Archives

THE BOSE-EINSTEIN EFFECT AND THE JET STRUCTURE
OF HADRONIC FINAL STATES

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A B S T R A C T

We argue that effects of Bose-Einstein statistics on two and possible three-pion correlations can provide a sensitive test of the mechanism through which hadronic final states come out in a variety of lepton and hadron induced processes

The peculiar predictions of gauge and dual theories in the topological expansion approach are derived and contrasted with those of other schemes.

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INTRODUCTION

It has been recently proposed ¹⁾ that a topological approach to the dynamics of quark and hadron interactions can lead to a unified picture of lepton-hadron and purely hadronic processes in a way which is consistent with the basic ideas of non-Abelian gauge theories (QCD), of the dual topological expansion (or dual unitarization) and of Gribov's Reggeon field theory.

Although the logical scheme has been worked out in detail only for mesons, where it is based on the systematic use of $1/N$ expansions, an extension to baryons, with many phenomenological consequences appears now possible both at the gauge theory level ²⁾ and in an S matrix approach ³⁾.

One of the main characteristics of the topological approach is that hadronic final states are obtained, in the sense of unitarity, by cutting diagrams which are locally planar. Globally, however, the unitarity cut goes through several planes (sheets). Their number changes according to the process considered and to the order of the contribution in the topological expansion. Thus we can have one-cut plane (e.g., $e^+e^- \rightarrow$ hadrons, Reggeon exchange, valence contribution to $e^-p \rightarrow e^- + X$), two-cut planes (Pomeron exchange, sea contribution to $e^-p \rightarrow e^- + X$), three-cut planes ($B\bar{B}$ annihilation) and even more (two-Pomeron cut, etc.). We call the number of cut planes the number of jets ^{*}). since, with respect to a certain axis, in a certain Lorentz frame, the final state associated with each cut plane is expected ⁴⁾ to have the typical jet structure of longitudinal scaling and limited transverse momenta.

A natural consequence of the scheme ^{1),2),4),5)} is that multiplicities in various processes are related, e.g., asymptotically,

$$\bar{n}_{3\text{jets}} \approx \frac{3}{2} \bar{n}_{2\text{jets}} \approx 3 \bar{n}_{1\text{jet}} = 3 \bar{n}_{e^+e^-}$$

^{*}) In the more standard terminology a jet is associated with a single quark. Here it is related to a high mass $q\bar{q}$ (or qqq) colour singlet state.

Effects of energy-momentum conservation and of relative motion of the centres-of-mass of each jet have to be taken into account, however, at finite energies. When this is done ⁵⁾, the above predictions are compatible with the data. An alternative scheme ⁶⁾, in which multi-hadron production (and hence multiplicity) is universal, appears ⁵⁾ to be less favoured, but not ruled out, by multiplicity data. In view of the above uncertainties, it seems important to have other more sensitive tests of the various mechanisms advocated for multiparticle production. In this paper we wish to point out that such a sensitive test can be provided by two and (perhaps) three-particle correlations, if one looks at effects of the type referred to in the literature as intensity fluctuations "à la Hambury-Brown-Twiss" ⁷⁾.

It is well understood that the Hambury-Brown-Twiss effect reflects the Bose-Einstein nature of the produced pions. Another phenomenon, the well-known GGLP effect ⁸⁾, is also a consequence of BE statistics and has been extensively explored in the last few years ⁹⁾. Furthermore, according to analyses of Kopylov-Podgoretsky, Cocconi and others ¹⁰⁾ and as already anticipated by the authors of Ref. 8), these effects can provide a useful tool for investigating the space-time extension of the interaction region. As in Refs. 7)-10), we also consider effects due to the identity of particles (bosons) on multiparticle correlation functions. It turns out that, when these effects are considered in the framework of the topological picture ¹⁾ mentioned above, definite trends are predicted which represent sensitive tests of the model.

Here we will just give the basic framework of our approach and will work out the immediate consequences for several two and three-pion correlation function in the topological expansion approach. We then discuss the results vis-à-vis the experimental situation and the predictions of other schemes.

