THE BETA MINUS STRENGTH FUNCTION OF NUCLEI FAR FROM STABILITY IN THE A = 90 MASS REGION

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Abstract

The existing experimental $\beta$-strength functions, $S_\beta(E)$, in the A = 90 mass region are reviewed and compared to nuclear model predictions. Systematic trends in $S_\beta(E)$ as a function of nuclear type and excitation energy, as well as the influence of shell effects and deformation are discussed.

1. Introduction

In recent years, the development of the Gammow-Teller (GT) strength in medium-heavy nuclei has been extensively studied via charge exchange reactions (see e.g.,$^{1-3}$ and $\beta$-delayed $\gamma$-ray and neutron spectroscopy$^{4-7}$). An important result of these investigations is that a considerable fraction of the total GT strength is concentrated in low-lying 'pygmy' resonances which appear to be well separated by several MeV from the main strength located in the collective GT giant resonance (GTGR) near the isobaric analogue state (IAS).

These findings are in contradiction to the old concept of a structureless GTGR developed nearly 20 years ago by Ikeda et al. $^{8}$, a concept which later also became the basis of the gross theory of $\beta$-decay (see e.g.,$^{9-10}$). While this model appeared useful for predicting the general variations of average $\beta$-decay properties like half-lives and $\beta$-delayed particle emission probabilities, it did not provide quantitative agreement and required different sets of parameters to describe different quantities. These inconsistencies indicate that the simple gross theory does not properly reproduce the GT strength at lower energies. As expected from general nuclear structure considerations$^{13,15}$ and confirmed by the recent experiments mentioned above, resonance-like structures in the tail of the GTGR which fall within the $Q_{\beta}$-window of isotopes far from stability become decisive in determining their $\beta$-decay properties.

Recently, more realistic shell model calculations of GT strength functions have shown to successfully describe the experimental situation$^{16,17}$ and, for the first time, seem to allow reliable extrapolations to far unstable regions still outside experimental reach.

Given this situation, it appears worthwhile to examine the present status of experimental and theoretical information on $S_\beta(E)$ of nuclei in the A = 90 mass region where the most complete data set is available by now.

2. Investigation of complete $\beta$-strength functions

For $\beta$-decay of nuclei far from stability, it has become a convenient approach to treat averages over a large number of highly excited states instead of analyzing individual transitions. Therefore, the general concept of a strength function in nuclear physics has been taken over by defining $S_\beta(E)$ as the energy distribution of the reciprocal $ft$ value$^{18}$, where the experimental quantities to be determined are the half-life, the isobaric mass difference $Q_\beta$ and the relative $\beta$-feeding per energy interval. The latter quantity, for the excitation energy range below the neutron binding energy $E_n$, is usually deduced from $\gamma$-ray spectroscopic studies applying well known coincidence techniques (see e.g.,$^{19,20}$). For the energy region above $E_n$, spectroscopy of $\beta$-delayed neutrons becomes essential. Since for nuclei far from stability the 'pygmy' resonances in $S_\beta(E)$ lie at medium excitation energies, we will restrict ourselves to briefly outline the experimental development of neutron spectroscopy during the last few years.

Besides technical problems, difficulties lie in the $\beta$-delayed neutron decay mode itself. Since it represents a two-step decay, it is not always straight-forward to derive unique information on the individual process of $\beta$-decay. In many cases, neutron emission can not only lead to the ground state but also to excited levels in the residual nucleus$^{21-23}$. This requires unfolding the singles neutron spectra into partial spectra to each final state by measuring delayed neutrons in coincidence with $\gamma$-rays depopulating those excited final levels$^{24-26}$.

