RECONSTRUCTION OF THE MOMENTUM OF A PARTICLE MOVING
IN AN AXIALLY SYMMETRIC MAGNETIC FIELD

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ABSTRACT

We describe a method which provides the coordinates along the trajectory of a particle traversing an axially symmetric magnetic field. The momentum of the particle is evaluated with a simple algorithm when the detectors which measure the coordinates of the particle trajectory are of cylindrical shape.

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1. INTRODUCTION

Several scattering experiments use a target which is positioned in a magnet with an axially symmetric field, e.g. the experiments making use of a solenoidal spectrometer and polarized target (PT) experiments¹⁾.

When the momentum of a scattered particle has to be measured in a PT magnet, the particle trajectory has to be detected over a region where the magnetic field varies greatly. The target has to be operated in a high field region (typically 25 kG) which, for practical reasons, has to be kept as small as possible, so that the magnetic field decreases rapidly with increasing distance from the target. The evaluation of the momentum usually requires a lengthy computer program for tracking the particle through the magnetic field.

In a series of experiments²⁻⁵⁾ of this kind we have developed an algorithm to compute the value of the momentum from the measured points along the trajectory. Such an algorithm takes advantage of the symmetry property of the field of the PT magnet, namely

- i) axial symmetry;
- ii) mirror symmetry about the median plane.

The method provides the coordinates along the trajectory as a series expansion in terms of the inverse momentum of the particle, of the distance of the particle trajectory from the symmetry axis, and of the initial inclination of the trajectory with the median plane. The momentum is evaluated with a simple procedure when the detectors are of cylindrical shape and concentric with the symmetry axis of the field.

In the present paper we describe this method and its accuracy for the PT spectrometer we have used in the experiments of Ref. 4 and 5. Details about the measurements of the field of the magnet and its parametrization in terms of pairs of Helmholtz coils are also given for completeness.

2. THE SPECTROMETER

The magnet we have used in the experiments of Refs. 4 and 5 is shown in Fig. 1. The axial symmetry is obtained with circular coils and magnet poles. Truncated-cone pole pieces provide the 25 kG field

at the centre, with a uniformity of $\pm 2.5 \times 10^{-4}$, required for the operation of the polarized target, over a region with a diameter of 15 cm.

The design of the detector (Fig. 2) matches the symmetry of the field and reflects the characteristics of the solution, which takes on a simple form in terms of azimuthal coordinates for the particle trajectory. These are provided, inside the magnet, by the cylindrically shaped wire spark chambers⁶⁾ $S_1, \dots, S_4$. Out of the magnet the particle direction is measured by the wire spark chambers⁷⁾ $S_5, \dots, S_8$ and an azimuthal coordinate is given on a reference cylinder of radius R . Chamber S_1 provides also a measurement of the elevation of the particle close to the target. The momentum band of interest is $0.25 < p < 1$ GeV/c.

3. MEASUREMENT OF THE FIELD OF THE MAGNET

The field of the magnet has been measured over the entire volume, including the stray field region up to the outer telescope, and the axial symmetry has been checked by comparing the measured field values at various meridian planes.

The measurements have been performed with an automatic scanning machine⁸⁾. Repeated measurements of each component of B at a given point have been obtained, and the measuring error has been estimated to be about ± 25 G. A total of about 4400 measurements were made.

Small corrections have been applied to the data for misalignments. Once these corrections have been applied, the data at various azimuths (0° , 45° , and 90°) have been compared, and the deviation from axial symmetry turned out to be at each point smaller than the measuring error. The field values in these planes have then been converted into radial (B_r) and vertical (B_z) components, for an overall of 542 (r, z) data points, some of which are plotted in Fig. 3. Use is made of a cylindrical coordinate system (r, θ, z) with the origin at the magnet centre and the z -axis pointing upwards, along the direction of the magnet field at the centre.

