I. INTRODUCTION

Quantum key distribution (QKD) \cite{1,2,3} allows two people, Alice and Bob, to communicate in secret, where the laws of quantum physics insure the total privacy of their communication. A secret key is generated by Alice transmitting quantum states to Bob, who performs measurements on the received states. An eavesdropper, Eve, who actively attacks the quantum channel, will disturb the quantum system and hence be detected. A passive eavesdropper, however, whose attack simulates the quantum channel will remain undetected, but the maximum information obtainable by this attack is known and can be bounded. Additionally Eve cannot intercept and perfectly copy the quantum states as a consequence of the no-cloning theorem of quantum information \cite{4}. After they have distilled a secret key, Alice and Bob can use this key to communicate secret information over a classical communication channel.

The first QKD protocol, known as the BB84 protocol, uses single randomly polarized photon states \cite{5}. Here Alice prepares and sends a random ensemble of single photon states over a quantum channel to Bob. Bob then measures the states by randomly switching between two non-commuting measurement bases - a compulsory step to insure the security of the protocol. Any loss to the environment or noise on the channel is attributed to Eve, who is only limited in her attack by the laws of physics. Later Alice and Bob produce a sifted key by discarding results where their bases are not the same. Alice and Bob then release a part of their raw key to test for channel transmission and errors in their correlated bit string. Reconciliation protocols \cite{5} are employed in order to correct any errors between Alice and Bob's correlated key. Finally privacy amplification \cite{6} is used to reduce Eve’s knowledge of the key to a negligible amount. Once a secret key has been generated it is used as a one-time pad \cite{7} to encrypt the message. The absolute security of the BB84 protocol has been proven in \cite{8,9,10}. Other single photon schemes proposed include the EPR protocol \cite{11} and the B92 protocol \cite{12}.

QKD using continuous variables was introduced in 1999 \cite{13} as an alternative to the original single photon schemes. Continuous variable \cite{14} QKD offer the advantages of higher detection efficiencies, compatibility with current technologies and faster communication speeds. In 2000, continuous variable QKD using squeezed states \cite{15} and EPR correlations \cite{16} were proposed. Further work included using squeezed \cite{17} and coherent states \cite{18} that generated Gaussian keys against individual Gaussian attacks. All of these protocols were originally only thought to be secure for line losses less than 50\% or 3dB of noise. However, it has been shown that this limit can be overcome by using either reverse reconciliation \cite{19} or post-selection \cite{20} techniques. Reverse reconciliation involves Alice (and Eve) estimating what states Bob has measured rather than the usual way of Bob (and Eve) trying to determine what states Alice has sent. In this sense the flow of classical information is in the reverse direction. The second protocol, post-selection, can also tolerate higher losses by Alice and Bob carefully selecting information for which they have an advantage over Eve. The unconditional security of continuous variable QKD has been proven for squeezed state protocols \cite{21} and Gaussian modulated coherent states using homodyne detection \cite{22}. Collective attacks using reverse reconciliation and their unconditional security was also discussed in \cite{23}. Continuous variable QKD has also been experimentally demonstrated in \cite{24,25,26}.

The random switching of measurement bases by Bob has been a fundamental step in QKD protocols using both single photon states and continuous variables. However recently we introduced a new coherent state continuous variable QKD protocol, against individual Gaussian attacks, that does not require switching. This new protocol, known as the no-switching protocol, involves Bob measuring both bases simultaneously. This was shown to offer higher information rates along with a simpler experimental setup than protocols that use switching \cite{27}. Since then the no-switching protocol has lead to various research, both theoretical \cite{28,29} and experimental \cite{30}. It was also shown in \cite{28} to be secure against collective eavesdropping attacks. In this paper we will expand on our analysis of the no-switching protocol and consider a more thorough eavesdropping attack. We will also discuss an equivalent no-switching protocol for discrete variables, in particular the BB84 protocol.

This paper is organized as follows. In Section II we discuss the no-switching protocol for coherent state continuous variable QKD. Section III discusses and compares two physical realizations of possible eavesdropping attacks. In Section IV...
we discuss an equivalent no-switching protocol for discrete variables. Section V concludes.