2.1 The singles neutron spectrum of $^{95}$Rb

Different neutron spectrometer types have come to application in delayed neutron spectroscopy, each detector having its advantages and disadvantages. However, for neutron spectroscopy of fission products with energy spectra in the range 0 - 3 MeV, high-resolution $^9$Be ionization chambers seem to be superior because of their overall properties. As an example for the quality of data now available, Figure 1 shows the neutron spectrum from $^{95}$Rb decay after correction for complete detector response$^{27}$. These, and most of the other $\beta$-data presented here, were obtained at the isotope separator DITIS$^{28}$ installed at the Grenoble high-flux reactor. The energy resolution for the $^{95}$Rb spectrum is 13 keV for thermal and about 23 keV for 1 MeV neutrons. This, together with the minimization of spectrum distortion effects$^{24,26}$ allowed to resolve the strong 13.6 keV line from the thermal neutron peak present in the original pulse height distribution.

The insert of Figure 1 shows the low-energy part of the $^{95}$Rb neutron spectrum measured with the time-of-flight (TOF) technique utilizing a Li-glass detector$^{27}$. The strong line at about 14 keV is clearly confirmed. With a half-width of 0.45 keV in this experiment, and the still rather low level-density$^{29}$ just above $E_n$ of 4.4 MeV$^{29}$ in $^{95}$Sr, this peak with very high probability represents a single neutron transition from an individual emitter level to the ground state of $^{94}$Sr. From its absolute intensity a $log ft$ value of 5.7 can be deduced for the $\beta$-branch preceding this neutron transition.

Improved neutron spectrum simulation techniques similar to the simple approaches$^{29-31}$ have indi-
cated that - in contrast to the belief of Hardy et al.\textsuperscript{30} - peak stripping analyses for at least the first few hundred keV of neutron spectra from precursors not too far away from stability, like e.g. $^{95}$Rb, are reasonable. It is worth to be mentioned in this context that our qld analysis of the $^{87}$Sr delayed neutron spectrum\textsuperscript{52} has been confirmed impressively by the recent neutron cross section measurements of Fogelberg et al.\textsuperscript{39}.

![Figure 1](image1)

**Figure 1**

Neutron spectrum from $^{95}$Rb decay after correction for detector response. The insert shows the low-energy part of the spectrum measured with a Li-glass detector.

2.2 Beta-delayed neutron decay from $^{95}$Rb to excited states in $^{94}$Sr

For $^{95}$Rb a large energy window ($Q_{\beta} - Q_{\gamma}$) = 4.9 MeV is available for neutron emission\textsuperscript{39} so that population of many excited states in the residual nucleus becomes energetically possible. Most of these levels in $^{94}$Sr and their spin assignments were already known from $\gamma\gamma$-coincidence and $\gamma\gamma$-angular correlation experiments of $^{93}$Rb $\beta$-decay\textsuperscript{30}. For defining the neutron branching $B_{n}$, $\beta$-delayed neutron spectra have been measured in coincidence with the strongest $\gamma$-lines depopulating the lowest final levels by applying $^{3}$He ionization chambers\textsuperscript{4,5,7,23} as well as the TOF technique with a NE213 liquid scintillator for neutron detection. In addition, for mass 95 time dependent and equilibrium $\beta\gamma$ and $\gamma\gamma$-coincidence spectra were recorded in order to determine absolute neutron branching ratios by normalizing the relative intensities of $\gamma$-lines originating from $^{95}$Rb($n_{p}$)$^{95}$Sr decay to known absolute $\gamma$-ray intensities of $^{95}$Rb-Sr-Y and $^{94}$Sr-Y. In Figure 2 the resultant scheme of neutron emission to the final states in $^{94}$Sr is shown. The main features are that strong neutron branches only lead to the ground and the first $2^+$ state, whereas all higher energy levels up to about 3.5 MeV are found to be weakly populated by delayed neutrons. Furthermore, it is interesting to note that from our $\gamma\gamma$-coincidence measurements we seem to have evidence for neutron feeding of the hitherto unknown excited $0^+$ states in $^{94}$Sr - in Figure 2 they are indicated by dashed lines - which have not been observed in $\beta\gamma$-decay studies of the $0^+ \rightarrow 3^+$ parent $^{94}$Rb. Our tentative spin assignment is based on the systematic lowering of the energies of excited $0^+$ states from $^{96,98}$Sr to $^{94,95}$Sr (see e.g. $^{19,20}$).