4. PARAMETRIZATION OF THE MAGNETIC FIELD

The main characteristics of the field, as apparent from Fig. 3, are the two plateaux, each followed by regions in which the vertical component of the field falls rapidly, and correspondingly the radial component shows pronounced peaks. These main features can be reproduced by superposing two pairs of Helmholtz coils of suitably chosen strength and with radii equal to the radial positions of the two maxima of B_r , namely 18 cm and 56 cm, placed symmetrically about the median plane. Figure 3 also shows the vertical and radial component of one pair of Helmholtz coils of 56 cm radius. By superposing a suitable number of these coils, a good description of the field can be obtained. This parametrization has the advantage that Maxwell's equations are satisfied. With a fitting program both the number of pairs of coils and their parameters have been optimized over the 542 data points. Good agreement with the measured field values has been obtained with 34 pairs of Helmholtz coils, whose parameters are listed in Table 1. The field at any point could then be computed in terms of those 68 parameters. To speed up computation, use was made of a tabulated array of values for the complete elliptic integrals⁹⁾ which enter the expression of B for a pair of Helmholtz coils.

5. EQUATIONS FOR THE PARTICLE TRAJECTORIES

The axial symmetry of the magnetic field implies for a particle moving across it the conservation of the component of the generalized angular momentum along the axis of symmetry. Moreover, the vector potential A has only one non-vanishing component, i.e. the azimuthal component $A(r,z)$.

In terms of $A(r,z)$, the corresponding first integral of motion is written as

$$rp \sin\alpha \cos\beta + rq A(r,z) = \text{constant} \quad (5.1)$$

where p is the conserved linear momentum, q the charge of the particle, α the angle between the component of $\vec{p}$ in the median plane and the radius vector $\vec{r}$, and β the inclination of $\vec{p}$ over the median plane (see Fig. 4).

The potential $A(r,z)$ can be obtained from the vertical and

radial components of the magnetic field B using

$$A(r, z) = \frac{1}{r} \int_0^r B_0(r) r dr - \int_0^z B_r(r, z) dz . \quad (5.2)$$

where $B_0(r) = B_z(r, 0)$.

The motion is completely described when equation (5.1) is complemented with

$$\frac{d}{dr} (\sin \beta) = \frac{q}{p} \operatorname{tg} \alpha \frac{\partial A}{\partial z} \quad (5.3)$$

which gives the variation of the z-component of the linear momentum. The equations for the coordinates of the trajectory $\theta(r)$ and $z(r)$, parametrized in terms of r , are then

$$\frac{d\theta}{dr} = \frac{\operatorname{tg} \alpha}{r} ; \quad \frac{dz}{dr} = \frac{\operatorname{tg} \beta}{\cos \alpha} . \quad (5.4)$$

We have searched for solutions which impose no limitation on r (i.e. for particles leaving the magnet) and for which the angle β with the median plane is small*).

Under these circumstances, if β_0 is the initial inclination of the particle, Eq. (5.2) can be written as

$$A(r, z) \approx A(r, 0) - \frac{1}{2} \frac{dB_0}{dr} z^2 \approx A(r, 0) - \frac{1}{2} \frac{dB_0}{dr} r^2 \operatorname{tg}^2 \beta_0 \quad (5.5)$$

Neglecting then terms of order β_0^2 the trajectory equations reduce to the set

$$r [p \sin \alpha + k(r)] = \text{constant} \quad (5.6)$$

$$\frac{d\theta}{dr} = \frac{\operatorname{tg} \alpha}{r} \quad (5.7)$$

*) Solutions can be found in the literature in the case of the Gaussian approximation, namely when the particles are close to the axis of symmetry (small values of r) and β is close to $\frac{\pi}{2}$.

The quantity $k(r) = qA(r,0)$ is the linear momentum associated with the field in the median plane and is shown in Fig. 5 for the magnet of Sec. 3.

6. ANALYSIS OF PLANAR TRAJECTORIES

For trajectories in the median plane, Eqs. (5.6) and (5.7) are exact. Referring to Fig. 6, let us choose the $\theta=0$ direction parallel to the tangent t to the trajectory at the point D at minimum distance from the origin. If d denotes this distance (positive if an observer moving along with the particle leaves the origin to the right), we can rewrite Eq. (5.6) as

$$\sin\alpha(r) = -\frac{d}{r} - \frac{\left[k(r) - \frac{d}{r}k(d)\right]}{p} \quad (6.1)$$