1. - TOPOLOGICAL CLASSIFICATION OF INCLUSIVE CROSS-SECTIONS AND
DERIVATION OF THE RELEVANT FORMULAE

A) - Two-particle correlations

We shall consider quantities like

$$\sigma_{in} ; \quad \rho_1(p_1) \equiv \frac{1}{\sigma_{in}} \frac{d\sigma_{in}}{dP_1} ; \quad \rho_2(p_1, p_2) \equiv \frac{1}{\sigma_{in}} \frac{d^2\sigma_{in}}{dP_1 dP_2} \dots$$

$$dP \equiv d^3\vec{P}/E$$

at high energy and with p_1, p_2 in the central region.

For hadron-hadron collisions, at FNAL, ISR energies, these processes are expected to be dominated by the bare Pomeron term which is, in the topological picture ¹⁾, a two-jet process. Total cross-sections and single particle inclusive cross-sections are immediately given by the diagrams of Fig. 1a,b. Corrections will have the same boundary structure but more handles, and will be down by $(1/N^2)^h$ (h = number of handles). As long as we deal with the leading term, factorization allows us to drop the particles initiating the process from the Regge-Mueller diagram.

When we go to two-particle distributions, the number of possible diagrams, even in leading order, is increased. The possible topologies are shown in Fig. 2. Let us discuss the meaning of the various terms. Figures 2a, b give the inclusive cross-section for either identical or non-identical particles when particle 1 is in one jet and particle 2 is in the other. Figures 2e,f give the case in which both particles are in the same jet but are not neighbours along the multiperipheral chain and Fig. 2i gives the case in which they are neighbours.

The remaining four diagrams 2c,d,g,h arise if particles 1 and 2 are of the same type and represent, in our framework, the Bose-Einstein effect. Contributions in which particles 1, 2 are in the same boundary and 1', 2' in a different one are expected to be strongly suppressed by multiperipheral dynamics and have been neglected.

A word is needed here to avoid confusion. When integrated over p_1 and p_2 , not all terms in Fig. 2 contribute to the bare Pomeron of σ_{in} . Indeed, exactly the BE interference terms give rise to topologies with one handle [i.e., $O(1/N^2)$ down] as it is obvious from the fact that, of the possible $N^2 \times N^2$ pairs of mesons, only N^2 are pairs of identical particles. That is to say, the bare Pomeron of the TE does not take into account BE effects, which are treated as corrections. However, for specific two-particle correlations, in particular configurations of the momenta, these effects are of the same order as (and sometimes even identical to) the leading ones and have to be kept.

We now introduce some notations.

Define from Fig. 1 :

$$\left(\frac{1}{\sigma} \frac{d\sigma}{dP_1} \right)_{1b} = 2 b(P_1, P_1) \equiv 2b(1,1) \quad (1.1)$$

where the factor 2 is inserted because of the two jets.

Then, by using the hypothesis of no correlations among the jets, we get

$$\left(\frac{1}{\sigma} \frac{d^2\sigma}{dP_1 dP_2} \right)_{2a,b} = 2 b(1,1) b(2,2) \quad (1.2)$$

For Figs. 2c,d we use again the no correlation assumption to write it in factorized form as :

$$\left(\frac{1}{\sigma} \frac{d^2\sigma}{dP_1 dP_2} \right)_{2c,d} = 2 b(1,2) b(2,1) \delta_{12} \quad (1.3)$$

where $b(1,2) = b(2,1)$ is a function of both p_1 and p_2 . For 1, 2 in the same jet we cannot assume factorization and we write instead :

$$\left(\frac{1}{\sigma} \frac{d^2\sigma}{dP_1 dP_2} \right)_{2e,f} = 2 b(1,2; 1,2) \quad (1.4)$$

$$\left(\frac{1}{\sigma} \frac{d^2\sigma}{dP_1 dP_2} \right)_{2g,h} = 2 b(1,2; 2,1) \delta_{12} \quad (1.5)$$

and finally

$$\left(\frac{1}{\sigma} \frac{d^2\sigma}{dP_1 dP_2} \right)_{2i} = 2 b_R(1,2;1,2) \text{Tr}(12 \bar{2} \bar{1}) + \\ + 2 b_R(1,2;2,1) \text{Tr}(12 \bar{1} \bar{2}) \quad (1.6)$$