2.3 The influence of $S_{\beta}(E)$ on $P_{\text{II}}^n$ and neutron spectrum envelopes

With the correction for $\gamma$-ray emission competing with neutron decay within the first few hundred keV above $Q_{\beta}$, the partial neutron spectra directly define by summation the strength distribution of the preceding $\beta$-decay to levels above $Q_{\beta}$ in the neutron emitter. Generally, the partial neutron spectra corresponding to strongly populated final levels can - although requiring long-term experiments because of the rare $\beta\gamma$-decay mode - be measured with sufficient counting statistics. For the weaker branches mainly feeding higher excited final states, which essentially define $S_{\beta}(E)$ near $Q_{\beta}$, however, this is very difficult or even impossible. In these cases one is limited to integral $P_{\text{II}}^n$-values without neutron energy information.

It is therefore interesting to test the influence of the shape of $S_{\beta}(E)$ on the $P_{\text{II}}^n$ and on the energy distributions of partial and singles neutron spectra. In an attempt to reproduce the experimental data of neutron-rich Rb precursors, we have performed calculations of the above quantities - their theoretical basis being outlined in\textsuperscript{31} - with different specifications of $S_{\beta}(E)$ and partial level widths for neutron emission. The main results of these calculations are that the $P_{\text{II}}^n$ and the spectrum envelopes are predominantly dependent on the shape of $S_{\beta}(E)$ and on the neutron transmission coefficients, and that structure effects in the neutron emission process are - with certain exceptions\textsuperscript{4,13,34} - of minor importance\textsuperscript{39}.

As an example, the results for $^{95}$Rb $\beta$-delayed neutron emission are summarized in Table 1 and Figure 3. From the comparison of experimental and theoretical $P_{\text{II}}^n$ in Table 1, it is evident that the oversimplified assumption of $S_{\beta}(E)$ proportional to level
Table 1
Comparison of experimental and theoretical β-delayed neutron branches \( p_n^l \) from the precursor \(^{95}\text{Rb}\) to the lowest-lying states in the residual nucleus \(^{94}\text{Sr}\)

<table>
<thead>
<tr>
<th>Levels in (^{94}\text{Sr})</th>
<th>( p_n^l(\text{exp}) ) in % of ( P_n^{\text{tot}} )</th>
<th>( p_n^l(\text{calc}) ) in % of ( P_n^{\text{tot}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J^l_f )</td>
<td>( E_L ) (keV)</td>
<td>( S_\beta(E)^{\text{exp}} )</td>
</tr>
<tr>
<td>( 0^+ )</td>
<td>g.s.</td>
<td>66.5 ± 2.5</td>
</tr>
<tr>
<td>( 2^+ )</td>
<td>837</td>
<td>28.8 ± 2.6</td>
</tr>
<tr>
<td>( 3^- )</td>
<td>1926</td>
<td>1.3 ± 0.4</td>
</tr>
<tr>
<td>( 4^+ )</td>
<td>2146</td>
<td>1.2 ± 0.4</td>
</tr>
<tr>
<td>( 2^\prime )</td>
<td>2271</td>
<td>0.9 ± 0.3</td>
</tr>
<tr>
<td>( 3^+, 4^\prime, \ldots ) \text{higher}</td>
<td></td>
<td>3.2</td>
</tr>
</tbody>
</table>