II. THE NO-SWITCHING PROTOCOL

A. Notation

Before leading into the steps of the no-switching protocol we will first briefly discuss the nomenclature used throughout this paper. Quantum states that we consider in this paper can be described using the state vector notation $|a⟩$. These quantum states can be described using the boson field annihilation operator $a$ which can be expressed in terms of the quadrature field operators as

$$a = \frac{1}{2}(\hat{X}^+ + i\hat{X}^-)$$  (1)

where $\hat{X}^+$ and $\hat{X}^-$ are the amplitude (+) and phase (-) quadrature operators respectively. The annihilation operator is not measurable in itself as it is non-Hermitian. However, we can express the real and imaginary parts of Eq. (1) as

$$\hat{X}^+ = \hat{a}^0 + \hat{a}$$  (2)
$$\hat{X}^- = i(\hat{a}^0 - \hat{a})$$  (3)

which are Hermitian and therefore measurable. These operators can be expressed in terms of a steady state and a fluctuating component as $\hat{X}^\pm = \langle \hat{X}^\pm \rangle + \delta \hat{X}^\pm$ with quadrature variances defined as $V^\pm = \langle (\delta \hat{X}^\pm)^2 \rangle$. In this paper all operators representing the amplitude and phase observables are denoted by a hat symbol.

B. No-Switching Protocol

In the original coherent state QKD protocols [17, 18] Alice first prepares a displaced vacuum state that will be sent to Bob. This is achieved by choosing two real random numbers $S^+$ and $S^-$ from a Gaussian probability distribution with zero mean $\langle S^\pm \rangle = 0$ and a variance of $V^\pm_S = \langle (S^\pm)^2 \rangle$. She then displaces the amplitude and phase quadratures of the coherent state by $S^+$ and $S^-$ respectively. The displaced coherent state can be represented by the state vector $|S^+ + iS^-⟩$. This state has corresponding operators $\hat{X}^\pm_A$ associated with the amplitude and phase observables of the quantum state. So typically we can express these operators and corresponding quadrature variances $V^\pm_A$ as

$$\hat{X}^\pm_A = S^\pm + \hat{N}^\pm_A$$  (4)
$$V^\pm_A = V^\pm_S + 1$$  (5)

where $\hat{N}^\pm_A$ is the operator associated with the amplitude and phase of the initial vacuum state $|0⟩$, which has a normalized variance $\langle (\hat{N}^\pm_A)^2 \rangle = 1$. Alice transmits this coherent state to Bob through a quantum channel with a channel transmission efficiency $\eta$. The losses in the channel couples to channel noise, which have corresponding quadrature operators denoted as $\hat{X}^\pm_B$ (and with corresponding quadrature variances $V^\pm_N$). Therefore the states that arrive at Bob’s station can be described by the quadrature operators and corresponding variances

$$\hat{X}^\pm_B = \sqrt{\eta}\hat{X}^\pm_A + \sqrt{1-\eta}\hat{X}^\pm_N$$  (6)
$$V^\pm_B = \eta V^\pm_A + (1-\eta)V^\pm_N$$  (7)

where the superscript ‘ indicates states entering Bob’s station. Once Bob receives these states he does not randomly switch between measurement quadratures, instead he simultaneously measures both the amplitude and phase quadratures of the state via a 50/50 beamsplitter and a pair of homodyne detectors. We denote Bob’s quadrature measurements by $\hat{X}^\pm_B$ with a quadrature variance of $V^\pm_B$, which can be expressed as

$$\hat{X}^\pm_B = \frac{1}{\sqrt{2}}(\sqrt{\eta}\hat{X}^\pm_A + \sqrt{1-\eta}\hat{X}^\pm_N + \hat{N}^\pm_B)$$  (8)
$$V^\pm_B = \frac{1}{2}(\eta V^\pm_A + (1-\eta)\langle (\hat{N}^\pm_B)^2 \rangle + 1).$$  (9)

where $\hat{N}^\pm_B$ is the vacuum noise entering into Bob’s 50/50 beamsplitter. Figure 1 gives a schematic of the no-switching coherent state QKD scheme using the no-switching protocol. AM: amplitude modulator, PM: phase modulator; $S^\pm$: random Gaussian numbers; $\hat{X}^\pm_A$: Alice’s prepared state; $\eta$: channel transmission; $\hat{X}^\pm_B$: channel noise; $\hat{N}^\pm_B$: describes the states entering Bob’s station; $\hat{N}^\pm_B$: Bob’s vacuum noise; $\hat{X}^\pm_B$: describes the states Bob measures after the beamsplitter.

![FIG. 1: Schematic of a coherent state continuous variable QKD scheme using the no-switching protocol. AM: amplitude modulator, PM: phase modulator; $S^\pm$: random Gaussian numbers; $\hat{X}^\pm_A$: Alice’s prepared state; $\eta$: channel transmission; $\hat{X}^\pm_B$: channel noise; $\hat{N}^\pm_B$: describes the states entering Bob’s station; $\hat{N}^\pm_B$: Bob’s vacuum noise; $\hat{X}^\pm_B$: describes the states Bob measures after the beamsplitter.](image)
and the overall security of the protocol. We will show that it is still secure against individual attacks and it results in higher information rates as a result of obtaining two simultaneous streams of information from both quadratures, instead of the usual one quadrature measurement.

