TRANSITION RADIATION SPECTRA FROM
RANDOMLY SPACED INTERFACES

C.W. Fabjan
CERN, Geneva, Switzerland

ABSTRACT

Spectra of X-ray transition radiation (XTR) were measured, which were emitted in the passage of relativistic electrons through different samples of polyethylene foam. Based on the good agreement with absolute theoretical predictions, possible TR detectors are optimized for the Lorentz factor $\gamma_L$ range, $\gamma_L = 200-1000$. Such radiators are predicted to offer useful discrimination between pions, kaons, and protons with momenta above 100 GeV/c.

Geneva - 18 April 1977

(Submitted to Nuclear Instrumentation and Methods)
1. INTRODUCTION

Recently, transition radiation detectors (TRDs) have been used in a particle physics experiment to identify electrons above 1 GeV/c. A large pion rejection (factor $\geq 20$) combined with high electron efficiency ($\eta_e \geq 0.9$) was measured for a single TRD \(^1\). This performance was achieved by a careful optimization of all parameters of the lithium-foil radiator and the X-ray detector, a multiwire proportional chamber (MWPC) filled with a Xe-CO$_2$ gas mixture\(^2\).

Several suggestions exist\(^3-7\) to extend the TRD concept to values of the Lorentz factor $\gamma_L < 1000$, where Čerenkov counters can only be used with very severe restrictions. For the conventional TRD (radiator followed by a MWPC measuring the TR superimposed on the ionization loss) to work in this $\gamma_L$-range, the relevant parameters such as foil-, gap-, and MWPC-thickness have to be scaled appropriately. For useful performances, many thousands of very thin foils (microns thick) are required to be spaced periodically, presenting a considerable practical difficulty\(^8\).

Here we study the performance of radiators consisting of randomly distributed interfaces (e.g. foam or suitably arranged granules). Measurements have been performed on the TR spectra obtained using different samples of polyethylene (PE) foam; the results of these measurements were found to agree closely with the predictions of the TR theory for randomly spaced interfaces\(^8,9\). Based on this agreement we apply the theory to the optimization of TRDs, consisting of randomly distributed lithium hydride (LiH) granules, followed by a MWPC. Interesting performance for $\pi/K/p$ identification for particle momenta above 100 GeV/c is predicted.

2. MEASUREMENTS AND RESULTS

The experimental arrangement used for the spectral measurements of the foam samples is shown schematically in Fig. 1. It was used in this form previously to measure TR spectra from periodic radiators\(^10\). The momentum-analysed beam was defined with small scintillators ($S_1$, $S_2$, $S_3$), and electrons were selected with a Čerenkov counter $\tilde{C}$. We measured the TR X-ray spectrum (XTR) with a Li-drifted silicon diode (SSD), giving an energy resolution of FWHM = 250 eV. Care was taken to minimize those instrumental effects which require corrections to the measured photon spectra: the gain of the linear electronics was frequently monitored and found to be stable to better than one percent; absorption between the radiator and the detector was small in the 566 cm-long He-filled drift tube, which separated the radiator from the detector. The detection efficiency of the Si detector required only a small correction below 5 keV and above 20 keV. In all samples studied, the average number of XTR photons emitted per electron was less than one, and hence the spectral distortion from two, not time-resolved, photons emitted by the same
particle was small. All measurements were carried out with electrons of \( p = 1.38 \text{ GeV/c momentum} \) \( (\gamma_L = 2700) \).

Samples of PE foam ("Ethafom") with three different densities were tested\(^*)\). In a previous study comparing a variety of commercially available foams, this brand of foam was found to give the highest XTR yield\(^11\). Some of the geometric parameters of the samples studied are given in Table 1. The distribution of gap spacings \( d_2 \) along the trajectory of the radiating particle, was measured with a transmission microscope; it is related but not identical to the distribution of foam bubble diameters.

The experimental results on the measured spectra are shown for two samples in Figs. 2 and 3 and are summarized in Table 2.

