Correlation Networks Among Currencies

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Abstract

By analyzing the foreign exchange market data of various currencies, we derive a hierarchical taxonomy of currencies constructing minimal-spanning trees. Clustered structure of the currencies and the key currency in each cluster are found. The clusters match nicely with the geographical regions of corresponding countries in the world such as Asia or East Europe, the key currencies are generally given by major economic countries as expected.

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Introduction

A value of currency is expected to reflect the whole economic status of the country, and a foreign exchange rate is considered to be a measure of economic balance of the two countries. In the real world there are several economic blocks such as Asia, but it is not clarified whether such economic blocks affect the foreign exchange rate fluctuations or not. From the viewpoint of physics, the foreign exchange market is a typical open system having interactions with all information around the world including price changes of other markets. Also, the mean transaction intervals of foreign exchange markets are typically about 10 seconds, and it is not clear how the market correlates with the huge scale information of a whole country or the economic blocks. In order to empirically establish the relations between microscopic market fluctuations and macroscopic economic states, it is important to investigate the interaction of currency rates in the high precision data of foreign exchange markets.

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The correlations among market prices have been analyzed intensively for stock prices by using minimal-spanning trees or self-organizing maps [1] [2] [3] [4] [5]. The interaction among stocks is expected to be caused by information flow, and direction of the information flow has been investigated from a cross-correlation function with a time shift [6][7][8]. L. Kullmann, et al. and J. Kertesz, et al. introduced a directed network among companies for the stocks [7][8]. We observe the interaction among foreign exchange markets using minimal-spanning tree.

We construct a currency minimal-spanning tree by defining correlation among foreign exchange rates as the distance. The minimal-spanning tree is a kind of currency map and is helpful for constructing a stable portfolio of the foreign exchange rates. We use correlation coefficient of daily difference of the logarithm rate in order to detect the topological arrangement of the currencies. The correlation coefficient is computed between all the possible pairs of rates in a given time period. We classify the currencies on the minimal-spanning tree according to the correlation coefficients, and find key currencies in each cluster. We analyze 26 currencies and 3 metals from January ’99 up to December ’03 provided by Exchange Rate Service [9].

**Method of hierarchical taxonomy of currencies**

We introduce a method of hierarchical taxonomy of currencies. We first define correlation function between a pair of foreign exchange rates in order to quantify synchronization between the currencies. We focus on a daily rate change $dP_i(t)$ defined as

$$dP_i(t) \equiv \log P_i(t + 1\text{day}) - \log P_i(t),$$

where $P_i(t)$ is the rate $i$ at the time $t$. Using the rate change, correlation coefficient between a pair of the rates can be calculated by cross-correlation function as

$$C_{ij} = \frac{\langle dP_i \cdot dP_j \rangle - \langle dP_i \rangle \langle dP_j \rangle}{\sqrt{(\langle dP_i^2 \rangle - \langle dP_i \rangle^2)(\langle dP_j^2 \rangle - \langle dP_j \rangle^2)}},$$

where $\langle dP_i \rangle$ represents the statistical average of $dP_i(t)$ for a given time. The correlation coefficient $C_{ij}$ has values ranging from $-1$ to $1$.

We get $n \times n$ matrix of $C_{ij}$ by calculating the cross-correlation function for all combinations among the given rates when $n$ kind of foreign exchange rates are given. It is clear that the matrix has symmetry $C_{ij} = C_{ji}$ with $C_{ii} = 1$ from the definition of Eq.(2). We apply the correlation matrix to construct a currency minimal-spanning tree (MST), and can intuitively understand network among the foreign exchange rates using the MST. The MST forms taxonomy for a
topological space of the \( n \) rates. The MST is a tree having \( n - 1 \) edges that minimize the sum of the edge distances in a connected weighted graph of the \( n \) rates. The edge distances satisfy the following three axioms of a Euclidean distance: (i) \( d_{ij} = 0 \) if and only if \( i = j \), (ii) \( d_{ij} = d_{ji} \), (iii) \( d_{ij} \leq d_{ik} + d_{kj} \). Here, \( d_{ij} \) expresses a distance for a pair of the rate \( i \) and the rate \( j \). We need Euclidean distances between the rates in order to construct the MST. However, the correlation coefficient \( C_{ij} \) does not satisfy the axioms. We can convert the correlation coefficient by appropriate functions so that the axioms can be applied [1]. One of the appropriate functions is

\[
d_{ij} = \sqrt{2(1 - C_{ij})},
\]

where \( d_{ij} \) is a distance for a pair of the rate \( i \) and the rate \( j \).