C. Mutual Information

In analyzing QKD protocols we are inevitably concerned with the net mutual information [30, 31] between Alice and Bob in the presence of Eve, i.e. the rate at which a secret key can be generated by Alice and Bob. In this paper we consider that Alice and Bob use the reverse reconciliation protocol to generate a secret key [19]. For the reverse reconciliation protocol the net information rate can be written as

$$\Delta I = I(B : A) - I(B : E)$$

(10)

where $I(B : A)$ is the mutual information between Bob and Alice and similarly between Bob and Eve, $I(B : E)$. We can define these quantities as

$$I(B : A) = H(B) - H(B|A)$$

(11)

$$I(B : E) = H(B) - H(B|E)$$

(12)

where $H(B)$ is Bob’s Shannon entropy and $H(B|A)$ and $H(B|E)$ are Alice’s and Eve’s conditional entropies relative to Bob’s measurement [31] respectively. The conditional entropy is a measure of how uncertain Alice and Eve are, on average, about Bob’s measurement result. These quantities can physically be thought of as noise, due to the quantum channel and the intrinsic quantum noise of Alice’s state. For the no-switching protocol, Alice encodes independent information onto both quadratures of a coherent state, which Bob then simultaneously measures. Subsequently we can describe the mutual information rate between Alice and Bob as the sum of the quadrature information rates as

$$\Delta I = \Delta I^+ + \Delta I^-$$

(13)

where $\Delta I^+$ is the information rate for the amplitude quadrature and $\Delta I^-$ the information rate for the phase quadrature. By substituting Eqs. (11,12) into Eq. (10) and assuming symmetry for both quadratures, we end up with the information rate for the no-switching protocol given by

$$\Delta I = 2[H(B|E) - H(B|A)]$$

(14)

Hence, in order to determine the final information rate, we need to determine both Alice’s and Eve’s conditional entropies.

D. Conditional Variances

We can express Alice’s and Eve’s conditional entropies of Bob’s quadrature measurements in terms of conditional variances [31]

$$H(B|A) = \frac{1}{2} \log_2(V_{A|B})$$

(15)

$$H(B|E) = \frac{1}{2} \log_2(V_{E|B})$$

(16)

where $V_{A|B}$ and $V_{E|B}$ are Alice’s and Eve’s conditional variances relative to Bob’s measurement respectively [32]. The conditional variance can be thought of as the uncertainty in Alice’s and Eve’s estimates of Bob’s quadrature measurement result. In general the conditional variance of $X$ given the event $Y$ can be written as

$$V_{X|Y} = var(X|Y) = \min_g((Y - gX)^2)$$

(17)

where $g$ is an optimal gain that minimizes the conditional variance. Therefore, the total information rate for the no-switching protocol given in Eq. (14), assuming symmetry of both quadratures, can be written in a simpler form in terms of conditional variances as

$$\Delta I = \log_2 \left( \frac{V_{E|B}}{V_{A|B}} \right)$$

(18)

E. Alice’s Conditional Variance

For the no-switching protocol Alice’s conditional variance of Bob’s measurements is defined as

$$V_{A|B}^\pm = \min_g \langle (\hat{X}^\pm_B - g^*_A S^\pm)^2 \rangle$$

(19)

where $\hat{X}^\pm_B$ is Bob’s quadrature measurement given by Eq. (8). $S^\pm$ is the quadrature displacement of Alice’s prepared state and $g^*_A$ is an experimental gain or Alice’s best estimate at what Bob has measured. This gain is then optimized to give a minimum conditional variance. The minimum gain is given by $g^*_A = \langle S^\pm X^\pm_B \rangle / \langle S^\pm S^\pm \rangle$ which is then substituted into Eq. (19) to give a conditional variance of

$$V_{A|B}^\pm = V_B^\pm - \frac{\langle S^\pm X^\pm_B \rangle^2}{V_S^2}$$

(20)

We now calculate Alice’s conditional variances for the no-switching protocol. To calculate the conditional variances we consider a more general protocol, where Alice can transmit to Bob displaced squeezed states instead of coherent states. This scenario leads to the best possible correlation between Alice and Bob for a particular quadrature measurement. In this case, the quadrature variance of the states prepared by Alice are given by

$$V_A^\pm = V_S^\pm + V_{sqz}^\pm$$

(21)