where b_R stands for "resonance" contribution (absent in $\pi^-\pi^-$, $\pi^+\pi^+$) and $\text{Tr}(12\bar{2}\bar{1})$, $\text{Tr}(12\bar{1}\bar{2})$ are the Chan-Paton trace factors for each contribution. It follows that :

$$\rho^+ = \rho^- = \rho^0 = 2 b(1,1) \quad (1.7)$$

$$\rho^{--} = 2 [b(1,1)b(2,2) + b^2(1,2)] + \\ + 2 [b(12;12) + b(12;21)] = \rho^{++} \quad (1.8)$$

$$\rho^{-+} = 2 [b(1,1)b(2,2) + b(12;12)] + \\ + 4 [b_R(12;12) + b_R(21;21)] \quad (1.9)$$

$$\rho^{-0} = \rho^{+0} = 2 [b(1,1)b(2,2) + b(1,2;1,2)] + \\ + 2 [b_R(12;12) + b_R(21;21) - \\ - b_R(12;21) - b_R(21;12)] \quad (1.10)$$

$$\rho^{00} = 2 [b(1,1)b(2,2) + b^2(1,2)] + \\ + 2 [b(12;12) + b(12;21)] + \\ + 2 [b_R(12,12) + b_R(21,21) + b_R(12,21) \\ + b_R(21,12)] \quad (1.11)$$

It is now a trivial algebraic calculation to extract from the above equations the correlation function

$$R_2 \equiv \frac{\rho_2(p_1, p_2)}{\rho_1(p_1) \rho_1(p_2)} - 1 \quad (1.12)$$

For two negative pions, one gets immediately

$$R^{--} = \frac{b(12)b(21)}{b(11)b(22)} + \frac{C(1,2)}{2} + \frac{b(12)b(21)}{b(11)b(22)} \frac{\tilde{C}(1,2)}{2} \quad (1.13)$$

where we have defined the correlation functions $C, \tilde{C}$ by

$$b(12,12) = b(11)b(22)[1 + C(1,2)]$$

with $C(1,2) > -1$, and

$$b(12,21) = b(12)b(21)[1 + \tilde{C}(1,2)] \quad (1.14)$$

By defining

$$K_{12}^2 \equiv \frac{b(12)b(21)}{b(11)b(22)} ; K_{12} \xrightarrow{p_1 \rightarrow p_2} 1 \quad (1.15)$$

and assuming $C(12) \simeq \tilde{C}(1,2)$, one obtains a more compact formula for R^{--} :

$$R^{--} = K_{12}^2 + \frac{C(1,2)}{2} [1 + K_{12}^2] \quad (1.16)$$

for one π^- and one π^+ one gets instead

$$R^{+-} = \frac{1}{2} [C(1,2) + 2 C_R(1,2)] \quad (1.17)$$

where

$$\frac{b_R(12,12) + b_R(21,21)}{b(11)b(22)} \equiv C_R(1,2) \quad (1.18)$$

and, for one $\pi^\pm$ and one π^0 ,

$$R^{-0} = R^{+0} = \frac{1}{2} [\bar{C}(1,2) + C_R(1,2) - \tilde{C}_R(1,2)] \quad (1.19)$$

where

$$\frac{b_R(12,21) + b_R(21,12)}{b(11) b(22)} \equiv \tilde{C}_R(1,2) \quad (1.20)$$

Finally, for two π^0 , one gets

$$R^{00} = K_{12}^2 + \frac{C(1,2)}{2} [1 + K_{12}^2] + \frac{1}{2} [\bar{C}_R(1,2) + \tilde{C}_R(1,2)] \quad (1.21)$$

One can easily generalize our calculations to the case of n jets ($n \geq 1$) and finds

$$R_{n,jets}^{--} = K_{12}^2 + \frac{C(1,2)}{n} [1 + K_{12}^2] \quad (1.22a)$$

$$R_{n,jets}^{+-} = \frac{1}{n} [C(1,2) + 2 C_R(1,2)] \quad (1.22b)$$

$$R_{n,jets}^{0-} = \frac{1}{n} [\bar{C}(1,2) + C_R(1,2) - \tilde{C}_R(1,2)] \quad (1.22c)$$

$$R_{n,jets}^{00} = K_{12}^2 + \frac{C(1,2)}{n} [1 + K_{12}^2] + \frac{1}{n} [\bar{C}_R(1,2) + \tilde{C}_R(1,2)] \quad (1.22d)$$

