WHAT IS THE WEAK CHARGE OF THE ELECTRON?

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ABSTRACT

The experimental results on \( \bar{\nu}_\mu \) electron scattering, when interpreted within \( SU_2 \times U_1 \) theories, put severe constraints on the weak charge of the right-handed electron. Under very general assumptions it is shown that the cases \( \frac{1}{3} \text{weak}(e_R^-) = 0 \) or \( -\frac{1}{3} \) are possible. The case \( \frac{1}{3} \text{weak}(e_R^-) = +\frac{1}{3} \) is found to be unlikely. All other assignments for \( e_R^- \) are excluded. Consequences for parity violation in atomic physics are discussed.
The conventional way to interpret the observed universality of leptons and quarks with respect to the weak interaction is to place the left-handed leptons and quarks into doublets of the weak isotopic spin SU\textsuperscript{weak}\textsubscript{2}

\[
\begin{pmatrix}
\nu_e & \nu_\mu \\
e^- & \mu^-
\end{pmatrix}_L,
\begin{pmatrix}
u_e & c \\
d \cos \theta + s \sin \theta & -d \sin \theta + s \cos \theta
\end{pmatrix}_L
\]

(L: left-handed, R: right-handed).

In the minimal SU\textsubscript{2} × U\textsubscript{1} theory\textsuperscript{1}), the right-handed fermions are assumed to be SU\textsuperscript{weak}\textsubscript{2} singlets. However, in many other theories, in particular the vector-like theories, at least some of the right-handed leptons and quarks do transform non-trivially under SU\textsuperscript{weak}\textsubscript{2} and are weak partners of new heavy leptons and quarks. [For a review, see Ref. 2)]. If the latter are chosen to be sufficiently heavy, the new charged currents, generated by the new SU\textsuperscript{weak}\textsubscript{2} representations, will not be active at low energies, below the corresponding thresholds. However, in SU\textsubscript{2} × U\textsubscript{1} theories the algebraic form of the neutral current depends critically on what kind of new weak multiplets exist. Thus the various neutral current cross-sections measured recently in neutrino scattering, when interpreted within SU\textsubscript{2} × U\textsubscript{1} theories, will pose constraints on the representation content of the right-handed valence quarks \(u_R, d_R\) and of the right-handed electron. In this note we investigate those constraints within a very general approach, especially with respect to the lepton system. We shall use only the data of the Gargamelle experiments\textsuperscript{3}) on \(\nu_\mu (\bar{\nu}_\mu) +\) nucleus \(\rightarrow\) anything and \(\bar{\nu}_\mu + e^+ \rightarrow \nu_\mu + e^+\):

\[
R^{\nu N} = 0.26 \pm 0.04,
\]
\[
R^{\bar{\nu} N} = 0.39 \pm 0.06, \quad (N: I = 0 \text{ nucleus})
\]

\[
\begin{pmatrix}
G^{\nu N} / G^{\bar{\nu} N}
\end{pmatrix} = 0.57 \pm 0.14;
\]

\[
G^{\bar{\nu} e^-} = 10^{+2.1}_{-0.9} \cdot 10^{-42} E_{\bar{\nu}} \text{[GeV]} \text{ cm}^2 / \text{electron},
\]

(90\% c.l.)
- 2 -

All other experimental results (on inclusive neutrino production, elastic neutrino-proton scattering, single pion production by neutrinos) are either associated with relatively large errors and are, therefore, less useful for providing constraints on the neutral current structure, or are carried out at very high energies, where it is difficult to estimate possible scaling violations or new thresholds.

We shall assume that all quarks have either charge $2/3$ or $-1/3$, and that

$$g = \frac{M_w^2}{M_Z^2 \cos^2 \theta_W}$$

($\theta_W$: $SU_2 \times U_1$ mixing angle) is a free parameter, unlike the situation in the minimal theory where $\rho = 1$. The effective weak Hamiltonian can be written as

$$H^w = \frac{4G}{f_{\pi}} \left( j^+_F j^-_F + g j^+_F j^n_F \right)$$

(2)

where $j^+_F, j^n_F$ are the charged (neutral) weak currents, respectively, and

$$j^+_F = j^3_F + x_w j^e_F; \quad x_w = \sin^2 \theta_W.$$

The most general structure of the quark doublets is (neglecting the Cabibbo angle):

$$\begin{pmatrix} u_L \cr d_L \end{pmatrix}; \quad \begin{pmatrix} u \cr b' \end{pmatrix}^\alpha; \quad \begin{pmatrix} t' \cr d \end{pmatrix}^\beta + \text{doublets involving non-valence quarks only}$$