where $V_{sqz}^\pm$ is the quadrature variance of the squeezed states prepared by Alice. To ensure that the quadrature variances of Alice’s transmitted state remains symmetric with variances $V_A^\pm$, we require that the maximum amount of squeezing is
limited by the variance of Alice’s transmitted states. This is given by the following inequality
\[
V_{sqz}^A \geq \frac{1}{V_A^-}
\]  
(22)
Substituting Eq. (9) into Eq. (20), with \(\langle S^+ X_B^\pm \rangle = \sqrt{\eta/2}V_S^\pm\), we can calculate Alice’s conditional variance to be
\[
V_A^\pm_{|B} = \frac{1}{2}\left(\eta V_{sqz}^A + (1 - \eta)V_N^A + 1\right)
\]  
(23)
where we have used the fact that \(V_S^\pm = V_A^\pm - V_{sqz}^A\) from Eq. (21). We point out that in the no-switching protocol Alice does not actually use squeezed states but rather coherent states. So eventually we will set \(V_{sqz}^A = 1\). We only consider that Alice uses squeezing for our analysis in order to give lower bounds for the quadrature conditional variances.

F. Uncertainty Relations

Before we explicitly calculate Eve’s conditional variance we first derive a general relationship between Alice’s and Eve’s conditional variances. We will then use this relation to bound Eve’s minimum conditional variance and hence Eve’s maximum mutual information with Bob. To calculate a relationship between Alice’s and Eve’s conditional variances of Bob’s measurement, \(V_{E|B}\) and \(V_{A|B}\), we define the operators that denote Alice’s and Eve’s inference of Bob’s measurement before his beamsplitter, expressed as
\[
\hat{X}_{E|B'} = \hat{X}_{B'} - g_E^+\hat{X}_E^+
\]  
(24)
\[
\hat{X}_{A|B'} = \hat{X}_{B'} - g_A^+\hat{S}_E^+
\]  
(25)
where \(g_E^+\hat{X}_E^+\) and \(g_A^+\hat{S}_E^+\) are Alice’s and Eve’s optimal estimates with optimal gains, \(g_E^+\) and \(g_A^+\). Finding the commutator of the above two equations, and using the fact that different Hilbert spaces commute, we find that
\[
[\hat{X}_{E|B'}, \hat{X}_{A|B'}] = [\hat{X}_{B'}, \hat{X}_{B'}] = 2i
\]  
(26)
This leads to \[33\] the joint Heisenberg uncertainty relation
\[
V_{E|B'}^\pm V_{A|B'}^\pm \geq 1
\]  
(27)
Therefore, there is a limit to what Alice and Eve can know simultaneously about what Bob has measured. Once again an important notational point is the superscript \(',\). Whenever this is used it implies that the equations are only dealing with the states before Bob’s beamsplitter. From the above inequality it is possible to determine the maximum information Eve can obtain about the state in terms of Alice’s conditional variances \(V_{A|B'}\).

G. Eve’s Conditional Variance

We now calculate Eve’s minimum conditional variance for an attack which is only limited by the joint Heisenberg uncertainty relation given in Eq. (27) and Alice’s conditional variance in Eq. (25). To find a lower bound on Eve’s conditional variance, we first consider her inference of Bob’s state prior to the 50/50 beamsplitter in Bob’s station. As with Alice’s conditional variance, Eve’s conditional variance is given by
\[
V_{E|B'}^\pm = \min\left(\langle \hat{X}_{B'}^\mp - g_E^\pm\hat{X}_E^\mp \rangle^2\right)
\]  
(28)
where \(\hat{X}_{B'}^\mp\) (defined in Eq. (22)) is the quadrature of Bob’s state that could be measured prior to the beamsplitter and the associated gain \(g_E^\pm\). Eve’s measurement variance after the beamsplitter conditioned on Bob’s measurement \(V_{E|B'}\) can be expressed in terms of the conditional variance before the beamsplitter \(V_{E|B'}^\pm\) as
\[
V_{E|B'}^\pm = \langle \left(\hat{X}_{B'}^\mp - g_E^\pm\hat{X}_E^\mp\right)^2\rangle
\]  
\[
= \langle \left(\frac{1}{\sqrt{2}}\left(\hat{X}_{B'}^\mp + \hat{N}_B^\pm - g_E^\pm\hat{X}_E^\mp\right)\right)^2\rangle
\]  
\[
= \frac{1}{2} \langle \left(\hat{X}_{B'}^\mp - \sqrt{2}g_E^\pm\hat{X}_E^\mp\right)^2\rangle + \frac{1}{2}
\]  
\[
= \frac{1}{2} \langle \left(\hat{X}_{B'}^\mp - g_E^\pm\hat{X}_E^\mp\right)^2\rangle + \frac{1}{2}
\]  
\[
= \frac{1}{2} \left(V_{E|B'}^\pm + 1\right)
\]  
(29)
where we have used the fact that Eve has no access to the beamsplitter in Bob’s station, and therefore has no knowledge of the vacuum entering through it. The uncertainty relation Eq. (27) tells us that there is a limit to what Alice and Eve can simultaneously know about what Bob has measured, i.e. \(V_{E|B'}^\pm \geq 1/V_{A|B'}^\pm\). We can now determine Alice’s conditional
variance of Bob, if Bob were to directly measure a single quadrature of his state before the 50/50 beamsplitter $\hat{V}_{A|B'}$. The derivation of $V_{A|B'}^\pm$ is based on the derivation given in [19], which goes as follows. We can use Eq. (20) with the following equations