We construct a MST for the \( n \) rates using \( n \times n \) matrix of \( d_{ij} \). One of methods which construct MST is called Kruskal’s algorithm [10][11]. The Kruskal’s algorithm is a simple method consisting of the following steps: In the first step we choose a pair of rates with nearest distance and connect with a line proportional to the distance. In the second step we also connect a pair with the 2\(^{nd} \) nearest distance. In the third step we also connect the nearest pair that is not connected by the same tree. We repeat the third step until all the given rates are connected in one tree. Finally, we achieve a connected graph without cycles. The graph is a MST linking the \( n \) rates.

We introduce maximal distance \( \hat{d}_{ij} \) between two successive rates encountered when moving form the starting rate \( i \) to the ending rate \( j \) over the shortest part of the MST connecting the two rates. For example, the distance \( \hat{d}_{ad} \) is \( d_{bc} \) when the MST is given as

\[
a - b = c - d,
\]

where \( d_{bc} \geq \max \{d_{ab}, d_{cd}\} \). The distance \( \hat{d}_{ij} \) satisfies axioms of Euclidean distance and a following ultrametric inequality with a condition stronger than the axiom (iii) \( \hat{d}_{ij} \leq \hat{d}_{ik} + \hat{d}_{kj} \) [12],

\[
\hat{d}_{ij} \leq \max \{\hat{d}_{ik}, \hat{d}_{kj}\}. \tag{4}
\]

The distance \( \hat{d}_{ij} \) is called Subdominant ultrametric distances [10][13]. A space connected by the distances provides a well defined topological arrangement that has associated with a unique indexed hierarchy. For a set of foreign exchange rates, we describe the hierarchy by constructing MST. The result will be elaborated in the next section.
Correlation networks among currencies

The traders in a foreign exchange market are always observing many other markets. Among them, they pay a special attention to the currencies of the countries which economically influences the country using the currency they are trading. For example, traders of Swiss Franc pay attention to Euro. Therefore, correlation between CHF(Swiss Franc)/USD and EUR(Euro)/USD is strong and there is a time delay of order less than a minute between the two rates because changes of EUR/USD feed back to CHF/USD of the future [6]. We investigated 26 currencies and 3 metals in New York market from January ’99 to December ‘03 as listed in Table.1. Probability density distributions of correlation coefficients among the currencies and the metals measured by USD(United States Dollar) for each year are shown in Fig.1. Here, the correlation coefficients are calculated from logarithm rate changes. The nontrivial various correlations are found for each year.

We clarify the market networks using MST. We first analyze the currencies and the metals measured by USD. The MST is constructed by the Kruskal’s algorithm using database of changes of the foreign exchange rates and the metal prices. We show the MST and an indexed hierarchical tree associated with the MST in Fig.2(a) and (b). We focus on EUR in Fig.2(a). Neighbors of the EUR are European currencies, such as Swiss Franc, Hungarian Forint, and Norwegian Krone. Other currencies also connect with the currencies of geographically close countries in the MST. From these results we notice that the currencies and the metals cluster with East Europe, West Europe, Oceania, South America, Asia, and metal.

We can find more clearly these clustered structures by observing the indexed hierarchical tree in conjunction with the MST in Fig.2(b). In the right side of the indexed hierarchical tree, the West Europe cluster (SEK-Swedish Krona, NOK-Norwegian Krone, CHF, EUR) and the East Europe cluster (SKK-Slovakian Koruna, CZK-Czech Koruna, HUF-Hungarian Forint) connect between EUR and HUF form Europe cluster. The Europe cluster also connects GBP-British Pound and PLZ-Polish Zloty. We can also clearly find the Oceania cluster (AUD-Australian Dollar, NZD-New Zealand Dollar), the Asian cluster (IDR-Indonesian Rupee, JPY-Japanese Yen, SGD-Singapore Dollar, THB-Thai Baht), the South American cluster (BRR-Brazilian Real, CLP-Chilean Peso, MXP-Mexican Peso), and the metal cluster (Au-Gold, Ag-Silver, Pt-Platinum). The key currencies which connect with some currencies are EUR, HUF, AUD, JPY and MXP in the clusters. Therefore, the clusters match nicely with the geographical regions of corresponding countries in the world, and the key currencies are generally given by major economic countries.

We next investigate relations between USD and other currencies. The metal cluster is almost independent of the currency clusters in Fig.2(a) and (b). Especially, the platinum most loosely connects with the currencies in the
metal cluster. We focus on the platinum with few influences to the currencies, and construct a MST using the currencies and the metals measured by the platinum. Fig.3(a) and (b) show the MST and an indexed hierarchical tree associated with the MST. Unlike Fig.2(a), one currency has big influence to other many currencies in Fig.3(a). The currency is USD as naturally expected; namely, USD has substantial weight in the global world economy. There are some European currencies centered around the EUR in the left side of Fig.3(a). The currencies (EUR, CHF, NOK, SEK, GBP) form the hierarchical tree which does not contain USD in Fig.3(b). Only European currencies are influenced by EUR rather than USD.