(3)

where $b'$ stands for an orthogonal linear combination of heavy quarks of charge $-1/3$, $t'$ for an orthogonal linear combination of heavy quarks of charge $2/3$, and $\alpha, \beta$ are parameters such that $|\alpha| \leq 1, |\beta| \leq 1$. The doublet structure denoted above represents a continuum of weak-interaction models. For example, for $\alpha = 1, \beta = 1$ the pure vector model is obtained. The choice $\beta = 0, \alpha = 1$ reproduces the model discussed, e.g., in Ref. 4). If $\alpha \neq 0,1$ or $\beta \neq 0,1$, or both, the $u_L^R, d_L^R$ quark
enters the weak interaction with a non-zero, but reduced strength, in which case flavour changing neutral currents involving the $u(d)$ quark will arise [see, for example, Ref. 5)].

In the valence quark approximation ($\bar{u} = \bar{d} = \bar{s} = s = 0$) one finds

$$R^{\nu N} = g^2 \left[ \frac{1}{2} + \frac{1}{12} (\alpha^\nu \beta^\nu) - x_w (1 + \frac{2}{3} \alpha^2 + \frac{1}{3} \beta^2) + \frac{20}{27} x_{\omega}^2 \right]$$

$$R^{\bar{\nu} N} = g^2 \left[ \frac{1}{2} + \frac{3}{4} (\alpha^\nu \beta^\nu) - x_w (1 + \frac{2}{3} \alpha^2 + \beta^2) + \frac{20}{27} x_{\omega}^2 \right].$$

(1)

The weak charge of the left-handed electron is fixed to be $I_3^{\text{weak}}(e^-) = -\frac{1}{2}$ by universality. The main purpose of our investigation is to present constraints for $I_3^{\text{weak}}(e^-)$ within $SU_2 \times U_1$ theories. The neutral leptonic current is, in general,

$$j^L_R = \frac{i}{2} (\bar{\nu}_e \nu_e - \bar{e}^- e^-) L + I_3^{\text{weak}} (e_R^-) (\bar{e}^- e^-) R +$$

$$+ \text{muonic contributions} + \text{heavy lepton contributions.}$$

The neutral current cross-sections for $\frac{(-)}{\nu \mu}$ electron scattering can be written as

$$G^{\nu e^-} = \frac{2 G_{\text{f}} M_e}{\pi} E \frac{g^2}{2} \left[ (-\frac{1}{2} + x_w)^2 + \frac{1}{3} \left( I_3^{\text{weak}} (e_R^-) + x_w \right)^2 \right]$$

$$G^{\bar{\nu} e^-} = \frac{2 G_{\text{f}} M_e}{\pi} E \frac{g^2}{2} \left[ \left( -\frac{1}{2} + x_w \right)^2 + \left( I_3^{\text{weak}} (e_R^-) + x_w \right)^2 \right].$$

(6)

In this paper we restrict ourselves to unmixed cases, i.e.,

$$I_3^{\text{weak}} (e_R^-) = \pm \frac{3}{2} \left( n = 0, 1, \ldots \right).$$
We consider the following ratios which are independent of the strength parameter \( p \) [see also Ref. 2]:

\[
R = \left( \frac{\sigma_{W}^+}{\sigma_{W}^0} \right)_N = \left( \frac{1}{\xi} + \frac{1}{4} (\alpha^+ \beta^+ - \alpha^- \beta^-) - I_{W} \left( \frac{1}{3} + \frac{1}{3} \xi^2 + \frac{1}{3} \beta^2 \right) + \frac{20}{27} x_{W}^2 \right) \left( \frac{1}{\xi} + \frac{1}{4} (\alpha^- \beta^- - \alpha^+ \beta^+) - I_{W} \left( \frac{1}{3} + \frac{1}{3} \xi^2 + \frac{1}{3} \beta^2 \right) + \frac{20}{27} x_{W}^2 \right)^{-1}
\]

\[
\tau = \frac{e^{-\frac{1}{2} \beta} \left( e^0 \right)}{E_{\gamma} \cdot 2 \alpha \langle m_e \rangle} \sqrt{R N} = \left( \frac{\alpha + x_{W}}{\xi} \right)^{1/2} + \left( \frac{I_{W} \langle e_{e} \rangle + x_{W}}{\xi} \right)^{1/2}
\]

Let us first concentrate on the hadronic ratio \( R \) [Eq. (7)]. In Table 1 we denote the range of values of \( x_{W} \) for the various choices of \( \alpha \) and \( \beta \), which is allowed on the basis of the data (1). Almost all values of \( \alpha \) and \( \beta \) can be admitted [see also Ref. 6)]. The pure vector case \( \left( \alpha = \beta = 1 \right) \) is excluded. It is curious to note the sensitivity of \( R \) to \( \alpha \) and \( \beta \). For example, the choice \( \alpha = 1, \beta = 0.9 \), which is very close to the pure vector case, is still allowed.

