{59}\). The early calculations of Seeger et al.\(^{15}\) used constant neutron densities and temperatures and \(\beta\)-strength functions \(S_{\beta} - \rho\) (see section 3). Schramm\(^{54}\) and Blake and Schramm\(^{55}\) discuss a dynamic \(r\)(n)-process where the competition of neutron captures and \(\beta\)-decays during the freeze-out phase should smooth the odd-even staggering occurring in the abundances for an \((n, \gamma)\) - \((\gamma, n)\) equilibrium but the neutron capture rates used were still very uncertain quantities. Kodama and Takahashi\(^{56}\) used the gross theory of \(\beta\)-decay (see section 3) and could show that \(\beta\)-delayed neutron emission is a very efficient process to smooth the final abundance curve. Hillebrandt et al.\(^{20}\) were the first to couple the \(r\)-process to an astrophysical site by performing a hydrodynamical calculation for the outer part of the dense stellar core during a supernova explosion. This treatment at that time seemed to allow one for the first time to explain all \(r\)-production peaks in one event.

An alternative site (the explosive He-burning) has been discussed by Hillebrandt and Thielemann\(^{25}\), Truran et al.\(^{16}\), Thielemann et al.\(^{61}\), Blake et al.\(^{13}\), Cowan et al.\(^{56}\), Wofel et al.\(^{13}\). They examined explosive carbon burning. Very recently Klapdor et al.\(^{44}\) reassessed the explosive He-burning by using microscopically calculated \(\beta\)-decay half-lives for the nuclei participating in the \(r\)-process. As a consequence of this improvement the explosive He-burning is found to be a very promising alternative to the "classical" \(r\)-process close to the supernova core, whose existence still is questionable.

3. The dependence of the \(r\)-process on nuclear systematics

The \(r\)-process is governed by neutron captures, photodisintegrations and \(\beta\)-decays. In an \((n, \gamma)\) - \((\gamma, n)\) equilibrium which is assumed in most of the models used up to now, the most important quantities are the \(\beta\)-decay half-lives and the mass law which control the speed and path of the \(r\)-process. Thus such calculations are most sensitive to these input data which makes their reliable prediction for neutron-rich nuclei up to the neutron drip line necessary.

When neutron captures, \(\beta\)-decays and \(\beta\)-delayed neutron processes are competing with each other (no \((n, \gamma)\) - \((\gamma, n)\) equilibrium), which is the case during freeze-out and processes with low neutron flux (\(r\)-processes), neutron capture has to be included explicitly, which means that reliable predictions of neutron capture cross sections are needed. This is necessary for an accurate prediction of isotopic ratios in the \(r\)-process path and thus is a precondtiion for any predictions of the production ratios of chronometric pairs. Though Schramm et al.\(^{54,55}\) claim that the uncertainties in these production ratios do not reflect uncertainties of isotopic ra-

tios in the \(r\)-process, a reliable \(r\)-distribution would reduce some large uncertainties.

All of these nuclear parameters have been treated in rather oversimplified ways in all \(r\)-process calculations existing up to now: As to the \(\beta\)-half-lives, they are based in the most advanced calculations on the gross-theory\(^{52}\) of \(\beta\)-decay. This theory cannot give a reliable description of \(\beta\)-decay half-lives, \(\beta\)-delayed neutron rates or \(\beta\)-delayed fission rates (the latter being decisive for a correct description of the decay back from the \(r\)-process path to the \(\beta\)-stability line for heavy nuclei and thus for the production ratios of chronometric pairs\(^{22}\)). This is discussed in detail by Klapdor et al.\(^{38,39,43,44}\); see also section 3.1.

As to the mass law, up to now sometimes (see, for example, Refs. 66,18) still the mass predictions of Truran et al.\(^{44}\) are used which seem to predict rather uncertain neutron separation energies. Another group\(^{43}\) unfortunately does not mention the mass law they are using. The situation is worse for neutron capture cross sections: There exist up to now no neutron-capture cross sections for nuclei far from stability in the literature. Holmes et al.\(^{15}\) and Hosley et al.\(^{72}\) published thermonuclear reaction rates for nuclei close to the stability line which base on very schematic models (see section 3.2). Unfortunately it is not stated in most \(r\)-process applications (e.g. Refs. 71,66,18) what procedure is used to calculate neutron cross sections; it seems however sure that it is not superior to the one outlined in Refs. 30,75.

In the next section an attempt is made to put the \(\beta\)-decay and neutron-capture calculations of nuclei far from stability on a reliable physical basis. This is done by a microscopic description of the beta strength function and by using a statistical model based on a "next to first principles" optical potential including effects of deformation for the calculation of the neutron capture rates.