$$S^\pm = \hat{X}_A^\pm - \hat{N}_A^\pm$$

$$V_S^\pm = V_A^\pm - \hat{V}_{sqz}^\pm$$

$$\hat{X}_{B'}^\pm = \sqrt{\eta} \hat{X}_A^\pm + \sqrt{1 - \eta} \hat{X}_N^\pm$$

$$\hat{V}_{B'}^\pm = \eta V_A^\pm + (1 - \eta) V_N^\pm$$

$$\langle S^\pm | \hat{X}_{B'}^\pm \rangle = \sqrt{\eta V_S^\pm}$$

The above equations are the same as the no-switching equations, e.g. Eqs. (32), except for the $1/\sqrt{2}$ and $\hat{N}_B^\pm$ that are due to Bob’s simultaneous quadrature measurements. Alice’s conditional variance using reverse reconciliation with switching is given by

$$V_{A|B'}^\pm \geq V_{A|B'}^\pm_{\text{min}} = \left( \eta/V_A^\pm + (1 - \eta)V_N^\pm \right)$$

Using this minimum value with Eq. (27) we can calculate Eve’s conditional variance when switching is used

$$V_{E|B}^\pm \geq (\eta/V_A^\pm + (1 - \eta)V_N^\pm)^{-1}.$$  

Again we emphasize that we have assumed that Eve can simultaneously measure both the amplitude and phase quadratures of her ancilla (or measuring) state without paying a “quantum duty”. This is in fact an unphysical assumption that allows Eve more information than what she is entitled to. We calculate a lower bound on Eve’s conditional variance for the no-switching protocol by substituting Eq. (36) into Eq. (29)

$$V_{E|B}^\pm \geq \frac{1}{2} \left( \left( \frac{\eta}{V_A^\pm} + (1 - \eta)V_N^\pm \right)^{-1} + 1 \right).$$

II. Secret Key Rate

We can now determine the final secret key rate for the no-switching protocol by substituting Alice’s and Eve’s conditional variances from Eqs. (23) and (37) (and symmetrize both quadratures) into Eq. (18). The final secret key rate is then given by

$$\Delta I \geq \log_2 \left( \frac{\eta/V_A^\pm + (1 - \eta)V_N^\pm - 1}{\eta + (1 - \eta)V_N^\pm + 1} \right)$$

where we have set $V_{sqz}^\pm = 1$ to indicate that we are using coherent states. Figure 2(b) shows a plot of Eq. (38) for varying channel transmissions and varying channel noise. We see that it is completely secure for vacuum noise and as the noise is increased the insecure region gets larger (as is the case for all QKD protocols). Figure 2(a) plots the information rate of Eq. (38) (for $V_N^\pm = 1$ and $V_A^\pm = 100$) against the channel transmission (dashed line). We can now compare the no-switching protocol (dashed line) to the switching protocol (dot dashed line) of [19]. Figure 2 shows that the no-switching protocol has a higher information rate than the switching protocol for all channel transmission losses.

We now turn our attention to physical implementations of eavesdropping attacks against the no-switching protocol and compare these attacks to the bound derived in Eq. (38).

III. EAVESDROPPING ATTACK

In the previous section we derived an upper bound on Eve’s maximum information for the the no-switching protocol. In this section we put a lower bound on Eve’s information by investigating a physical eavesdropping attack. A feed-forward scheme was discussed in [27] as a possible eavesdropping attack to the no-switching protocol for varying channel noise (i.e. not just vacuum noise). Here Eve used a beamsplitter to gather information from the quantum channel (see Fig. 1). She then measured both quadratures simultaneously and then feed forward these altered states onto Bob. We denote this attack as the coherent feed-forward attack. We will now consider a more sophisticated attack that incorporates additional entanglement. We show that by giving Eve additional resources, e.g. the use of entanglement, that she gets no additional information compared to the coherent feed-forward attack. We then discuss reasons for thinking that the coherent feed-forward attack might be optimal for individual Gaussian attacks.