Density still used by some authors\(^{29,30,36}\) fails to account for the experimental data. The strong overestimation of the \( p_1^0 \) to excited states reflects the much too steep slope of this assumption compared to the real shape of \( S_\beta(E) \). The \( p_1^0 \) derived from the gross theory of \( \beta \)-decay are in reasonable agreement with experiment, except for the population of the final states above about 2.3 MeV in \(^{94}\text{Sr}\), indicating that this model underestimates \( S_\beta(E) \) above 7 MeV in \(^{94}\text{Sr}\) (see Figure 4). Best overall agreement with the experimental \( p_1^0 \) is obtained when applying an analytic fit to the experimental \( S_\beta(E) \) - a constant strength between \( B_n \) and 7 MeV and a Gaussian function to model the resonance near 8.5 MeV. Nearly identical results are obtained when using the shell model strength distribution shown in Figure 4. In all these calculations, partial level widths for neutron emission deduced from the Auerbach-Perley transmission coefficients based on a Woods-Saxon potential\(^{37}\) were used. In order to demonstrate the influence of the partial level widths for neutron decay, in Table 1 and Figure 3 results of calculations with transmission coefficients from the square well approximation\(^{38}\) are added. As expected, poorer agreement between the predictions from this approach and experiment is achieved.

Attempts to reproduce the shapes of the experimental singles and partial neutron spectra with the above calculations yield similar results. As is seen from Figure 3 for the \(^{95}\text{Rb}\) singles neutron spectrum, good agreement with experiment is obtained when using the real shape of \( S_\beta(E) \) and the Auerbach-Perley transmission coefficients. In the case of \(^{95}\text{Rb}\) decay, all neutron waves are defined by angular momentum rules from the known spins and parities of the levels connected in \( \beta \)-delayed neutron emission (see Figure 2).

With such comparisons between experiment and calculations as presented here, one is able to prove the consistency of different measured quantities and to deduce first reliable estimates on the shape of \( S_\beta(E) \) from a few integral delayed neutron properties.

3.4 Neutron-gamma angular correlations

For many \( \beta \)-delayed neutron emitting systems the information on spins and parities of initial, intermediate and final states is still scarce. One method, complementary to directly measuring nuclear spins, e.g. by atomic-beam magnetic resonances\(^{39}\) or by \( \gamma \)-angular correlations\(^{19,20}\), may be to determine the neutron wave type of partial spectra to individual states in the residual nucleus. As mentioned in Sec. 2.3, when knowing the real shape of \( S_\beta(E) \), first indications on the \( l_n \)-waves can already be obtained from the neutron spectrum envelopes, their average energies and the \( p_1^0 \)-values. The optimum method, however, to get information on the neutron wave types and their pureness is to measure angular correlations between delayed neutrons and \( \gamma \)-lines depopulating excited final states.

In a first experiment at the OSITIS separator facility, we have performed \( \gamma \)-coincidence measurements at 4 different angles by using two TOF systems and a fast NaI(d)-detector. We have chosen as a test case \( \beta \)-delayed neutron decay from \(^{95}\text{Rb}\) to the 2\(^+\) state in \(^{94}\text{Sr}\) because of its rather strong neutron

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Figure 3
Comparison of experimental and calculated envelopes of the \(^{95}\text{Rb}\) singles neutron spectrum, plotted in 100 keV intervals;
Experiment: data shown in Figure 1,
Calc. I: \( S_\beta(E)^{\text{exp}} \) and \( r_n \) from\(^{37}\),
Calc. II: \( S_\beta(E)^{\text{calc}} \) (gross th.) and \( r_n \) from\(^{37}\),
Calc. III: \( S_\beta(\text{exp}) \) and \( r_n \) from\(^{37}\),
Calc. IV: \( S_\beta(\text{calc}) \) and \( r_n \) from\(^{38}\).
The average neutron energies of the spectra are given in parenthesis.
branch and the known restriction to $p$-wave neutron emission. The preliminary result of the angular distribution of the $\left(3/2,5/2,7/2\right)^{1+} \rightarrow 2^{+} \rightarrow 0^{+}$ $\gamma$-cascade obtained from the integral $\gamma\gamma$-coincidence spectra is shown in Figure 4. From the $W(\theta)$ fit function one can deduce a 15% admixture of $T_{n}^{\pi}=3$ neutrons in the distribution of $T_{n}^{\pi}=1$ neutrons. The data indicate that from such—notably not straightforward—experiments one may, indeed, obtain new information on the $\beta$-delayed neutron emission process.