3.1 Calculation of beta strength functions and half-lives for neutron-rich nuclei

The shape of the Gamow-Teller beta strength function \(S_\beta\) in nuclei far from the line of beta stability is not only of importance for astrophysical applications - one of which is discussed in this paper - but also for various problems in nuclear physics as the interpretation of reactor neutrino oscillation experiments\(^{56}\), the production of transuranium elements by thermonuclear explosions, the problem of determining fission barriers from beta-decayed fission, and for reactor control (see Klapdor et al.\(^{38-43}\)).

Surprisingly, in spite of this importance of \(S_\beta\) in all of the above mentioned applications up to now, the calculation of beta decay half-lives \(\beta\)-delayed particle emission and fission rates has been made by use of strongly oversimplified assumptions on the beta strength function \(S_\beta\), which is defined as

\[
S_\beta(E) = \frac{1}{\sqrt{\rho(E)}} T_{\beta} \frac{E^2}{m} \frac{d^3p}{d^3E} \cdot D
\]

with \(\rho\) denoting the level density in the daughter nucleus and \(B\) a reduced transition probability. The assumptions used were \(S_\beta(E) \sim \rho(E), \ S_\beta = \text{const}\), or the gross theory of \(\beta\)-decay or combinations of these. The first two of these assumptions (though still defended in Ref. 25; see, however, Refs. 39,47) have no physical basis and are just relics from a time of limited experimental possibilities (compare, e.g. the \(S_\beta\) measured for \(^{198}\text{yr}\) with a more recent measurement\(^{47}\) or the \(S_\beta\) measured\(^{40}\)) for \(^{87}\text{Br}\)
with a more precise measurement \(^{39}\)). The third of these assumptions \(^{39}\) was a progress when it was introduced a decade ago; it fulfills sum rules but from its concept already it is unable to describe low-lying structure in \(S_G(E)\). For these structures, however, which govern the beta decay properties, there is experimental evidence not only directly from \(\beta\)-decay \(^{37,39,42,42,42,42}\) but also from the iso-vector \(M_1\) decay of isobaric analogue states \(^{37,42}\) and from \((p,n)\) charge exchange reactions \(^{57,40,43,42}\) (for a detailed discussion see Klapp et al. \(^{39,42,42}\)).

It was thus felt necessary to find a more realistic description of \(S_G\), which at the same time would still be simple enough (with respect to computer time) to allow the application to the physical problems mentioned above, i.e., to allow computation of \(S_G\) for several thousands of nuclei.

### 3.1.1 The model

We combine the single particle shell model, the Brown-Bolsterli model \(^{113}\) and the macroscopic relation between the excitation energies of isobaric analogue and antianalogue states in the daughter nucleus. The states in the daughter nucleus which are generated by Gamow-Teller transitions are restricted to be core-polarized \((|v_J\rangle\langle v_J|)\), spin-flip \((|v_J\rangle\langle v_{J+1}|)\), and back-spin-flip \((|v_{J+1}\rangle\langle v_{J}|)\) states relative to the \(\beta\)-decaying parent state \(^{39}\)).

The single particle model predicts core-polarized states higher by the pairing energy \(\Delta_p\) than the antianalogue state.

\[\Delta_{\text{pair}} = \langle 3J|J)^Γ|3J\rangle\Gamma = \sqrt{2J+1} \sqrt{2J+1} G\]

where \(G = 20/A\),

\[E_{\text{CPS}} = E_{\text{IAS}} + \Delta_{\text{pair}}.\]

The energy of the AAS is connected by the Lane potential with the energy of the isobaric analogue state \((E_{\text{IAS}})\) of the parent ground state:

\[E_{\text{IAS}} = E_{\text{IAS}} - V_0 T_0/A\]

where \(T_0\) is the isospin of the parent nucleus and the strength \(V_0\) of the Lane potential is found to be \(-120\) MeV from the IAS-AAS splitting in nuclei. The excitation energy of the IAS in the daughter nucleus can be expressed as

\[E_{\text{IAS}} = \Delta E + Q_y - (M_n - M_p).\]

The Coulomb displacement energy \(\Delta E\) is given \(^{39}\) within \(\pm 100\) keV (for not too strongly deformed nuclei) by

\[\Delta E = 1.444Z^2/A^{1/3} - 1.13\] MeV,

where \(Z = (Z_1 + Z_f)/2\). In the single particle model, the spin-flip (SFS) and back-spin-flip (BSFS) configurations can be generated by Gamow-Teller configurations by the spin-orbit splitting \(\Delta_{\text{LS}}\):

\[E_{\text{SFS,BSFS}} = E_{\text{CPS}} \pm \Delta_{\text{LS}}.\]

Following Bohr and Mottelson \(^{12}\), we estimate

\[\Delta_{\text{LS}} = 10(21 + 1)A^{-2/3}\] MeV.