The relation $R^{--} = R^{00} + R^{0-} - R^{+-}$ holds as a consequence of isospin conservation and of the $I=0$ nature of Pomeron exchange. The following relation follows from (1.22)

$$2 R_{2,jets}^{--} - R_{1,jet}^{--} = \frac{3}{2} R_{3,jets}^{--} - \frac{1}{2} R_{1,jet}^{--} = K_{12}^2 \quad (1.23)$$

and, within the validity of our scheme, could be used to measure directly the function $K_{12}^2(p_1, p_2)$ which is, as we shall see, of physical interest.

We now turn to a comparison of unlike and like pion correlations. If one wants to limit himself to charged pions, a subtraction of the "resonance" contribution to R^{+-} (e.g., from the ρ^0) is necessary. One then easily gets

$$\frac{\rho^{--}}{\rho^{*+-}} = \frac{R^{--} + 1}{R^{*+-} + 1} = 1 + \frac{b_{(1,2)}^2}{b_{(1,1)} b_{(2,2)}} = 1 + K_{12}^2 \xrightarrow{P_1 \rightarrow P_2} 2 \quad (1.24)$$

where ρ^{*+-} indicates subtraction of the resonance contribution.

For ρ^{--}/ρ^{-0} there is no resonance contribution at $P_1 = P_2$ and we expect

$$\rho^{--} \xrightarrow{P_1 \rightarrow P_2} 2 \rho^{+0} \quad (1.25)$$

Accurate measurements of ρ^{+0} would be quite desirable in order to test these predictions.

Another very interesting question is the dependence of ρ^{--}/ρ^{+-} on $(P_1 - P_2)$. Our result, Eq. (1.24) should be compared with the formula of Kopylov and Podgoretsky (KP) ¹⁰⁾ which reads

$$\frac{\rho^{--}}{\rho^{+-}} = 1 + \frac{b_{(1,2)}^{KP} b_{(2,1)}^{KP}}{b_{(1,1)}^{KP} b_{(2,2)}^{KP}} \simeq 1 + \frac{\{2 J_1(R q_T) / R q_T\}^2}{1 + (\tau q_0)^2} \quad (1.26)$$

where $b^{KP}(i,j)$ is the mutual coherence function of KP ¹⁰⁾,

$$q_T = \frac{|(\vec{P}_1 - \vec{P}_2) \times (\vec{P}_1 + \vec{P}_2)|}{|\vec{P}_1 + \vec{P}_2|}, \quad q_0 = |E_1 - E_2|$$

and R, τ are two parameters which, following the Hambury-Brown-Twiss effect ⁷⁾, have been interpreted by the authors as a measure of size of the interaction region (assumed to be a sphere of radius R) and as the lifetime of the pion sources, respectively. The strong similarity between Eq. (1.24) and Eq. (1.26) of KP suggests that our definition of $b(1,2)$ is the S matrix analogue of the mutual coherence function of KP ¹⁰⁾. Notice that Eqs. (1.22), (1.23) could provide measurement of K_{12} and, therefore, of the R, τ parameters of KP.

It would be interesting to analyze the expected behaviour of $K_{12}^2(p_1, p_2)$ in a Regge-Mueller approach like the one we have adopted. This should tell us about the possible variables upon which K_{12} can depend and could lead us to an S matrix analogue of the parameters R and τ . Because of the sort of coherent emission implied by multiperipheral dynamics it is unlikely that a result as simple as Eq. (1.26) will emerge.