The TR yield expected from such radiators has been evaluated using the theory of TR emission from randomly spaced radiators\(^8,9\). For simplicity, the foam structure was modeled to consist of spheres with diameters \( d \), Gaussian-distributed about the average \( \langle d \rangle \) and with variance \( \sigma(d) \). The diameter \( d \) was chosen to reproduce the density and the average measured gap spacing of the foam sample studied, whereas \( \sigma(d) \) is adjusted to approximate the distribution of measured gap spacings. In the calculation these spheres are randomly separated, and the interface distributions effective for TR emission are generated by tracking particles through this "foam". The two distributions of "plate" thickness and gap separation are obtained and are well fitted to functions of the form

\[
f(x) = \beta^\alpha x^{\alpha-1} \exp(-\beta x)/\Gamma(\alpha),
\]

which conveniently permits analytic averaging over all interfaces\(^9\). The subsequent calculation follows Ref. 9, which, however, treats only the case of TR emission from plates in vacuum. An estimate for the corresponding correction was obtained by comparing the TR yield from equivalent, periodically spaced, PE radiators in air and vacuum, and found to be less than five percent. The final results are corrected for several small experimental effects mentioned earlier, and permit a comparison with the measured spectral distributions (Figs. 2 and 3) and with the measured total detected energy and the average number of photon counts per electron (Table 2).

The comparison with the theoretical expectation is quite favourable, with agreement to within \( \pm 10\% \) for all three samples measured. For the \( \gamma_L \) value and the dimensions of the radiators studied here, and with values of the "irregularity"

\(^*)\) Ethafom is manufactured by the Dow Chemical Company. Dow Chemical Europe S.A. kindly provided us with the samples.
parameter $\xi = d_i / \sigma(d_i) \leq 0.5$ (i = 1, 2), interference effects between the interfaces are not expected to modulate the spectra significantly; one is essentially probing the single-interface TR spectrum

3. PERFORMANCE OPTIMIZATION OF LiH TRDs FOR LOW-$\gamma_L$ VALUES

With the formalism as described in the previous section, a low-$\gamma_L$ LiH radiator was studied. Lithium hydride is a potentially very attractive choice of radiator material with a mass absorption coefficient comparable to that of lithium metal and with a higher plasma frequency. Transition radiation from LiH powder has been observed\(^\text{12}\). For this optimization we considered the radiator again to consist of granules with diameter $d$, and variance $\sigma(d)$, and of adjustable granule density.

Some of the results are summarized in Table 3. The diameter of the granules was optimized for each $\gamma_L$ value. The radiator length $R_L$ was adjusted to give on the average 2000 granules traversed, for two different densities of the granules (5% and 10% of the bulk density). Compared to the saturation energy radiated from an infinitely long radiator, the detected energy varied from about 60% at $\gamma_L = 200$ to 75% at $\gamma_L = 1000$.

A conventional MWPC, filled with Xe and having a thickness as given in Table 3, detected the XTR. The absorption of a 10 $\mu$m-thick mylar window, assumed to separate the radiator from the MWPC, was included in this simulation.

Multiple TR detectors of the kind described would allow identification of particles with momenta above $\sim 100$ GeV/c, that are virtually inaccessible to other identification techniques. Results of a Monte Carlo study for a TRD system are shown in Figs. 4a and 4b. A maximum-likelihood method was chosen to discriminate optimally between the XTR signals superimposed on the ionization loss deposited by the different particles. A relatively small number ($\leq 10$) of TRDs is sufficient to discriminate between $\pi$'s and $K$'s down to 40 GeV/c with a kaon rejection adequate for many experimental investigations. Similarly, kaon-proton separation becomes practical for particle momenta above $\sim 100$ GeV/c.

4. CONCLUSIONS

Measurements of the XTR spectra obtained from PE foam radiators agree well with the absolute theoretical predictions of XTR emission from random interfaces. Theoretical expectations for a randomly spaced LiH granular radiator indicate useful pion, kaon, and proton identification for particle momenta above 100 GeV/c.
The help in data taking given by Dr. W. Struczinski is gratefully acknowledged. I enjoyed many stimulating discussions with Professor W.J. Willis on the concepts of such transition radiation detectors.
REFERENCES


5) W.J. Willis, unpublished notes.


9) G.M. Garibian, L.A. Gevorgian and C. Yang, Nuclear Instrum. Methods 125 (1975) 133. In formula (4) the expression for \( Q \) should read

\[
Q = 0.5(1 + p)(1 + h_a h_b) - h_a - p h_b.
\]