In the Brown-Bolsterli model, the matrix elements of the effective two-body interaction are given by the products of the matrix elements for the involved transitions, in this case by the products of the Gamow-Teller matrix elements.

In summary, our Hamiltonian is assumed to be

\[H = G_0 \varepsilon(\tau) + \varepsilon_{\text{LS}} + \varepsilon_{\text{pair}} + G_1 \varepsilon(\sigma)(\tau)\]

where \(G_0 = V_y/A\). The parameter \(G_1\) is extracted from the recent experimental location of the GT giant resonance by charge exchange reactions (see section 3.1.2). After the configuration of the decaying state is defined, the possible configurations for the GT-decay in the daughter nucleus can be determined immediately. Then the Hamiltonian \((9)\) is diagonalized in those configurations consisting of CPS, SFS and BSFS.

After the diagonalization of the Hamiltonian, the reduced transition rate of the GT decay to the \(l\)-th level in the daughter nucleus is given simply by

\[B^l_i = 1/(2I_1 + 1) \sum_{k} k_i \sum_{k} k_i U_{k}^{\pm} \text{CPS or SFS or BSFS}\]

where the \(U_{k}^{\pm}\) are reduced matrix elements of the Gamow-Teller transitions between parent state and the corresponding "pure" final configuration, and where \(\nu_k\) are amplitudes of the configurations in the actual final states.

The sequence of single particle levels used in the calculations takes into account deformation according to a deformed Nilsson potential. For further details of the calculations, see Klapp et al. \(^{42}\).

### 3.1.2 Results for \(S_G\) and the Beta decay half-lives

The GT beta strength functions and beta decay half-lives have been calculated using the procedure described in section 3.1.1 for the neutron-rich nuclei with \(Z = 24-115\) between the line of beta stability and the neutron drip line. In this section we shall give a description of the general result and present some typical examples. The main features of the GT \(S_G\) are: The GT residual interaction shifts the main part of the GT strength into a collective state near the isobaric analogue state of the beta decaying state, whose wave function mainly consists of spin-flip and core-polarized configurations (see Refs. 42, 44). This GT giant resonance lies by the spin flip split \(\Delta T_0\) of the collective \(M_1\) mode below the IAS of the main \((T_0)\) part of the \(M_1\) giant resonance in the parent nucleus.

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![Fig. 1](image-url)  
**Fig. 1.** Beta strength function calculated for the GT \(\beta\)-decay of \(^{86}\)B \((B = (4\beta/g_9)\mu G)\). Solid line: this work; dashed line: gross theory; histogram: experiment (Kratz et al., this conf.). The lowest-lying peak corresponds to \(g_{9/2} \rightarrow g_{9/2}\) back-spin-flip transition; the pygmy resonance just below \(Q_y\) is collectively built up by core-polarized and spin-flip configurations.
of Figs. 9,10, when the gross theory is used instead of the microscopically calculated half-lives (Fig. 11 has to be compared to Fig. 9(b)). The second abundance peak cannot be produced as a result of the longer half-lives of the gross theory. The same situation is found for all other cases shown in Fig. 9 (see Klappor et al. 44)).

The question whether the explosive He-burning is a site for an r-process producing solar abundances thus is extremely sensitive to a correct treatment of a standard nuclear physics quantity as β-decay half-lives. The new site avoids the problem of overproduction of heavy elements, because the mass fraction per star of r-elements synthesized in explosive He burning is about 3 orders of magnitude smaller than in the "classical" r-process.

4.3 Additional sites for an r(n)-process

As pointed out already, neutron abundances necessary for an r-process can only be obtained in an explosive process. This could be flashes but the most probable cases are stellar explosions. Novae undergo explosive hydrogen burning and consequently no neutrons are released. Thus we have to look into different sites in a supernova explosion. The outcome with respect to neutron fluxes in explosions of supermassive objects 11) (if they existed at all) are completely unknown.

Cowan et al. 19 emphasize the well-known neutron source 12C(a,n)16O where the general difficulty in obtaining 12C and He at the same site is tried to be overcome by mixing hydrogen into the helium layer. They examine especially helium flash calculations in a degenerate stellar core. Their calculations also are very sensitive to the choice of β-decay half-lives. This scenario, especially the 12C abundance, has still to be checked in consistent models.