The experimental $S_{\beta}(E)$ is dominated by two pronounced resonances centered at about 3.7 MeV and 8.5 MeV with a nearly constant distribution in between. It may be of interest here to convert the sum rules strength concentrated in these two peaks back into $\beta$-feeding. We then find that about 64% of $^{95}$Rb $\beta$-decay populates levels in the energy range 3.0 to 4.5 MeV, and that the high-lying resonance which contains about two times the strength of the lower peak, represents a $\beta$-feeding of only 0.06% to the energy range 7.5 MeV to $Q_{\beta}$. This, on the one hand, reflects the very strong influence of the Fermi function on the $\beta$-strength and, on the other hand, demonstrates the importance of weak neutron branches to highly excited states in the residual nucleus for determination of the correct shape of $S_{\beta}(E)$ near $Q_{\beta}$.

The experimental situation is very well reproduced by the shell model which locates the strength of the $\nu_{9/2} \rightarrow \nu_{7/2}$ back spin flip (BSF) transition at 3.6 MeV and a collective 'pygmy' resonance built up by several core polarized (CP) and spin flip (SF) configurations around 9 MeV in $^{95}$Sr. For the gross theory approach, it is evident that it cannot reproduce these resonances simply because nuclear structure effects are neglected in this model.

With the obvious differences in the shapes of the different model strength functions of $^{95}$Rb decay,

![Figure 4](image)

**Figure 4**

Experimental data points and $W(\theta)$ fit function for the $(3/2,5/2,7/2)^{1+} \rightarrow 2^{+} \rightarrow 0^{+}$ $\gamma$-cascade in $^{95}$Rb($\beta\gamma$)$^{94}$Sr$^{*}$

![Figure 5](image)

**Figure 5**

The experimental $\beta$-strength function of $^{95}$Rb decay, plotted as reduced GT transition probability $B'(GT)$ per 100 keV against excitation energy. For comparison, also the theoretical $S_{\beta}(E)$ from the assumption $S_{\gamma}(E)$ of the gross theory and the recent Heidelberg microscopic shell model calculations are shown.

![Diagram](image)
it might be interesting to see how well average $\beta$-decay properties like half-life and $P_n$-value are reproduced for this single case. In Table 2 the results are compared to the experimental data. It is evident that, concerning the sum rule strength inside the $Q_B$-window of $^{95}$Rb, experiment, shell model and gross theory agree within about 10%. Also the experimental $P_n$-value is reproduced here by the shell model (within 10%) and the gross theory (within 30%), whereas for the $\beta$-decay half-life the latter model overestimates this quantity by a factor of five due to the 'missing' strength of the $g_{9/2}$ excitation at 3.7 MeV. as was proposed by Nyman. Figure 6 shows the systematics of the average GT strength for Br, Kr and Rb isotopes as a function of mass number. Obviously, the apparent odd-even effects are not reliably reproduced by the gross theory. Furthermore, the experimental average GT strength for the odd-proton isotope sequences Br and Rb seem to be lower than the gross theory values near $\beta$-stability and slowly approach the model predictions when going away from stability.

Table 2

<table>
<thead>
<tr>
<th>Property</th>
<th>Experiment</th>
<th>Shell model</th>
<th>Gross theory</th>
<th>$S_B(E)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B'(GT)$</td>
<td>1.40</td>
<td>1.58</td>
<td>1.34</td>
<td>not defined</td>
</tr>
<tr>
<td>$0&lt;E&lt;Q_B$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{1/2}$ (s)</td>
<td>0.38</td>
<td>0.33</td>
<td>1.9</td>
<td>not defined</td>
</tr>
<tr>
<td>$P_n$ (%)</td>
<td>8.5</td>
<td>9.5</td>
<td>11.3</td>
<td>28</td>
</tr>
</tbody>
</table>

From a comparison of the detailed model strength functions and their predictions of average $\beta$-decay parameters, as done for $^{95}$Rb, one must conclude that agreement between experimental and theoretical gross quantities like $\beta$-decay half-life (which according to its definition reflects the average $\beta$-decay within the $Q_B$-window) and $P_n$-values (which represent ratios of cumulative $\beta$-branchings to certain excitation energy ranges) should not be overinterpreted, since they may be rather intensive to details in the underlying $\beta$-strength distribution. Therefore, from such average properties alone, in particular when measured for single nucleus, no conclusions should be drawn about the shape of $S_B(E)$ and absolute strength values, as this was done for example in Refs. A, B.