### Table 1
Geometrical parameters of foam samples studied

<table>
<thead>
<tr>
<th>Sample</th>
<th>Density (g/cm³)</th>
<th>Length (cm)</th>
<th>Average gap spacing (measured) (d₂) (cm)</th>
<th>Variance of gap spacing (measured) (σ(d₂))</th>
<th>Average plate thickness (computed) (d₁) (cm)</th>
<th>Variance of plate thickness (computed) (σ(d₁))</th>
<th>Number of plates traversed (computed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ethafoam 220</td>
<td>0.037</td>
<td>5.0</td>
<td>0.087</td>
<td>0.031</td>
<td>0.0035</td>
<td>0.0014</td>
<td>56</td>
</tr>
<tr>
<td>Ethafoam 400</td>
<td>0.053</td>
<td>5.0</td>
<td>0.070</td>
<td>0.034</td>
<td>0.0044</td>
<td>0.0017</td>
<td>71</td>
</tr>
<tr>
<td>Ethafoam 600</td>
<td>0.111</td>
<td>5.0</td>
<td>0.055</td>
<td>0.024</td>
<td>0.010</td>
<td>0.0031</td>
<td>80</td>
</tr>
</tbody>
</table>

### Table 2
Measured and calculated results

<table>
<thead>
<tr>
<th>Sample</th>
<th>Measurement (\langle E_{\text{total}} \rangle_{\text{detect.}}) (keV)</th>
<th>Calculation (\langle E_{\text{total}} \rangle_{\text{detect.}}) (keV)</th>
<th>Measurement (counts)</th>
<th>Calculation (counts)</th>
<th>Calculation (photons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ethafoam 220</td>
<td>6.96</td>
<td>7.07</td>
<td>0.45</td>
<td>0.46</td>
<td>0.51</td>
</tr>
<tr>
<td>Ethafoam 400</td>
<td>7.81</td>
<td>8.14</td>
<td>0.49</td>
<td>0.51</td>
<td>0.56</td>
</tr>
<tr>
<td>Ethafoam 600</td>
<td>6.62</td>
<td>6.01</td>
<td>0.40</td>
<td>0.37</td>
<td>0.40</td>
</tr>
<tr>
<td>$\gamma_L$ (cm)</td>
<td>Granule diameter (cm)</td>
<td>No. of granules traversed</td>
<td>Thickness of MWPC (cm)</td>
<td>Density of granules</td>
<td>5%</td>
</tr>
<tr>
<td>----------------</td>
<td>-----------------------</td>
<td>---------------------------</td>
<td>------------------------</td>
<td>--------------------</td>
<td>-----</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>0.0004</td>
<td>2000</td>
<td>0.2</td>
<td>Length of radiator (cm)</td>
<td>10.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\langle E_{TR} \rangle$ detected (keV)</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\langle$ Photons detected $\rangle$</td>
<td>0.27</td>
</tr>
<tr>
<td>400</td>
<td>0.0006</td>
<td>2000</td>
<td>0.4</td>
<td>Length of radiator (cm)</td>
<td>15.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\langle E_{TR} \rangle$ detected (keV)</td>
<td>4.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\langle$ Photons detected $\rangle$</td>
<td>1.19</td>
</tr>
<tr>
<td>600</td>
<td>0.0008</td>
<td>2000</td>
<td>0.4</td>
<td>Length of radiator (cm)</td>
<td>21.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\langle E_{TR} \rangle$ detected (keV)</td>
<td>9.44</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\langle$ Photons detected $\rangle$</td>
<td>2.04</td>
</tr>
<tr>
<td>1000</td>
<td>0.0016</td>
<td>2000</td>
<td>0.4</td>
<td>Length of radiator (cm)</td>
<td>42.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\langle E_{TR} \rangle$ detected (keV)</td>
<td>17.85</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\langle$ Photons detected $\rangle$</td>
<td>3.11</td>
</tr>
</tbody>
</table>
Figure captions

Fig. 1: Experimental layout for the absolute spectral TR yield measurements. Not to scale.

Fig. 2: The TR spectrum measured from $\gamma_L = 2700$ electrons traversing 5.0 cm of "Ethafom 220" (solid line) and the theoretical expectation (dotted line). The spectrum was accumulated for 5000 electrons.

Fig. 3: As Fig. 2 with a 5.0 cm long "Ethafom 600" sample. The spectrum was accumulated for 2000 electrons.

Fig. 4: a) Pion-kaon separation for 60 GeV/c particles as a function of the number of TRDs, for a $\pi$ efficiency of $\eta_\pi = 0.9$ and $\eta_\pi = 0.8$. Each radiator is 15.9 cm long with, on the average, 4000 granules with average diameter $d = 0.0006$ cm traversed per radiator.

b) Kaon-proton separation for 120 GeV/c and 160 GeV/c particles as a function of the number of TRDs (a kaon efficiency of $\eta_K = 0.8$ was assumed). Each radiator is 10.7 cm long with, on the average, 4000 granules traversed per radiator (average granule diameter $d = 0.0004$ cm).