Wefel et al. 21) investigated explosive carbon burning. That means processing of material in a shock front which underwent complete hydrostatic helium burning before. This is similar to the explosive processing of the burning layers except for the point that the He needed for the 12C(a,n)16O reaction is produced in explosive carbon burning via the reaction 12C(12C,p) 20Ne. The authors used either solar values or values from complete core He-burning (Lamb et al. 40) for the initial abundances of heavy nuclei, which gives only a strong s-enhancement for the first peak. Consequently the result does not resemble the solar r-pattern - similar as in explosive He-burning with little s-processing.

There might be an additional site of great interest which can be concluded from recent results by Thielemann and Arnett 44). They find that a strong s-process occurs on the already He-s-processed seeds also in hydrostatic carbon burning. This process occurs via the following reactions:

\[
\begin{align*}
\alpha(0.50) & 20Ne \\
12C(12C, p 0.50) & 2Na \\
n(10^{-3}) & 22Mg
\end{align*}
\]

leading to a neutron flux as strong as the α- and p-channel in the 12C+12C-reaction. Such a strongly s-processed seed distribution from partial C-burning might be of great interest for explosive C-burning and would also lead to a kind of (n)-process.

In explosive Ne-burning where Ne is photodisinTEGRATED via 20Ne(γ,α)16O, (α,n) reactions will also play an important role. Those alternative sites should also be examined and as they probably lead to smaller neutron fluxes the solution of an r(n)-process network including neutron capture rates will become necessary.

5. Conclusions

It is shown that the astrophysical r-process and the question of its site are very sensitive to "standard" nuclear physics parameters like the beta decay properties and neutron capture rates. Since for these quantities in almost all r-process calculations up to now, and also in all estimates of the production rates of chronometric pairs, only very rough assumptions have been made, it is attempted in this paper to present procedures which put the calculation of these quantities for nuclei far from stability on a reliable physical basis. This is done by a microscopic description of the beta strength function and by using a statistical model based on a "next to first principles" optical potential including effects of deformation for the neutron capture rates. The β-decay rates for -6000 nuclei between the β-stability line and the neutron drop line are calculated. The heavy element synthesis by explosive He burning then is calculated using these β-rates and using realistic star models, treating the supernova explosion hydrodynamically. It is shown that a consequence of the revised β-decay rates - explosive He burning seems at present to be the most convincing site of an r-process leading to a solar r-element distribution in one event.

References

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ically accessible in $\beta^-$ decay are the "pygmy" resonances in the low-energy tail of $S_2$ produced for neutron-rich nuclei mainly by back-spin-flip transitions (see Fig. 1). The wave functions of the low-lying states are relatively pure compared to the collective highest-lying states.

Some additional strength in the $Q_0$ window which will occur for neutron-rich nuclei with $v=(3N)^{2/3}/(3N)^{1/3} \approx 1$ from the low-lying $E1(\Delta T=-1, \mu_z=-1, \Delta N=-1)$ mode $^{21}$ is excited by first-forbidden transitions will not be discussed here.

The sensitivity of the position of the calculated GTGR on $G_2/G_1$ allows one to fix the parameter $G_2$ from the recently published $(p,n)$ and $(^3\text{He},t)$ charge exchange reactions$^{37,44,51}$ These data show in contrast to the theoretical expectation of Ref. 14 but confirming an earlier expectation$^{21}$) a concentration of the GT strength in a high-lying resonance, and are reproduced by our calculations$^{44,51}$ The fact that the total GT strength measured in these reactions exhausts only about 25% of the calculated GT sum rule is the first direct indication that a renormalization of the axial beta current is necessary$^{52}$ This is discussed with its consequences in a separate paper$^{52}$), together with its connection to the M1 multipole strength and the renormalization of the axial $g$-factor in $^{235}\text{Pb}$, e.g. only $15\%$ of the M1 sum rule strength is observed by $(n,\gamma)$ and $(\gamma,n)$ reactions$^{21,52}$, being below the sensitivity limit of the $(e,e')$ reaction$^{22}$.