B) - Three-particle correlations

The case of three-particle correlations can be dealt with in a similar way, it is just more cumbersome. Defining, as usual, the normalized three-particle correlation function $R^{\alpha\beta\gamma}(p_1, p_2, p_3)$ by

$$\frac{\rho^{\alpha\beta\gamma}}{\rho^\alpha \rho^\beta \rho^\gamma} = 1 + R^{\alpha\beta}(p_1, p_2) + R^{\beta\gamma}(p_2, p_3) + R^{\gamma\alpha}(p_3, p_1) + R^{\alpha\beta\gamma}(p_1, p_2, p_3) \quad (1.27)$$

one finds, for the general case of n jets,

$$\begin{aligned} R_{n \text{ jets}}^{---} &= 2 K_{12} K_{23} K_{31} + \\ &+ \frac{2}{n} \left[K_{12}^2 C_{13,23} + (\text{cyclic}) + K_{12} K_{23} K_{31} C_{13,32} + (\text{cyclic}) \right] + \\ &+ \frac{1}{n^2} \left[C_{123,123} + K_{12}^2 C_{123,213} + (\text{cyclic}) + K_{12} K_{23} K_{31} (C_{123,312} + C_{123,231}) \right] \end{aligned} \quad (1.28)$$

Here K_{ij} is defined as in Eq. (1.15), $C_{\alpha\beta, \gamma\delta}$ are the analogue of the two-particle correlation functions $C(1,2)$ of Eq. (1.14) generalized to more than two independent momenta [in this notation $C(1,2) \equiv C_{12,12}$] and, finally, $C_{\alpha\beta\gamma, \lambda\mu\nu}$ are three-particle correlation functions appearing when the three-particles come from the same jet. For instance, $C_{123,123}$ is defined by

$$\frac{b(123,123)}{b(11)b(22)b(33)} = 1 + C(12) + C(23) + C(31) + C_{123,123} > 0 \quad (1.29)$$

where $b(123,123)$ is the non-Bose-Einstein contribution to the three-particle inclusive cross-section in which all three-particles are in the same jet.

Two-particle correlations come in with a $1/n$ factor and three-particle correlations with a $1/n^2$ factor. This is exactly what one expects since it is less and less likely to have two or three particles in the same jet as the number of jets, n , increases.

The case of R^{--+} can be obtained from Eq. (1.28) by setting $K_{13} = K_{23} = 0$. One gets

$$R_{n\text{jets}}^{--+} = \frac{2}{n} K_{12}^2 C_{13,23} + \frac{1}{n^2} [C_{123,123} + K_{12}^2 C_{123,213}] + \text{"Resonance contributions"} \quad (1.30)$$

As in the case of two-particle correlations we denote by "resonance contributions" those coming from two or more initial (or final) particles in the same boundary (as in Fig. 2i). Similarly :

$$R_{n\text{jets}}^{-+0} = \frac{1}{n^2} C_{123,123} + \text{"Resonance contributions"} \quad (1.31)$$

We note the following equality to hold

$$9R_{3\text{jets}}^{---} + R_{1\text{jet}}^{---} - 8R_{2\text{jets}}^{---} = 4K_{12}K_{23}K_{31} \quad (1.32)$$

This equation can be combined with Eqs. (1.23) to provide an amusing connection between two and three-particle correlations.

The case of $\vec{P}_1 \approx \vec{P}_2 \approx \vec{P}_3$ is of particular interest because that is where BE effects are expected to be largest. We easily find :

$$R_{n\text{jets}}^{---} \xrightarrow{P_i \rightarrow P} 2 + \frac{12}{n} C_2 + \frac{6}{n^2} C_3 \quad (1.33)$$

where $C_3 = \lim_{P_i \rightarrow P} C_{123,123}$.

Similarly :

$$\begin{aligned} R_{n\text{jets}}^{-++} &\xrightarrow{P_i \rightarrow P} \frac{2}{n} C_2 + \frac{2}{n^2} C_3 + (\text{Resonance}) \\ R_{n\text{jets}}^{-+0} &\xrightarrow{P_i \rightarrow P} \frac{1}{n^2} C_3 + (\text{Resonance}) \end{aligned} \quad (1.34)$$

We close this section by noticing that, in the limit $n \gg 1$ (which can be called the limit of incoherent emission) our Eq. (1.28) becomes

$$R_{n\text{jets}}^{---} \xrightarrow{n \gg 1} 2 K_{12} K_{23} K_{31} \quad (1.35)$$

in agreement with the formula given by Kopylov et al. ¹¹⁾.