3. Systematic trends in $S_B(E)$ of nuclei in the $A = 90$ mass region

The best way to decide between the validity of different nuclear models is to compare detailed experimental $S_B(E)$ with theoretical predictions. Since such information is, however, still scarce, also systematic investigations of more global quantities over many isotopes may be helpful in indicating certain trends in nuclear structure.

Most data on $S_B(E)$ exist so far in the $A = 90$ region which represents the top of the low mass peak in thermal neutron induced fission of $^{235}$U. In the following, this mass range will be reviewed in some detail.

3.1. Average GT strength

In an attempt to parametrize the $\beta$-strength in a simple way, we have calculated the average GT strength above the pairing gap $C$ per energy interval by the relation

$$B'(GT) = \frac{\sum B'_i(GT)/Q_B - C}{C < E < Q_B}$$

and have compared the experimental values with the predictions from the gross theory. This attempt represents an alternative way to treating average transition rates normalized to the level density.

A similar odd-even effect in the $B'(GT)$ as for neutrons is also seen for protons. The systematics for $N = 51$ to 56 shown in Figure 7 indicates that the average GT strength is higher for nuclei with even proton number than for isotones with odd proton number. This odd-even staggering appears more pronounced in experiment than in the gross theory predictions, in particular for the odd isotope chains. The average experimental GT strength seems to increase relative to the model predictions when approaching the $N = 56$ sub-shell.

Figure 6

Systematics of the average experimental GT strength for the Br, Kr and Rb isotope sequences, in comparison with values calculated from the gross theory of $\beta$-decay.
Already from these systematics we can conclude that the observed deviations of the experimental data from the gross theory predictions—which are consistent with the discrepancies observed when considering other gross $\beta$-decay properties—indicate that this simple model obviously does not correctly describe $S_\beta(E)$ at lower excitation energies.

3.2 Detailed $\beta^-$-strength distributions

In the following we will show that high-resolution spectroscopic data allow to identify details as well as systematic trends of shell structures in $S_\beta(E)$ in the $A = 90$ region. These structures can be understood from the microscopic strength function calculations including deformation, which allow us to attribute explicitly the configurations to the peaks observed in the experimental $S_\beta(E)$.

In Figure 9 we show the experimental strength functions of the isotone series $N = 54$ ranging from $Z = 34$ to 39. These data were obtained from $\gamma$-ray and delayed neutron spec- troscopic data. Regarding the isotones with even proton number, $^{92}$Se, $^{90}$Kr, and $^{88}$Se, the lowest-lying strength at about 1.4 to 1.9 MeV is due to the $\nu\pi_1/2 + \nu\pi_3/2$ back spin flip (BSF) transition. Its strength is seen to decrease with decreasing pro-
ton number. For $^{92}$Sr this BSFS is the only one contributing to allowed $\beta$-decay. With increasing $Q_\beta$, in $^{90}$Kr and $^{88}$Se further GT strength in the daughter nuclei becomes accessible, $\pi P_{1/2}^+ P_{1/2}^-$ and $\pi P_{3/2}^+ P_{3/2}^-$ corepolarized states (CPS) around 3.5 MeV, and in $^{88}$Se beyond $B_n$ additional mixed CPS appear. Concerning the isotones with odd proton number, a similar strength pattern is observed with comparable relative energy distances from $Q_\beta$. In $^{99}$Y no allowed strength falls within the $Q_\beta$-window, explaining the long $\beta$-decay half-life of 10 h.