A renormalization of the axial beta current is not done in this paper; the necessary reduction is performed here for the low-lying part of $S_2$ schematically by corresponding choice of $G_2(G_2 \approx 2/3-1)$. The calculated $S_2$ and beta decay half-lives are in good agreement with experiment where a comparison to experimental results is possible (see Figs. 1,2; for further examples see Ref. 14). Shell effects, for example, the sharp decrease in $T_{1/2}$ in the Rb isotopes due to the beginning of the occupation of the $g_{9/2}$ neutron shell, are reproduced (Fig. 2). In the beta strength function they in general lead to strongly different $T_{1/2}$ values (and $\alpha$-delayed fission rates; see Klapdor and Oda$^{52}$) than expected from the gross theory. This is evident from Fig. 1, which compares the $S_2$ of $^{235}\text{Rb}$ calculated by our model with the gross theory and with experiment: The position of the lowest-lying peak in $S_2(E) (g_{9/2} \rightarrow g_{9/2}$ back-spin-flip transition) here is decisive for the $P_0$ value.

In the same way as our calculations reproduce the $\beta$-decay properties of $^{235}\text{Rb}$, they predict, e.g. the systematics of $\beta$-decay properties found experimentally in the particularly well-investigated mass range around $A=90$ (K.L. Kratz et al., this conference, or the systematics in the neutron-rich Na isotopes (Ziegert et al., this conference)).

The general result for the half-lives obtained is as follows: The half-lives for neutron-rich nuclei are systematically shorter, up to an order of magnitude, than those predicted by the gross theory$^{22}$ of $\beta$-decay (see Fig. 2), which on the other hand tends to underestimate the half-lives for nuclei close to the beta stability line. The difference between the predictions of the two theories depends in detail on $P_0$ and $A$. Figure 2 shows some examples of our calculations. The whole body of our results will be given elsewhere$^{52}$.

### 3.2 Neutron capture rates for heavy nuclei

As some compromise between accuracy and limited computer time we calculated the neutron capture cross sections by use of the statistical model$^{59}$. We thus have to calculate the expression for the total cross section of $\sigma^{12}(E)$ of the reaction $n$(j,k)$\gamma$, where target i (reaction product 1) is in the excited state $\mu(\nu)$.

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**Fig. 2** Microscopically calculated $\beta$-decay half-lives (in sec) against mass number of neutron-rich nuclei of the elements Rb, J, Mo, Cs, La, Sr (middle of the solid lines). Squares denote experimental half-lives; dotted lines, gross theory$^{52}$.
\[
\begin{align*}
\sigma_{ij}^{\mu\nu}(E_j) &= \frac{\pi n^2/2m_1}{\Delta(J+1)\Delta(J+1)} \sum_{J_{1s}} (2J+1) \times \\
T_j^{(1)}(E_j, J_{1s}, E_{i1}, J_{i1}) &= \sum_{J_{1s}} \sum_{s=1}^{J_{1s}} \gamma_j^{(1)}(E_j, J_{1s}, E_{i1}, J_{i1}) \\
T_j^{(E_j)} &= \sum_{E_j} E_j^{(1)}(E_j, J_{1s}, E_{i1}, J_{i1}) \\
T_j^{(E_j, J_{1s}, E_{i1}, J_{i1})} &= \sum_{J_{1s}} \sum_{s=1}^{J_{1s}} \gamma_j^{(1)}(E_j, J_{1s}, E_{i1}, J_{i1}) \\
T_j^{(E_j, J_{1s}, E_{i1}, J_{i1})} &= \sum_{J_{1s}} \sum_{s=1}^{J_{1s}} \gamma_j^{(1)}(E_j, J_{1s}, E_{i1}, J_{i1}) \times \\
\rho(E_j, J_{1s}, E_{i1}, J_{i1}) \text{d}E_i
\end{align*}
\]

Thus the important ingredients are the transmission coefficients \(T\) for the particle channels, the \(\gamma\)-transmission coefficients and the level densities \(\rho\).

### 3.2.1 Particle transmission coefficients

We used an optical model potential for neutrons and protons obtained from infinite nuclear matter calculations with Brückner-Hartree-Fock approximation and Reid's hard core interaction by applying the local density approximation to finite nuclei (Jekoune et al.\(^{39}\)).

Such a "next to first principle calculation" on theoretical grounds should allow a more reliable extrapolation into the region of unstable nuclei than phenomenological models fitted to stable nuclei. The results for the s-neutron strength function \((S_0^{\mu\nu}/D_0 \text{ g} y) = (1/2\pi \ln(1/1-T)) \text{E}_{11} \text{ g} y\) obtained with different potentials are shown in Fig. 3a. The "equivalent square well" result usually used in astrophysical applications\(^{17, 23}\) is the straight line.