2. - DISCUSSION OF THE RESULTS

A) - Two-particle distributions

We now consider our results on two-particle correlations [Eq. (1.22a)] with our identification of $e^+e^- \rightarrow \text{hadrons}$ ^{*)} as a single jet process ($n=1$), of ordinary hadronic inelastic processes as a two-jet process ($n=2$) and of $\bar{p}p$ annihilation as a three-jet process ($n=3$). We have already mentioned that one can extract the interesting function $K_{12}(P_1, P_2)$ by forming suitable combinations. In more physical terms, Eq. (1.23) can be rewritten as

$$K_{12}^2(P_1, P_2) = 2 R_{\bar{p}p}^{--} - R_{e^+e^-}^{--} = \frac{3}{2} R_{(\bar{p}p)_a}^{--} - \frac{1}{2} R_{e^+e^-}^{--} \quad (2.1)$$

Measurements of the quantities on the right-hand side of (2.1) would lead to clean estimates of the space-time extent of the interaction region ¹⁰⁾.

*) For our purposes this process can be replaced by the valence contribution to $e^-p \rightarrow e^- + X$, i.e., at $x_{BJ} \simeq 1$.

A different way to proceed is to eliminate K_{12}^2 from Eq. (1.22a). One gets

$$\left[R_{e^+e^-}^{--} - R_{\pi p}^{--} \right] = \frac{3}{4} \left[R_{e^+e^-}^{--} - R_{(\bar{p}p)a}^{--} \right] \quad (2.2)$$

In the limit $P_1 \approx P_2$ ($S_{12} = (P_1 + P_2)^2 \approx 4m_\pi^2$) one has ($K_{12} \rightarrow 1$)

$$\left[R^{--} \right]_{e^+e^-} : \left[R^{--} \right]_{\pi p} : \left[R^{--} \right]_{(\bar{p}p)a} = 1 : \frac{1}{2} : \frac{1}{3} \quad (2.3)$$

This last set of equations becomes interesting if the quantities in brackets are significantly different from zero, i.e., from Eq. (1.22a), if

$C_2 \equiv \lim_{P_1 \rightarrow P_2} C(1,2)$ is different from zero by a reasonable amount ^{*)}. The only available data are in πp or pp ¹²⁾ and they indicate a value

$$\rho^{--}/(\rho^-)^2 \xrightarrow{P_1 \rightarrow P_2} 1.5 \pm .1 ; R^{--} \rightarrow .5 \pm .1 \quad (2.4)$$

This implies $C_2 \approx -0.5 \pm 0.1$, in agreement with our expectations. From this, we predict :

$$\begin{aligned} R_{e^+e^-}^{--} &\xrightarrow{P_1 \rightarrow P_2} .0 \pm .2 \\ R_{(\bar{p}p)a}^{--} &\xrightarrow{P_1 \rightarrow P_2} .67 \pm .07 \end{aligned} \quad (2.5)$$

We see that the BE effect is predicted to be small in e^+e^- and largest in $\bar{p}p$ annihilation. This comes physically from the fact that two π^- in the same jet (dual multiperipheral chain) cannot be neighbours ; as a consequence, we expect a dynamical negative correlation between the two π^- , which suppresses the probability ^{**) of their sitting at the same P and therefore reduces drastically the BE effect. On the contrary, if two identical pions are in different jets they are essentially dynamically uncorrelated and the BE effect becomes fully effective. Thus processes with many jets give a larger R^{--} than those with few jets. In the limit $n \gg 1$ we approach $R^{--} = 1$ (i.e., $\rho^{--} = 2\rho^-\rho^-$) which is the natural value for a non-correlated (statistical emission) scheme.}

^{*)} Remember that, from the definition (1.14), $C_2 > -1$ and that one expects $C_2 < 0$ (i.e., repulsion of two π^- in the same chain).