A. Entanglement Feed-Forward Attack

Figure 3 shows a schematic of the entanglement feed-forward attack. This attack goes as follows: Eve creates two Einstein-Podolsky-Rosen (EPR) [34] entangled beams by interfering two squeezed beams $\hat{X}_{sqz1}^\pm$ and $\hat{X}_{sqz2}^\pm$ on a 50/50 beamsplitter. The quadratures of the entangled beams are described by the following operators

$$\hat{X}_{epr1}^\pm = \frac{1}{\sqrt{2}} (\hat{X}_{sqz1}^\pm + \hat{X}_{sqz2}^\pm)$$

$$\hat{X}_{epr2}^\pm = \frac{1}{\sqrt{2}} (\hat{X}_{sqz1}^\pm - \hat{X}_{sqz2}^\pm)$$

Eve retains the second of the entangled beams $\hat{X}_{epr2}^\pm$, which she simultaneously measures both quadratures via a 50/50 beamsplitter and two perfect homodyne detectors. We assume that Eve has quantum memory and that she performs these measurements only after Bob has received the states. These resulting quadrature operators and corresponding variances are given by

$$\hat{X}_{E1}^\pm = \frac{\hat{X}_{sqz1}^\pm - \hat{X}_{sqz2}^\pm}{\sqrt{2}} - \hat{N}_{E1}^\pm$$

$$V_{E1}^\pm = \frac{(V_{sqz1}^\pm + V_{sqz2}^\pm + 2)}{4}$$
Bob would be expecting a signal described by the quadrature operator $\sqrt{\eta} \hat{X}_A^\pm$ due to losses in the quantum channel. Now Eve knows that she is sending Bob a signal of $\sqrt{\eta} \hat{X}_A^\pm + g_E^E \sqrt{(1-\epsilon)/2} \hat{X}_A^\pm$. Therefore she wants $\sqrt{\eta} \hat{X}_A^\pm$ to equal $\sqrt{\eta} \hat{X}_A^\pm + g_E^E \sqrt{(1-\epsilon)/2} \hat{X}_A^\pm$. This leads to a gain of

$$g_E^E = \sqrt{2(\sqrt{\eta} - \sqrt{\epsilon})} / \sqrt{1-\epsilon}$$

(44)

We point out that this is the same gain as given for the coherent feed-forward attack [27]. Now because Eve has reduced the transmission of the quantum channel, she will need to add additional Gaussian noise $\mathcal{N}_{sqz}^\pm$ (with variance $V_{sqz}^\pm$) to remain undetected (which she can subsequently infer out of her estimate of Bob’s quadrature measurement with arbitrary precision). Once Bob receives these altered quantum states from Eve, he then measures both quadratures simultaneously

$$\hat{X}_{E2}^\pm = [\sqrt{\eta} \hat{X}_A^\pm - \sqrt{(1-\epsilon)/2} (\hat{X}_{sqz1}^\pm + \hat{X}_{sqz2}^\pm)]$$

(45)

This has a corresponding variance of

$$V_{E2}^\pm = \eta V_A^\pm / 2 \pm V_{sqz1}^\pm + V_{sqz2}^\pm$$

(46)

We are now in a position to calculate Eve’s conditional variance $V_{E1,E2|B}^\pm$ for this entanglement feed-forward attack, as a function of the beamsplitter transmission $\epsilon$. Once we have calculated this conditional variance we can then proceed as before to determine the secret key rate. The conditional variance in this case will be a tripartite conditional variance defined as

$$V_{E1,E2|B}^\pm = \langle(\hat{X}_{E2}^\pm - g_1^E \hat{X}_{E1}^\pm - g_2^E \hat{X}_{E2}^\pm)^2 \rangle$$

(47)

as we need to accommodate both of Eve’s measurements in her estimate of Bob’s quadrature measurements. Minimizing the two gains, we have

$$V_{E1,E2|B}^\pm = V_{E2}^\pm = \frac{V_{E1}^E \langle\hat{X}_{E1}^E \hat{X}_{E2}^E\rangle^2 + V_{E2}^E \langle\hat{X}_{E2}^E \hat{X}_{E1}^E\rangle^2}{V_{E1}^E \langle\hat{X}_{E1}^E \hat{X}_{E2}^E\rangle^2 + V_{E2}^E \langle\hat{X}_{E2}^E \hat{X}_{E1}^E\rangle^2}$$

(48)

The above equation is a function of the two channel transmissions $\eta$ and $\epsilon$. To calculate one in terms of the other we need to consider how much information Eve is allowed to have before she is detected by Alice and Bob. This is related by the following inequality

$$(1-\eta)V_{E1}^\pm \geq (\langle V_{sqz1}^\pm + V_{sqz2}^\pm + 2\epsilon \rangle$$

(49)

$$- 2\sqrt{\eta} (2 + V_{sqz1}^\pm + V_{sqz2}^\pm) + \eta (2 + \epsilon (V_{sqz1}^\pm + V_{sqz2}^\pm))) / (2(1-\epsilon))$$