In the limit of $p \rightarrow \infty$, Eqs. (6.1) and (5.7) have as solution the coordinates of the tangent t

$$\theta_t(r) = -\alpha_t(r) = \arcsin \frac{d}{r}.$$

For finite values of p , the trajectory is bent away from this straight line, and the quantity

$$\psi(r) = \theta(r) - \arcsin \frac{d}{r} \quad (6.2)$$

is related to the amount of bending. As long as the trajectory passes close to the origin (small d), the distance, at a given r , between the point $P(r,\theta)$ on the trajectory and the point $P'(r,\theta_t)$ on the tangent t , is roughly proportional to $1/p$. One can expect, therefore, that the product $p\psi(r)$ is only a function of r , to first order in $1/p$ and d . From Eqs. (6.2), (5.7) and (6.1) a differential equation for $p\psi(r)$ can be written as

$$p \frac{d\psi}{dr} = p \frac{\operatorname{tg}\alpha}{r} + \frac{pd}{r^2 \sqrt{1 - \frac{d^2}{r^2}}} =$$

$$= - \frac{\left[k(r) - \frac{d}{r} k(d) \right] + p \frac{d}{r}}{r \sqrt{1 - \left\{ \frac{\left[k(r) - \frac{d}{r} k(d) \right]}{p} + \frac{r}{d} \right\}^2}} + \frac{p \frac{d}{r}}{r \sqrt{1 - \frac{d^2}{r^2}}} \quad (6.3)$$

Making a series expansion in $1/p$ and d of the right hand term of Eq. (6.3), one gets

$$p \frac{d\psi}{dr} = -\frac{k(r)}{r} \left\{ 1 + \frac{1}{2} \frac{k^2(r)}{p^2} + \frac{3}{2} \frac{k(r)}{p} \frac{d}{r} + \frac{1}{2} \left[3 - \frac{qB_0(o)r}{k(r)} \right] \frac{d^2}{r^2} + \dots \right\} \quad (6.4)$$

where use is also made of the relation $k(d) = \frac{1}{2} qB_0(o)d$, which is valid when the point D is in the uniform field region. The negative sign in front of the right-hand side of Eq. (6.4) is due to the fact that ψ decreases for positive particles, B_z being positive.

Equation (6.4) can be integrated between the radius r and the reference radius R to give

$$p \left[\psi(r) - \psi(R) \right] \approx F(r; p, d) = D_{11}(r) + \frac{1}{2} \left[3D_{13}(r) + q \frac{B_0(o)}{r} \right] d^2 + \frac{3}{2} D_{22}(r) \frac{d}{p} + \frac{1}{2} D_{31}(r) \frac{1}{p^2} + \dots \quad (6.5)$$

where we have introduced the double integrals

$$D_{ik}(r) = \int_r^R \frac{k^i(x)}{x^k} dx = q^i \int_r^R \frac{dx}{x^{i+k}} \left\{ \int_0^x y B_0(y) dy \right\}^i \quad (6.6)$$

To show qualitatively the validity of a series expansion for $p\psi(r)$, we compare in Fig. 7 its true value, obtained from Eq. (6.2), where $\theta(r)$ is computed via the tracking program, with the leading term (full line) in its series expansion

$$K(r) = \int_0^r \frac{k(x)}{x} dx \quad (6.7)$$

for some values of $1/p$ and d (dashed lines). Although the values of the parameters are rather large, the dashed lines differ little from the full line, which means that the higher order terms in Eq. (6.5) represent a small fraction of the leading term.

Although the approximation

$$p \frac{d\psi}{dr} \approx -\frac{k(r)}{r}$$

holds only if $d^2 \ll r^2$ and $k^2(r) \ll p^2$, after the integration one has got a series expansion which is rapidly converging also for rather large values of the parameters.

The trajectory equation (6.5) can be utilized in a fitting program, where one needs to know both $\theta(r)$ and its derivatives as functions of the parameters of the trajectory, namely $1/p$, d , and $\theta(R)$. When the trajectory is measured with cylindrical chambers, the quantities D_{ik} are constants, which have to be computed just once for each chamber.

7. ACCURACY OF THE SOLUTION

We have given in Eq. (6.5) the solution to the equations of motion in the form of a series expansion. The accuracy of the solution depends on the number of terms taken into account. It turns out that the series converges rapidly, so that a good accuracy is already obtained by keeping only the terms up to second order, as shown.