The deviation of our calculation from experiment around 2 A=160 is due to deformation. As for deformed nuclei a coupled channel calculation is not possible within this framework, we performed the following approximation. Averaging the deformed potential (deformation \(\alpha\) taken from the droplet model) over all possible incident particle angles is equivalent\(^{16}\) to a spherical potential of larger diffuseness and the latter is used\(^{25}\) to reproduce the behavior around A=160 as shown in Fig. 3b.

### 3.2.2 \(\gamma\)-transmission coefficients

We include E1 and M1 transitions. The M1 transitions are treated as in Holmes et al.\(^{39}\); the most important E1 transitions can be described by

\[
T_{E1}(E) = \frac{8 e^2}{3 m c^2} \text{N} \text{Z} \left( \begin{array}{c}
\frac{1}{2} c_i \frac{1}{2} c_i \frac{1}{2} c_i \frac{1}{2} c_i \frac{1}{2} c_i
\end{array} \right)
\]

where \(E_m\) and \(E\) are the energy and the width of the giant dipole resonance (GDR). For deformed nuclei the GDR splits into two components for oscillations parallel and perpendicular to the axis of rotational symmetry. Then the coefficients \(C_1 = 1/3, C_2 = 2/3\) have to be applied. The energy of the GDR can be predicted very accurately by the droplet model\(^{26}\).

### Fig. 3a Neutron strength functions calculated from different optical potentials (\(\alpha\) equivalent square well potential (Holmes et al.\(^{39}\)), \(\alpha\) Moldauer\(^{27}\), \(\alpha\) Becchetti and Greenlees\(^{17}\), \(\alpha\) Wilmore and Hodgson\(^{27}\), Jekoune et al.\(^{39}\)); experimental values are taken from Lynn\(^{46}\).

### Fig. 3b Same as Fig. 3a, but including an "equivalent spherical potential" for deformed nuclei, where the deformation parameter \(\alpha\) is taken from a droplet model\(^{26}\).

For the width we used a macroscopic-microscopic treatment \((\Gamma = \Gamma_0 + \Delta\Gamma)\) which includes a part \(\Gamma_0\) due to viscous damping (one-body damping model)\(^{19}\) and a broadening term \(\Delta\Gamma\) due to coupling to quadrupole surface vibrations which explains the shell structure effect in the width\(^{67}\). The results shown in Fig. 4a give very good agreement (for A>110 generally within 0.5 MeV) and are again safer for nuclei far from stability than the purely empirical treatment by Holmes et al.\(^{39}\) (see Fig. 4b) who also do not include the splitting for deformed nuclei.

### 3.2.3 Level densities

The back-shifted Fermi gas formula which is used for example in Refs. 30,75 is known to fit level densities quite well at high excitation energies (see, e.g. Ref. 24). But at low excitation energies which is most important for the \(r\)-process, where typical excitation energies are of the order of a few MeV, this formula is not applicable. Gilbert and Cameron\(^{26}\) proposed the constant temperature formula for low excitation energies.

We perform a microscopic treatment using a model of fermions interacting via the pairing force and treating this interaction within the BCS approximation (Arnold et al.\(^{24}\), this conference). This leads to better predictions in particular at low excitation energies and for deformed nuclei.

### 3.2.4 Some results

Some typical results for neutron capture rates of stable nuclei are shown in Fig. 5. The agreement is usually within a factor of 2. The same procedure can also be applied for
Fig. 4a Width of the giant dipole resonance as calculated by Thielemann and Arnold. For deformed nuclei the GDR splits into two parts. Experimental values from Ref. 7.

Fig. 4b Prediction of Holmes et al. of the GDR widths. Deformation is not included; thus the splitting for deformed nuclei cannot be predicted.

Fig. 5(a,b) Calculated total $(n,\gamma)$ cross sections using the procedure given in the text. Experimental values from Ref. 6. The agreement is generally within a factor of 2.

Fig. 6(a,b) Calculated total $^{51}V(p,\gamma)$ and $^{51}V(p,n)$ cross sections. The inclusion of width fluctuation corrections and realistic neutron strength functions prevent a too large drop in the $(p,\gamma)$ cross section at the neutron threshold. Experimental values from Ref. 76.

charged particle reactions (see Fig. 6) where the inclusion of width fluctuation corrections prevents the much too deep drop at channel thresholds obtained by Woosley et al. 75).

4. The site of the r-process

Since neutron abundances necessary to shift the abundance peaks in each isotopic chain 10-15 mass numbers out of the valley of a-stability cannot be obtained on long time scales, the r-process has to be connected with explosive events. Up to now, mainly two sites have been discussed in the literature, which could produce the necessary neutrons. They are discussed in the following two subsections together with the new results obtained for the explosive He-burning in this paper. The third subsection briefly discusses further eventually possible sites.