FIG. 3: Schematic of two possible eavesdropping attacks: (a) Coherent feed-forward attack and (b) entanglement feed-forward attack.
where the left hand side is the amount of noise for an arbitrary quantum channel. This noise must be greater than or equal to the total noise Eve has put onto the channel (as can be seen from Eq. (45)). We numerically minimize $V^\text{FF}_{E1,E2|B}$ for all $\epsilon$ between $\epsilon_{\text{min}}$ and $\epsilon_{\text{max}}$. Alice’s conditional variance remains the same as before (i.e. Eq. (22)) as Eve has applied the correct gain. Now that we have Eve’s and Alice’s conditional variances we can use Eq. (19) to numerically calculate the secret key rate. We find that the secret key rate for the entanglement feed-forward attack is the same as the secret key rate for the coherent feed-forward attack.

In Fig. 2 we plot both the entanglement feed-forward attack and the coherent feed-forward attack (both the same top solid line). We conjecture that this higher bound is in fact the optimal bound for the no-switching protocol against individual attacks. In principle the best Eve can do is to optimally clone the states she intercepted from Alice and then send these cloned copies back onto Bob - which is exactly what the coherent feed-forward attack does. Also by giving Eve more resources, such as entanglement, does not in any way give her an information advantage. This suggests we have found an optimal information bound.

IV. BB84 AND THE NO-SWITCHING PROTOCOL

As we have seen, the no-switching protocol works successfully in the continuous variable regime using coherent states. It eliminates the need to randomly switch bases resulting in simplicity and higher information rates. We now ask the question: can the no-switching protocol be applied to the discrete variable regime and give the same benefits? To determine if this happens we consider an equivalent no-switching protocol for single photon states. This involves introducing a quantum cloning machine in Bob’s station that allows him to imperfectly clone (due to the no-cloning theorem [4]) what Alice has sent into two sets of copies. Bob can then measure both polarization bases simultaneously. This cloning machine will negate the need for Bob to randomly switch measurement bases. As a note, there have been other discrete variable proposals that do not rely on the random switching of measurement bases but rather the random switching of state manipulation, such as the “ping pong” protocol [36].

A. How the No-Switching Protocol for BB84 Works

Alice sends a random ensemble of linearly polarized photons to Bob. These photons are from a set of 4 possible polarizations: horizontal (H) $\equiv |0\rangle$, vertical (V) $\equiv |1\rangle$, diagonal (D) $\equiv (|0\rangle + |1\rangle)/\sqrt{2}$ and anti-diagonal (A) $\equiv (|0\rangle - |1\rangle)/\sqrt{2}$. Bob then uses a quantum cloning machine [35] that takes one input state and outputs two identical clones. Bob can therefore measure the first set of clones with the rectilinear bases and the other set with the diagonal bases. This guarantees that Bob always measures in the correct basis. The classical steps of reconciliation and privacy amplification follows. Figure 4 gives an illustration of the protocol.

FIG. 4: The steps of the BB84 scheme using the no-switching protocol. (a) Alice sends to Bob a random ensemble of photons chosen from 4 possible polarizations. (b) Bob clones the incoming photons from Alice using a universal quantum cloning machine. This produces the two clones $b_2$ and $b_3$. (c) Bob then measures one clone in the horizontal/vertical basis (H/V) and the other clone in the diagonal/antidiagonal (D/A) basis. After communicating classically with Alice, Bob can then discard those times where he used the wrong basis.

B. Bob’s Quantum Cloning Machine

We assume that Bob has a universal quantum cloning machine at his station. This enables him to clone with a maximum fidelity of 5/6 or $\approx 83\%$ of the original input state. The universal quantum cloning machine is state independent so will copy any input state with the same fidelity. Therefore it is suited to QKD as there is no prior knowledge of which of the four polarization states Alice has sent.

To illustrate how the universal quantum cloner works we suppose that Bob wants to copy an arbitrary pure state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ (this pure state corresponds to any one of the four photon polarizations). In our protocol Bob’s inputs a given qubit $b_1$ into his cloning machine which outputs the identical clones $b_2$ and $b_3$. Here the state of the cloning machine is given by $|C\rangle_6$. Bob wants to copy the bases $|0\rangle$ and $|1\rangle$ of the arbitrary pure state. This initial preparation by Bob of his
cloning machine is given by

\[ \begin{aligned}
|0\rangle|0\rangle|C\rangle_b \\
|1\rangle|0\rangle|C\rangle_b
\end{aligned} \right) \tag{50} \]