^{**) Positive correlation effects due to clusters (mainly $A_2^0 \rightarrow \pi^-\pi^-\pi^+$) are expected not to upset this negative correlation.}

It is interesting, at this point, to compare our predictions Eqs. (2.5) with those of a "universal emission" model of the type discussed in Ref. 6). This complete universality will also imply a universal value for R^{--} in striking disagreement with our predictions (2.5). It looks to us that an accurate measurement of R^{--} in $e^+e^- \rightarrow \text{hadrons}$ would then be of great value for deciding between these two (and probably other) production mechanisms.

Our predictions can also be tested in $\pi^+\pi^-$ distributions. There we expect, from Eq. (1.22b), at all p_1, p_2

$$R_{e^+e^-}^{+-} : R_{\pi p}^{+-} : R_{(pp)_a}^{+-} = 3 : 3/2 : 1 \quad (2.6)$$

Besides, we get the predictions (1.24), (1.25) for ρ^{--}/ρ^{+-} and ρ^{--}/ρ^{+0} but these are not peculiar of our scheme.

B) - Three-pion distributions

As we have said in Section 1B, one can obtain again from three-particle correlations information about the KP functions K_{12}, K_{23}, K_{31} , from Eq. (1.32). This equation clearly requires data which will not be available in the near future. It seems more appropriate instead to compare our predictions for R^{---} [Eq. (1.33)] with the data which are becoming available ¹³⁾ from a 205 GeV/c pp experiment at FNAL.

For pp and $P_1 \sim P_2 \sim P_3$ we use Eq. (1.33) with $n=2$ to obtain :

$$R_{pp}^{---} \xrightarrow{P_i \rightarrow P} 2 + 6C_2 + \frac{6}{4}C_3 \quad (2.7)$$

From the definition (1.29) of C_3 we see that

$$C_3 > -1 - 3C_2 \quad (2.8)$$

We also expect $b(123,123)$ in the left-hand side of Eq. (1.29) to be smaller than $b(12,12) b(3,3)$ at $P_i \sim P$ and this gives

$$C_3 < -2 C_2 \quad (2.9)$$

The two limits (2.8), (2.9) turn out to be rather close [remember $C_2 \approx -0.5$ from Eq. (2.4)]. Let us take, for instance, the average value

$$C_3 = -\frac{1}{2} - \frac{5}{2} C_2 \quad (2.10)$$

Inserting such value in Eq. (2.7) we get

$$R_{pp}^{---} = \frac{5}{4} + \frac{9}{4} C_2 \quad (2.11)$$

to be compared with [Eq. (1.22a) at $P_1 \sim P_2$] :

$$R_{pp}^{--} = 1 + C_2 \quad (2.12)$$

Unfortunately the calculation of R^{---} from the data is extremely sensitive to normalizations (such as the height of the plateau ρ^- , which increases with energy) and it involves cancellations among large numbers. It seems more appropriate to re-express our predictions (2.11), (2.12) in terms of ρ^{---} , ρ^{--} and ρ^- . One gets the relation :

$$\frac{\rho(\bar{P}_i \sim P)}{(\rho^-)^3} = \frac{21}{4} \left(\frac{\rho(\bar{P}_i \sim P)}{(\rho^-)^2} - 1 \right) = \frac{21}{4} R^{--} \quad (2.13)$$

If we take $R^{--} \simeq 0.6$ we predict $\rho^{---}/(\rho^-)^3 = 3.15$. The experimental value of $\rho^{---}(P_i \sim P)$ appears to be around 1.45 which agrees with our result with $R^{--} = 0.6$ for $\rho^- = (1.45/3.15)^{\frac{1}{3}} = 0.77$ a very reasonable value. A small change in the value of ρ , however, could worsen the agreement considerably. It seems to us that an effort should be made in order to provide accurate values for ρ^- , ρ^{--} , ρ^{---} at values of $s_{ij} = (P_i + P_j)^2$ near threshold, so that rather precise predictions, as the ones given here, can be tested.