The value $\theta(r)$ computed with Eq. (6.5) can be compared with that given by the tracking program, at the chamber S_3 (in Fig. 2), by means of Δs_3 , the difference between the computed and the true value of the curvilinear coordinate $s_3 = r_3\theta(r_3)$. In the experiment the radius r_3 ($= 254$ mm) was so chosen that on s_3 the sagitta for the arc of trajectory between $r_1 = 62$ mm and $R = 1200$ mm was close to maximum.

The quantity $p \cdot \Delta s_3$ gives the accuracy of the algorithm. Figure 8 shows this quantity as a function of $1/p$ for three values of d . When D_{31} the coefficient of the quadratic correction, is given the value of Eq. (6.6), $p \cdot \Delta s_3$ (dotted line) increases rapidly with $1/p$, due to the effect of the higher order terms (mainly of order $1/p^4$ in the momentum range shown). In order to extend the range of validity of Eq. (6.5) towards the low- p values, the coefficient D_{31} has been slightly increased (full curves), and this explains the shape of the full curves in Fig. 8.

The quantity $p \cdot \Delta s_3$ stays within a band which is 0.1 (mm $\cdot$ GeV/c) wide in the whole range $0.25 < p < 1$ GeV/c. This means that the accuracy of the algorithm is at least ten times better than the precision with

which ψ_3 is eventually fitted in the experiment.

The decision to keep in the series expansion (6.5) only terms up to second order is bound to the particular experiments we refer to. In a different situation, higher order terms can be added in a straightforward manner, either by further differentiating equation (6.3) and evaluating the corresponding integrals, or by fitting the residuals with some standard technique¹⁰⁾.

8. ANALYSIS OF NON-PLANAR TRAJECTORIES

For trajectories in the median plane a series expansion has been obtained which is the exact solution of the equations of motion. For the general case of trajectories out of the median plane a solution has been worked out with terms up to second order, and the procedure for evaluating higher order terms is less straightforward than in the previous case.

It is useful to derive the β -dependence of $p\psi(r)$ when $d=0$ and p is large. Since focusing is a second-order effect in $1/p$, the inclination β in equation (5.1) can be kept constant and equal to β_0 , the initial inclination. Moreover, we have $\cos\alpha \approx 1$ and $z \approx z_0 + r \operatorname{tg}\beta_0$. Using the approximation (5.5), one gets

$$\lim_{1/p \rightarrow 0} p\psi(r) = - \frac{K(r) + qH(r; \beta_0, z_0)}{\cos\beta_0} \quad (8.1)$$

where

$$H(r; \beta_0, z_0) = \frac{1}{2} \operatorname{tg}^2\beta_0 \left[\int_0^r B_0(x) dx - rB_0(r) \right] - z_0 \operatorname{tg}\beta_0 B_0(r) \quad (8.2)$$

having neglected the z_0^2 term, which turns out to be very small.

The solution (8.1) is a good approximation as long as the vector potential does not vary too much versus z along the particle trajectory, so that the approximation (5.5) is a good one. This does not necessarily imply that z is small; it just requires $z^2 (dB_0/dr) \ll A(r, 0)$, so that z can also be large as long as the vertical component of the field does not change too much with r . In this sense the algorithm is applicable to a more general class of problems, for instance the motion of a particle in a solenoidal magnet.

When d and $1/p$ are both different from zero, equation (6.5) suggests modifying equation (8.1) into

$$p \left[\psi(r) - \psi(R) \right] \approx \frac{F(r;p,d) + qH(r;\beta_0,z_0)}{\cos \beta_0} \quad (8.3)$$

This expression turns out to be quite accurate; in particular the β_0 dependence, which is an order of magnitude more important than the z_0 dependence, is well accounted for, as apparent from Fig. 9, which shows $p\Delta s_3$, with s_3 now given by equation (8.3). At a fixed momentum value the dependence of $p\Delta s_3$ on d for various β_0 and z_0 is shown in Fig. 10. Again, the residues are rather small, so that Eq. (8.3) can be regarded as a good description of the projection of the trajectory of the particle onto the median plane. There is a great complication with respect to Eq. (6.5), namely that now four parameters (apart from $\theta(R)$) need to be known, i.e. $1/p$, d , β_0 , and z_0 , as contrasted with the previous two-parameter situation.