The same systematic trend in the experimental $S_\beta(E)$ is also seen for the $N = 55$ isotope sequence when comparing the locations of the resonances relative to $Q_\beta$. In Figure 10 the reduced GT transition probabilities of $^{90}$Br up to $^{94}$Y are shown together with the gross theory strength distributions.

Another possibility for studying systematics in $S_\beta(E)$ is to view isotope chains. First indications of the variations in the shape of $S_\beta(E)$ as a function of the $Q_\beta$-window can, for example, be obtained

*Figure 10*

Comparison of experimental $S_\beta(E)$ from the $N = 55$ isotope series with predictions from the gross theory of $\beta$-decay.
when comparing the Sr and Y isotopes presented in Figures 9 and 10. The picture becomes more clearly, however, when regarding longer isotope chains. This is done in Figure 11 for the $\beta$-strength functions of the isotope series $^{87-93}$Kr. For the lighter isotopes $^{87-89}$Kr, the low-lying strength is predicted by the shell model to be mainly due to $v_{p1/2} + \pi_{3/2}$ CPS. In $^{90-92}$Kr at lower energies the $v_{g7/2} + \pi_{3/2}$ ESFS appears, which may already weakly occur in the isotopes $^{88,89}$Kr. In $^{93}$Kr with $N = 57$, occupation of the $v_{g7/2}$ sub-shell starts which leads to the $v_{g7/2} + \pi_{9/2}$ BSFS transition below the $\pi_{3/2} - \pi_{1/2}$ excitation. With increasing neutron number and $Q_\beta$ value further core-polarized states partly mixed with spin flip states become accessible to $\beta$-decay.

MeV in addition to the BSFS and CPS seen in the lighter Rb isotopes. Furthermore, a strong resonance formed by several CP and SF configurations appears just below $Q_\beta$ in $^{93}$Rb. Probably, its low-energy tail is already indicative in $^{92}$Rb. In an attempt to fit these two strong resonances by Gaussian distributions, one finds rather narrow half-widths of about 0.3 MeV for the lower and roughly 0.5 MeV for the higher peak. The strong collective 'pygmy' resonance around 9 MeV occurs also in $^{92}$Rb. Below this strength, a qualitative change of the shape of $S_0(E)$ occurs when going from $^{92}$Rb to $^{95}$Rb. Such a change could be attributed to the sudden onset of strong deformation for this $N = 60$ isotope suggested from recent experimental findings and shell model calculations. With

![Graphical representation of $\beta$-strength functions from isotopes $^{87-93}$Kr.](image)

**Figure 11**

Experimental $\beta$-strength functions from the isotopes $^{87-93}$Kr. The shell model configurations attributable to the peaks in $S_0(E)$ are included (see also text).

In Figure 12 the $S_0(E)$ of the odd-mass Rb isotopes between $A = 89$ and 97 are shown. Within this sequence of nuclei we cross the $N = 56$ sub-shell and enter the region of strong deformation around $N = 60$ (see Refs. 44-47). For the lighter Rb isotopes up to mass 93, the lowest-lying strength corresponds as in the respective longer-lived Kr isotopes - to the $v_{p1/2} + \pi_{3/2}$ BSFS. In $^{95}$Rb with $N = 58$, a strong $v_{g7/2} + \pi_{9/2}$ transition occurs at about 3.5 MeV in addition to the BSFS and CPS seen in the lighter Rb isotopes. Furthermore, a strong resonance formed by several CP and SF configurations appears just below $Q_\beta$ in $^{93}$Rb. Probably, its low-energy tail is already indicative in $^{92}$Rb. In an attempt to fit these two strong resonances by Gaussian distributions, one finds rather narrow half-widths of about 0.3 MeV for the lower and roughly 0.5 MeV for the higher peak. The strong collective 'pygmy' resonance around 9 MeV occurs also in $^{92}$Rb. Below this strength, a qualitative change of the shape of $S_0(E)$ occurs when going from $^{92}$Rb to $^{95}$Rb. Such a change could be attributed to the sudden onset of strong deformation for this $N = 60$ isotope suggested from recent experimental findings and shell model calculations. With

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the experimental $S_b(E)$ would, however, be an indication of the odd proton in the $[431/2]_2^+$ Nilsson orbital, consistent with a $J^N = 3/2^+$ ground state spin and parity of $^{97}$Rb.