4.1 The "classical" r-process

The "classical" r-process is connected with the outer edge of the core of a massive star which undergoes a supernova explosion, where neutron number densities up to $10^{28}$ cm$^{-3}$ and temperatures ranges of $10^{15}$ K are reached. During the preceding gravitational collapse the heavier nuclei underwent photodisintegration and electron capture by the degenerate electron gas, which increased the total neutron to proton ratio. In the subsequent explosion for temperatures above $5 \times 10^{9}$ K the heavier "seed nuclei" for the r-process are built up almost instantaneously in a nuclear statistical equilibrium, which is governed by the temperature, density and the total neutron to proton (N/P) ratio. For outer zones, at lower temperatures, the buildup of seed nuclei has to be followed explicitly in a network calculation.

For a special choice of the radial dependence of the N/P ratio, and of the position of the masscut between the remaining neutron star and the ejected envelope, Hillebrandt et al. 28) were able to obtain the relative solar r-abundances in one event. However, the N/P ratio was a rather unknown quantity at that time. Further, the assumption that this masscut, necessary for reproducing the solar r-abundances, is the same for all supernovae leads to a tremendous overproduction (by a factor of $10^{3}-10^{4}$) of r nuclei 77). It might be noted here that the
4.2 Explosive helium burning

This type of r-process independently proposed by Hillebrandt and Thielemann[6] and Truran et al.[7] to avoid the serious problems of the r-process discussed in the previous section utilizes the fact that all $^4\text{He}$ left from hydrogen-CN burning is transformed into $^{22}\text{Ne}$ during hydrostatic helium burning via the reaction chain $^{14}\text{N}(\alpha,\gamma)^{19}\text{F}(p)^{20}\text{Ne}(\alpha,\gamma)^{22}\text{Ne}$. The reaction $^{22}\text{Ne}(\alpha,n)^{25}\text{Mg}$ is driving the s-process during hydrostatic helium burning. When the burning shell is hit by a supernova shock front, the neutron number density produced by this reaction rises to $10^{11}$ cm$^{-1}$. Figure 7 shows, for a

![Graph showing temperature and density as functions of time for different mass zones in the He burning shell](image)

The different maximum neutron abundances are not so much the result of different density and temperature profiles but reflect the uncertainty in the $^{22}\text{Ne}$ abundance. At the high temperatures ($3.4 \times 10^8$ K) in the He-burning shell a massive s-process will occur which leads to a partial depletion of $^{22}\text{Ne}$, but on the other hand a constant mixing of $^{14}\text{N}$ into the outer zones into the He-shell. This will counteract this effect. This means that curve c in Fig. 8 has probably to be taken as a lower limit. In all cases an $(n,\gamma)-(\gamma,n)$ equilibrium for nuclei with A > 100 will be achieved ($n_0 > 10^{14}$ cm$^{-3}$) for the first 0.2 s. Thus the buildup of heavy elements is governed by two equations:

$$\frac{Y_{Z,A+1}}{Y_{Z,A}} = \frac{G(z,A+1)}{G(z,A)} \frac{\nu_A}{2} \frac{\nu_A}{2} \frac{3/2}{\exp(\varepsilon/kT)}$$

with $Y_{Z,A}$ abundance of nucleus $A_Z$, $Y_n$ neutron abundance, $G$ partition function, $\nu$ density, $N_A$ Avogadro number, k Boltzmann constant, T temperature and Sn neutron separation energy, describes the abundance relations in one isotopic chain.

$$\gamma_{Z} = \sum_A \gamma_{Z,A} = -\sum_A \gamma_{A,Z}$$

describes the changes of abundances by $\beta$-decays.

In the results of the calculations described below, the mass formula is that of Hilt et al.[6]. Before showing the results a characteristic feature of an r-process during explosive helium burning should be noted:

The seed nuclei even for large A's are already existing because of previous s-processing in the He-burning shell. Thus this process is more a "shaping" of an s-distribution into an r-distribution than the building up of heavy elements from the iron peak. We would further like to note already here that the results obtained for an r-process in explosive He-burning will show up to be sensitive mainly
to the following quantities: From the astrophysics side 1. to the amount of $^{22}$Ne available and the degree of its depletion (for which no calculations exist up to now), 2. to the preshock element distribution in the He burning shell - from the nuclear physics side to the beta decay rates.