**Discussion**

Each country’s currency influences currencies of neighboring countries. We showed correlation networks among currencies by using MST. We found some clusters in the correlation networks. The clusters match nicely with the geographical regions of corresponding countries in the world. The key currencies are generally given by major economic countries in the clusters. It was confirmed that USD is virtually the standard global currency because especially USD has big influence to other currencies. Therefore, minor currency depends on USD and the key currency of the region where it belongs. Traders generally handle the job of only one foreign exchange rate in exchange markets. Therefore, the dependence among currencies means that the traders are always observing not only the market they trade in, but also markets of USD and the key currencies. They feed back the changes of the currencies in their own trading.

In financial engineering, many models independently describe a foreign exchange rate, assuming that interactions among different currencies can be processed by random noises of the exchange rate based on the concepts of a mean field approximation. The theory should be improved for minor currencies.

We expect that the hierarchical taxonomy of currencies is helpful for the improvement. We finally discuss the correlation networks from a standpoint of monetary systems. In international trades between two traders, one of the both traders has to exchange currency in an exchange market except when the both traders are using the same currency. Therefore, there are exchange risks, such as exchange fee, in the international trades. Governments are interested in regional currency without the exchange risks such as the Euro because they want to invigorate international trades in a region. When introducing the regional currency, we have to determine basket ratio of the regional currency in consideration of economical dependency among the countries. The economical dependency can be clarified using our theory of the correlation networks. Therefore, we expect that the correlation networks are helpful in future monetary system.
Acknowledgement
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References


Table 1 The set of Daily data for 26 currencies and 3 metal
<table>
<thead>
<tr>
<th>Code</th>
<th>Currency</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD</td>
<td>Australian Dollar</td>
</tr>
<tr>
<td>BRR</td>
<td>Brazilian Real</td>
</tr>
<tr>
<td>GBP</td>
<td>British Pound</td>
</tr>
<tr>
<td>CAD</td>
<td>Canadian Dollar</td>
</tr>
<tr>
<td>CLP</td>
<td>Chilean Peso</td>
</tr>
<tr>
<td>COP</td>
<td>Colombian Peso</td>
</tr>
<tr>
<td>CZK</td>
<td>Czech Koruna</td>
</tr>
<tr>
<td>EUR</td>
<td>Euro</td>
</tr>
<tr>
<td>HUF</td>
<td>Hungarian Forint</td>
</tr>
<tr>
<td>IDR</td>
<td>Indonesian Rupiah</td>
</tr>
<tr>
<td>JPY</td>
<td>Japanese Yen</td>
</tr>
<tr>
<td>MXP</td>
<td>Mexican Peso</td>
</tr>
<tr>
<td>NZD</td>
<td>New Zealand Dollar</td>
</tr>
<tr>
<td>NOK</td>
<td>Norwegian Kroner</td>
</tr>
<tr>
<td>PEN</td>
<td>Peruvian New Sole</td>
</tr>
<tr>
<td>PHP</td>
<td>Philippines Peso</td>
</tr>
<tr>
<td>PLZ</td>
<td>Polish Zloty</td>
</tr>
<tr>
<td>RUR</td>
<td>Russian Ruble</td>
</tr>
<tr>
<td>SGD</td>
<td>Singapore Dollar</td>
</tr>
<tr>
<td>SKK</td>
<td>Slovakian Koruna</td>
</tr>
<tr>
<td>ZAR</td>
<td>South Africa Rand</td>
</tr>
<tr>
<td>KRW</td>
<td>South Korean Won</td>
</tr>
<tr>
<td>SEK</td>
<td>Swedish Krona</td>
</tr>
<tr>
<td>CHF</td>
<td>Swiss Franc</td>
</tr>
<tr>
<td>THB</td>
<td>Thai Baht</td>
</tr>
<tr>
<td>USD</td>
<td>U.S. Dollar</td>
</tr>
<tr>
<td>Au</td>
<td>Ounce of Gold in New York market</td>
</tr>
<tr>
<td>Ag</td>
<td>Ounce of Silver in New York market</td>
</tr>
<tr>
<td>Pt</td>
<td>Ounce of Platinum in New York market</td>
</tr>
</tbody>
</table>
Fig.1 PDF of correlation coefficient for currencies measured by US dollar for every year.

Fig.2 (a): MST for the currencies and the metals measured by USD. (b): Indexed hierarchical tree obtained for the currencies and the metals.

Fig.3 (a): MST for the currencies and the metals measured by ounce of platinum. (b): Indexed hierarchical tree obtained for the currencies and the metals.
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