The vertical coordinate of the trajectory can be calculated as a function of the parameters by successive approximations. Starting from Eq. (6.1) one can write

$$\alpha(r) \approx - \frac{k(r)}{p} - \frac{d}{r}$$

and, to second order,

$$\sin \beta(r) \approx \sin \beta_0 \left\{ 1 - \frac{1}{p^2} V(r) - \frac{d}{p} q[B_0(0) - B_0(r)] \right\}, \quad (8.4)$$

neglecting the small z_0 dependence and taking $\sin \beta_0 \approx \text{tg} \beta_0$. Substituting this into Eq. (5.4) and integrating, one gets

$$z(r) \approx r \text{tg} \beta_0 \left\{ 1 - \frac{1}{p^2} G_1(r) - \frac{d}{p} G_2(r) - \frac{1}{2} \frac{d^2}{r^2} \right\} \quad (8.5)$$

where

$$V(r) = - \frac{q}{\text{tg} \beta_0} \int_0^r k(x) B_r[x, z(x)] dx \sim q^2 \int_0^r x B_0^2(x) dx - q r B_0(r) k(r)$$

$$G_1(r) = \frac{1}{r} \left[\int_0^r V(x) dx - \frac{1}{2} \int_0^r k^2(x) dx \right]$$

$$G_2(r) = \frac{1}{r} \left[q B_0(r) - K(r) \right].$$

Equation (8.5), like equation (8.4), is correct only to second order, and it might not be accurate enough to give the expectation value of $z(r)$ in a fitting program. Eventually higher order terms have to be considered in Eq. (5.4), or, more simply, the residuals can be fitted in terms of the parameters.

Actually, knowing the other parameters, β_0 is best estimated from the inclination of the trajectory out of the magnet, so that what one really needs is Eq. (8.4). Fig. 11 shows the accuracy with which the focusing effect is described by the second order term for the magnet of Sec. 3.

9. CALCULATION OF MOMENTUM

A pattern recognition program requires the momentum of a particle to be computed from knowing the coordinates of three points along its trajectory. This would mean solving Eq. (8.3) for the parameters, which is not straightforward. For the experiments²⁻⁵⁾ we have computed the parameters of a trajectory, given the coordinates $\theta(R)$ and $z(R)$, the direction of the particle outside the field $\alpha(R)$ and $\beta(R)$, and a second azimuthal coordinate $\theta(r)$ as close to the magnet centre as possible ($r = 62$ mm). Equations (5.1), (8.3) and (8.4) can be rewritten as

$$\left\{ \begin{array}{l} \frac{1}{p} = - \frac{\sin\alpha(R) \cos\beta(R) + \frac{d}{R} \cos\beta_0}{qA[R, z(R)] - \frac{d}{R} k(d)} \\ d = r \sin \left\{ \theta(r) - \theta(R) + \arcsin \frac{d}{R} - \frac{F(r; p, d) + qH(r; \beta_0, z_0)}{p \cos\beta_0} \right\} \\ \sin\beta_0 = \sin\beta(R) \left\{ 1 - \frac{1}{p^2} V(R) - \frac{d}{p} qB_0(o) \right\}^{-1} \end{array} \right. \quad (9.1)$$

The three equations are solved for the parameters by an iterative method. First one sets $d = 0$ and $\beta_0 = \beta(R)$ and gets $1/p$ from the first equation. This value, inserted in the second gives a first estimate of d , which is then fed, together with $1/p$, into the third to get a new β_0 . The new d , β_0 set is then fed again into the first equation, and so on. The initial elevation z_0 was very small in the experiment ($|z_0| < 10$ mm), so the z_0 -dependence was neglected in

Eq. (9.1). The procedure converges very fast, as shown for a typical situation in Fig. 12, so that it is repeated only five times. After the five iterations, the values of d , $1/p$ and β_0 so obtained are inserted in Eq. (8.3) to obtain the values of θ at the radii of the other cylindrical chambers, which can then be compared in the pattern recognition program with the measured values.

Once the measured coordinates along the trajectory have been singled out by the pattern recognition program, the parameters of the trajectory are fitted in a standard way by using Eq. (8.3), as mentioned in Sec. 6.

10. CONCLUSIONS

We have shown that the problem of reconstructing the trajectory of a particle of given momentum moving across a magnetic field can be solved in a rather simple way when

- a) the field has a symmetry about an axis, and a mirror-symmetry about a median plane,
- b) the detectors measuring the coordinates of the particle trajectory are of cylindrical shape and concentric with the symmetry axis of the field.