where \(|0\rangle\) is the blank state (e.g., the blank paper in copying machines) that is used to make the clones with. The resulting output of the cloning machine in Eqs. (50, 51) is given by [35]

\[
\begin{aligned}
\cdots \rightarrow \sqrt{\frac{5}{3}}|00\rangle|\uparrow\rangle + \sqrt{\frac{1}{6}}|+\rangle|\downarrow\rangle \\
\cdots \rightarrow \sqrt{\frac{5}{3}}|11\rangle|\downarrow\rangle + \sqrt{\frac{1}{6}}|+\rangle|\uparrow\rangle
\end{aligned} \right) \tag{52} \]

where \(|+\rangle = (|10\rangle + |01\rangle)/\sqrt{2} \) and \(|\uparrow\rangle\) and \(|\downarrow\rangle\) represent the output states of the quantum cloning machine. The resulting clones are identical, with each being of worse quality than the original qubit that Bob used as the input state. We are then able to perform a partial trace over the cloning machine states which allows us to look at any subsystem of the original density operator. After performing a partial trace over the cloning machines we are left with a density operator describing the resulting two output clones \(\hat{\rho}_{\text{out}}^{b}\). To find the density operator of either of the two clones individually, we can perform a partial trace over either of the two clones. The resulting reduced density operator can then be written as

\[
\hat{\rho}_{\text{out}}^{b} = \frac{5}{6} |\psi_{1}\rangle\langle\psi_{1}| + \frac{1}{6} |\psi_{2}\rangle\langle\psi_{2}| \tag{54} \]

where \(|\psi_{1}\rangle = \alpha|0\rangle + \beta|1\rangle\) and \(|\psi_{2}\rangle = \beta|0\rangle - \alpha|1\rangle\). Eq. (54) is equal to the other density operator \(\hat{\rho}_{\text{out}}^{b}\) due to the symmetry of the system. Eq. (54) tells us that the clones comprise of 5/6 of the original input state and 1/6 of the errors of the universal quantum cloning machine. Therefore by having these clones of the original states sent by Alice, Bob can abandon switching and measure both bases at the same time.

C. Information Rate

Finally to determine whether the no-switching protocol works in the BB84 case we again need to consider information rates. Unlike the continuous variable information rates we previously used, we now need to use a (discrete) binary symmetric channel given by [30]

\[ I_{AB} = 1 + p_{e}\log_{2}p_{e} + (1 - p_{e})\log_{2}(1 - p_{e}) \tag{55} \]

where \(p_{e}\) is the error probability between Alice and Bob’s mutual information. In the normal switching BB84 protocol (where we neglect Eve for the moment), Alice and Bob have an error probability of 25%. However after key siftin,where

they discard incorrect basis measurements, they have zero error probability leading to \(I_{AB} = 1\) bits/signal. But in actual fact \(I_{AB} = 0.5\) bits/signal as Alice and Bob have, on average, thrown away half of their original data in the classical communication step.

D. To Switch Or Not To Switch

In our no-switching protocol for the BB84 protocol we have an error probability of 1/6 or \(\approx 17\%\) (from Bob’s cloning machine), compared with an error rate of zero for the BB84 protocol, after Alice and Bob discard the results for the incorrect measurement basis. Substituting this 17% error into Eq. (55) gives an information rate of \(I_{AB} = 0.35\) bits/signal.

This information rate is slower than the \(I_{AB} = 0.5\) bits/signal for the BB84 protocol and thus it is more beneficial to use the original BB84 scheme. It is important to note that the 17% error is from Bob’s station and not the quantum channel (i.e. not from Eve’s tampering). Normally an error rate of 17% from the quantum channel is too high and would mean that Bob and Alice would not be able to distill a secure key. In our case, before Bob starts cloning, he and Alice can test the quantum channel for eavesdropping and provided it is below a safe threshold, Bob can continue cloning. In any event randomly switching the measurement bases when using discrete variable QKD gives a higher information rate than using the no-switching protocol.

V. CONCLUSION

Quantum key distribution has long made use of the random switching of measurement bases to ensure the security of the protocol. We have shown that switching is not a necessary requirement for coherent state continuous variable quantum key distribution. This was demonstrated via the no-switching protocol. The no-switching protocol gives higher information rates than protocols that use switching and also offers a simpler experimental setup.

We investigated a physical implementation of the eavesdropping scheme in the form of an entangling feed-forward attack. We showed that this attack was no more effective than a simpler coherent feed-forward attack that did not use entanglement. We then conjectured that the coherent feed-forward attack was the optimal attack assuming individual Gaussian attacks.

Finally we have shown that there is no advantage by applying the no-switching protocol to the original BB84 protocol which employs single photon states. This is an interesting result as it highlights the differences between QKD with single photon states compared to continuous variables.

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