We note that, with the value (2.10) for C_3 , our predictions for e^+e^- and $(\bar{p}p)_a$ become [from Eq. (1.33) again with $C_2 \approx -0.4$]

$$\begin{aligned} R_{e^+e^-}^{---} &\xrightarrow{P_i \rightarrow P} -1 - 3C_2 \approx .2 \\ R_{(\bar{p}p)_a}^{---} &\xrightarrow{P_i \rightarrow P} \frac{1}{3} (5 + 7C_2) \approx .7 ; R_{pp}^{---} \approx .35 \end{aligned} \quad (2.14)$$

Another relation, which can be easily derived from the general formulae, but which is not easy to check experimentally, is :

$$\left[R^{---} - 6R^{--} + 4 \right]_{e^+e^-} : \left[\text{same} \right]_{pp} : \left[\text{same} \right]_{(\bar{p}p)_a} = 1 : \frac{1}{4} : \frac{1}{9} \quad (2.15)$$

at $P_i \sim P$.

Finally, more relations can be derived involving $(+--)$, $(+-o)$, $(--o)$ and other charge configurations. Again here we encounter the problem of subtracting the "resonant" contributions to non-exotic channels. For the experimentally interesting channel $(\pm^{++})$ we get, in the limit $P_i \sim P$, and after subtraction of "resonant" contributions,

$$\begin{aligned} R_{pp}^{--+} &= C_2 + C_3/2 \approx -\frac{1}{4} (1 + C_2) \approx -1/8 \\ R_{e^+e^-}^{--+} &= 2C_2 + 2C_3 \approx -1 - 3C_2 \approx 1/2 \\ R_{(\bar{p}p)_a}^{--+} &= \frac{2}{3} C_2 + \frac{2}{9} C_3 \approx -\frac{1}{9} (1 + C_2) \approx 0 \end{aligned} \quad (2.16)$$

($C_2 = -.5$ here)

These correlations should in any case satisfy the relation

$$9 R_{(\bar{p}p)_a}^{--+} + R_{e^+e^-}^{--+} - 8 R_{pp}^{--+} = 0 \quad (2.17)$$

which follows from Eq. (1.30) and should not be altered by "resonant" contributions.

CONCLUSIONS

In this paper we have emphasized the importance of obtaining precise data on two, and possibly three-pion correlations in order to obtain information on the dynamics of multihadron production in various lepton and hadron induced processes. The effect which is most sensitive to the underlying picture turns out to be that of Bose-Einstein correlations for identical pions. The point here is that strong ordering in rapidity tends to wash out the effect, whereas a purely statistical emission enhances it. In certain models, such as the gauge-dual approach to hadrodynamics in the topological expansion framework, different processes correspond to different combinations of coherent and incoherent emission depending on the "number of jets" in the process. In this framework, quantitative predictions can be obtained both for two and for three-pion correlations. The most interesting prediction is probably the one concerning $e^+e^- \rightarrow \text{hadrons}$. We expect $R^{--}(P_1 P_2)$ at $P_1 \sim P_2$ to be much smaller there than in pp or πp collisions, whereas other schemes would predict about the same value. Measurement of such a quantity should be within reach.

On three-particle correlations better data are certainly needed and the region $P_1 \sim P_1 \sim P_3$ should be carefully measured in the $(---)$ $(+++)$ $(--+)$ $(++-)$ charge configurations. Correct normalization of the inclusive cross-sections is very important.

On the more theoretical side our approach could provide an S matrix, momentum space alternative to the standard space-time description of the Hambury-Brown-Twiss effect.

We can provide a Regge-Mueller definition of the mutual coherence function of Kopylov-Podgoretsky. This could provide new tools for the study of Bose-Einstein effects in multihadron final states.

ACKNOWLEDGEMENTS

We wish to thank A. Pais for an interesting discussion. One of us (A.G.) wishes to thank the Theoretical Physics Division at CERN for the warm hospitality extended to him during the period in which this work was done. INFN financial support is also acknowledged.

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FIGURE CAPTIONS

Figure 1 Total (a) and single particle inclusive (b) cross-sections in
the topological expansion.

Figure 2 Components of the two-particle inclusive cross-sections.
(c), (d), (g), (h) provide the Bose-Einstein effect.

FIG.1

BOSE - EINSTEIN
EFFECT

(1 ↔ 2)
(1' ↔ 2')

(1 ↔ 2)
(1' ↔ 2')
BOSE - EINSTEIN
EFFECT

(1 ↔ 2)
(1' ↔ 2')

FIG. 2