A convenient parametrization for such a magnetic field is in terms of a suitable number of Helmholtz coil pairs, and we have described such a procedure applied to the magnet we have used in the experiments of Refs. 4 and 5.

The symmetry properties of the field imply the conservation of the component along the axis of symmetry of the generalised angular momentum of the system, particle plus field. Such a conservation results in a first integral of the equations of motion, which are most conveniently written in a cylindrical coordinate system in terms of the vector potential associated with the magnetic field.

By introducing a suitable variable ψ , related to the azimuthal coordinate $\theta(r)$ along the projection of the trajectory onto the median plane, it has been possible to isolate the leading term for $\theta(r)$. The solution is then given as a series expansion versus the inverse momentum of the particle, the distance of the trajectory from the symmetry axis

and the inclination over the median plane. By considering the leading term plus the next-order correction terms only (quadratic in the parameters), one obtains precisions of a few parts in a thousand, quite adequate for the experiments of Refs. 2 - 5. Hints are given for computing higher-order correction terms.

The coefficients in the series expansion are just radial integrals of the vector potential up to the radius at which the azimuthal coordinate is needed. In the case b) they are just constants, and both the computation of the coordinates, given the parameters, and the inverse problem of computing the parameters from the knowledge of the coordinates at three points along the trajectory, turn out to be particularly fast.

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Table 1

Parameters of the 34 pairs of Helmholtz coils

Radius (mm)	Strength (arbitrary units)
112.2	- 3.3142
158.0	2.0179
160.3	20.6040
196.7	- 0.7165
205.7	14.1790
226.6	4.9523
254.9	9.3423
267.4	4.5480
307.6	10.5780
328.6	- 6.4029
358.7	6.0380
382.2	4.2107
419.4	- 11.9060
432.3	- 4.5875
440.1	- 46.1070
488.1	22.4340
495.9	82.8930
508.0	12.1780
551.1	34.5100
584.0	- 53.9310
605.7	74.5760
649.8	11.5130
674.0	1.7356
675.7	0.5733
714.0	- 7.8287
725.8	5.3267
770.4	12.5090
903.3	11.6200
1021.0	6.0634
1198.0	5.2496
1290.0	10.9610
1633.0	- 13.2250
1794.0	29.5040
2486.0	11.6930

FIGURE CAPTIONS

- Fig. 1 : Top and side view of the Polarized Target magnet which has been used in the experiments of Refs. 4 and 5.
- Fig. 2 : Plan view of the PT magnet and the detector used in the experiments of Refs. 4 and 5; $S_1 \dots S_4$ are cylindrically shaped wire spark chambers operating inside the magnet, in a region of high field; $S_5 \dots S_8$ are plane wire spark chambers.
- Fig. 3 : Measured values for vertical (B_z) and radial (B_r) components of the magnetic field of magnet of Fig. 1. Also shown are the same quantities (full lines) for a pair of Helmholtz coils of 56 cm radius.
- Fig. 4 : Reference system.
- Fig. 5 : Plot of $k(r) = q A(r,0)$ for the magnet of Fig. 1
- Fig. 6 : Definition of $\psi(r)$, the difference between the azimuthal coordinate of the trajectory and of its tangent t at D , the point at minimum distance from the origin.
- Fig. 7 : Comparison between $p \cdot \psi$ given by the tracking program (dashed lines) and $K(r) = \int_0^r \frac{k(r)}{r} dr$.
- Fig. 8 : Momentum dependence of $p \Delta s_3$. Full and dashed lines correspond to two different values of the coefficient of the $1/p^2$ term.
- Fig. 9 : Momentum dependence of $p \Delta s_3$ for non-planar trajectories passing through the magnet centre.
- Fig. 10 : Dependence of $p \Delta s_3$ on the distance d of the trajectory from the magnet centre (positive if the centre of the magnet stays to the right of an observer moving along the particle trajectory).
- Fig. 11 : Momentum dependence of the focusing effect. Points are from the tracking program whereas the straight line has the slope $V(R)$ from Eq. (8.4).
- Fig. 12 : Precision in calculating the momentum by solving with an iterative procedure the system (9.1).

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12