Summarizing this section, we can conclude that with the present sophisticated experimental techniques, structures in $S_b(E)$ of nuclei around $A = 90$ can be observed up to high excitation energies, and that they seem to be well understood in terms of a microscopic shell model. It is demonstrated in Figure 12 that shell model calculations, above being able to reproduce experimental data in detail, should allow reliable extrapolations of $\beta$-decay properties into regions still out of reach. As an example, Figure 13 shows the calculated $\beta$-decay half-lives for Rb isotopes up to the neutron drip line. Perfect agreement in the overall $T_{1/2}$ slope and the absolute values is observed when compared to the existing experimental data, in particular above the sub-shell closure at $N = 56$ where a sudden drop in half-life occurs.

**Figure 13**
Comparison of experimental $\beta$-decay half-lives with predictions from a microscopic shell model and the gross theory of $\beta$-decay.

4. Conclusion

When looking back to the situation in this field at the Cargèse conference in 1976 and considering the general progress since then, it seems natural that the theories are being replaced by more sophisticated models yet as the ability of measuring microscopic quantities increases. In this sense, also the well known "parademonium", which according to its authors was mainly developed for pedagogical reasons, should be viewed - perhaps with the final remark that one might ask with a ghost of a malicious smile who was the virtual pedagogue and who the scholar in the lectures on decay properties of tails....

**Figure 12**
Experimental $S_b(E)$ of the odd-mass $^{89-97}$Rb isotopes in comparison with predictions from a shell model and the gross theory of $\beta$-decay.

Ref. 403
Acknowledgement

We would like to acknowledge stimulating discussions with many colleagues, especially with S.G. Prussin who has greatly participated in the development of this field during the past years. We thank C. Ekström, P. Hoff, and M. Zendel for making available data, partly prior to publication, and we are indebted to P. Pauser for providing us with a special s-detector for our latest T0F-experiment at OS-IS. We furthermore acknowledge financial support from the Institut Lauve - Langevin and from the German Bundesministerium für Forschung und Technologie.

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DISCUSSION

P.G. Hamaen: When you say that you used the shell model for 87Rb, which is strongly deformed, do you mean that you used the Nilsson model?

K.L. Knuts: Yes; as pointed out by Klapdor et al., Z.Physik A299 (1981) 213, the spectra and the wave functions for each nucleus are calculated in the deformed Nilsson potential.

K. Takahashi: Could you comment on 89Rb which, in the shell model you mentioned, has already a 3g_{7/2} neutron?

K.L. Knuts: 89Rb is, in fact, not completely clear. Its experimental strength function does show a strong resonance at 4 MeV, which could be attributed to the 5g_{7/2} - 4h_{9/2} transition. Since 89Rb is not yet deformed - or only weakly deformed - in the shell model we used the 3g_{7/2} orbital may be filled before the 1g_{9/2} shell. This is a case where the comparison of experiment with shell model predictions may help to make a unique Nilsson orbital assignment, similar to the case 97Rb I showed.

E.W. Otten: If the structure in either the neutron spectra or the shell strength function is not accidental but reflects some significant physical phenomenon, it should be repeated in some systematic fashion in neighbouring nuclei. I understand that around 95Rb you have the problem of rapidly changing nuclear structure. But could you observe some regularity in regions of relatively stable nuclear structure?

K.L. Knuts: Yes, we could. Such regularities are, for example, seen in the Sr and Kr isotope series, and also in the N=51 to 55 isotope sequences. In all these cases, the 8-strength functions show a similar pattern when regarding the relative energy distance of "pygmy" resonances from the Q_{8}-values.