![Graph showing computed r-process abundance distributions in explosive He burning at freeze-out (crosses) and solar system r-process abundances (the latter from Ref. 16, shown as dots). All abundances are normalized to log $Y(\text{Si}) = 6$. β decay half-lives from this work. Model: Density $\rho(t) = 10^6 \exp(-t/\tau)$ with $\tau = 1$ s and adiabatic expansion. Preshock He shell composition: a) solar, modified by hydrostatic CNO and He burning. b) Like in a) but s-processed according to Ref. 67. c) Like in b), but with additional enhancement of the Ba-peak by a factor of 10.

Figure 9 shows the results for a density-temperature profile corresponding to a peak temperature of $8 \times 10^8$ K and a peak density of $10^6$ g cm$^{-3}$ and exponential adiabatic expansion on a time scale of 1 s. In Fig. 9a) a solar preshock composition of heavy elements (Z > 24) is used. Figure 9b) uses solar preshock abundances enhanced according to Truran and Iben$^{27}$ by s-processing during hydrostatic burning. This latter assumption was motivated by the fact that shell-He-burning of massive stars releases neutrons with densities typically of the order of at least $10^{11}$ cm$^{-3}$ over time scales of a few hundred years. The preshock neutron exposure is therefore even larger than the one obtained$^{(67)}$ for thermally pulsating medium mass stars. In addition one can expect from the rather high continuous neutron flux that the heavy s-nuclie (Ba-peak) are even more enhanced relative to the other ones$^{(68)}$.

Using this assumption leads to the r-process distribution shown in Fig. 9c). A hydrodynamic supernova explosion calculation of the Weaver-Woosley 25 M$_\odot$ mass star with a lower bound for the initial $^{22}$Ne abundance (see above) leads to the result shown in Fig. 10. All cases use the microscopically computed β-decay half-lives (see section 3.1 and Klapdor et al.$^{44}$).

Fig. 10 Computed r-process abundances in explosive He-burning at freeze-out. Model characteristics: Hydrodynamically computed densities, temperatures and neutron concentrations for the 25 M$_\odot$ star model of Ref. 70. Initial composition: s-enhanced (as in Ref. 67). β-decay half-lives from this work.

Fig. 11 Same as in Fig. 9b), but β-decay half-lives from gross theory of β-decay.

The use of these β-rates removes the problems of obtaining a solar r-distribution still existing in the work by Thielemann et al.$^{(61)}$, who used β-rates from the gross theory of β-decay. Figure 11 shows a typical example of what is obtained instead.
DISCUSSION

H. Morinaga: I do not think it is justified to use statistical model for estimating $\sigma(n,\gamma)$ along the r-process path where $E_{\gamma} = 2$ MeV, because both level density and $\Gamma_\gamma$ become much smaller than those for stable nuclei. There are also experimental evidences. In order to explain very many neutron capture reported by D. Hofman in the Lysekil conference with statistical model, Cameron has to introduce very strange mass formula which is not correct. Introduction of direct neutron capture can explain the multiple neutron capture. H. Morinaga (Proc. Conf. on Medium-Light Nuclei, Firenze, 1977).

F.-K. Thielemann: The preliminary r-process calculations shown here are still performed assuming an $(n,\gamma)$-equilibrium, thus these results are not affected. But for the freeze-out and low neutron abundances neutron captures have to be taken into account explicitly. When this is done I completely agree with you that the direct capture processes should be included in those parts of the r-process path where they give a large contribution to the total cross-section. Unfortunately, there exist no reliable models yet for predicting direct capture cross-sections for the nuclei of relevance in this problem.

M. Arnould: I want to emphasize that in the attempt to explain the solar system r-process abundance distribution in the framework of a shell (He) burning r-process model, one does not have to forget that the A ≤ 65 abundance distribution is in great danger of becoming severely messed up [large overproduction (with respect to solar) of relatively neutron-rich isotopes, a typical example being $^{46}\text{Ca}$]. This might put constraints on the possible net contribution of such a type of r-process to the solar system material.

F.-K. Thielemann: It is right that A ≤ 65 abundances will also consist of relatively neutron-rich nuclei. But there have also been seen as isotopic anomalies in meteorite inclusions (Ca, $^{40}\text{Ar}$, $^{52}\text{Ti}$), which underlines the possibility of this process.

$^{46}\text{Ca}$ (overproduced in this calculation) is a special case also causing trouble in other places. This discrepancy could perhaps be traced back to the experimentally unknown neutron capture cross-section of $^{46}\text{Ca}$ and not to the astrophysical site.