We now include the leptonic data in our analysis. According to the data Eq. (1) we have, using the relation \( 20^2 m_e E_{\gamma}/\pi \equiv 1.7 \times 10^{-41} E_{\gamma} [\text{GeV}] \) cm^2:

\[
\tau = 0.23 \pm 0.49 - 0.22 \quad (90\% \text{ confidence level}).
\]

In Table 2 we describe the allowed range of \( \alpha \) and \( \beta \) for the case \( I_{W} \langle e_{e} \rangle = 0 \). The forbidden region in the \((\alpha, \beta)\) space increases slightly. In Table 3 the same analysis is carried through for \( I_{W} \langle e_{e} \rangle = -\frac{1}{3} \). Again the forbidden region increases only slightly. Note, however, that in both cases the inclusion of the leptonic data provides stronger bounds for \( x_{W} \).

In Table 4 we continue our analysis for the case \( I_{W} \langle e_{e} \rangle = +\frac{1}{3} \). The forbidden region increases strongly, only the region \( \alpha \geq 0.7 \) is still allowed, and only very small values of \( x_{W} \) are possible. If we require that the leptonic neutral current
cross-section is within 1½ standard deviation of the data Eq. (1), all values of α and β are forbidden. Thus the case $I_3(e_R^-) = +\frac{1}{2}$ is rather unlikely.

Finally, we investigate the cases $|I_3(e_R^-)| \geq 1$. It is easy to see that such cases are very unlikely [see also Ref. 7]). For example, the case $I_3 = +1$ requires $r > 1.6$ for all choices of $\alpha$ and $\beta$. This is excluded on a high confidence level. The case $I_3 = -1$ requires $r > 1.0$ for all choices of $\alpha$ and $\beta$ and is also excluded on a relatively high confidence level. In all cases with $|I_3(e_R^-)| > 1$ one obtains $r \geq 2$; such assignments are excluded.

Summarizing our conclusions we can say that under the very general assumptions stated above, the right-handed electron can either be a $SU_2^{weak}$ singlet, or the $I_3 = -\frac{1}{2}$ member of a $SU_2^{weak}$ doublet. It is unlikely that $e_R^-$ in the $I_3 = +\frac{1}{2}$ member of an $SU_2^{weak}$ doublet [such a theory was recently discussed in Ref. 8]). All other assignments of $e_R^-$ are excluded.

Our conclusion rules out many lepton theories discussed recently, in particular theories in which the right-handed electron belongs to an $SU_2$ triplet$^2,7$).

Recently the ratio $R^{lept.} = \sigma^{\nu\bar{\nu}}(e^-)/\sigma^{\nu\bar{\nu}}(e^-)$ has been measured in a counter experiment [see, for example, Ref. 9]], with the result $R^{lept.} = 1.3 \pm 80\%$. Both the vector case $I_3(e_R^-) = -\frac{1}{2}$ and the case $I_3(e_R^-) = 0$ are allowed by this data.

Clearly better data for $\nu^\nu$ electron scattering are needed in order to distinguish between the vector case and the "standard" case $I_1(e_R^-) = 0$. In the "standard" case parity violation in atomic physics should exist, and the effects would add coherently in case of heavy atoms. [For a discussion of the experimental situation, see Ref. 10).] The coherent effects would be absent in the vector case.

If new, refined experiments designed to search for parity violation in atomic physics should fail to show an effect, and if the new $\nu^\nu$ electron scattering experiments should rule out the vector case, all $SU_2 \times U_1$ theories would be ruled out, and one would be forced by experiment to discuss theories involving several neutral intermediate bosons. As discussed in Ref. 11) [see also Ref. 12)], theories with at least two Z bosons can describe a situation in which parity is conserved by the neutral current Hamiltonian, and yet the cross-sections for $\nu^\nu$ and $\nu^\nu$ neutral current scattering are not equal.
Acknowledgement

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References


   H. Fritzsch, Caltech preprint CALT-68-584 (to be published in: Proc. of the Orbis Scientiae 1977, Univ. of Miami, Florida) and references quoted therein.


9) A. Faissner, Talk given at the Tbilissi Conference (1976).


11) H. Fritzsch and F. Minkowski, Nuclear Phys. B103, 61 (1976);

Table 1

Allowed range of $x_w = \sin^2 \theta_w$ as a function of $\alpha$ and $\beta$.

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Table 2

Allowed range of $x_w$ as a function of $\alpha$ and $\beta$ in case $I_2(e^+_R) = 0$

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Table 3

Allowed range of $x_w$ as a function of $\alpha$ and $\beta$ in case $I_{z}(e^{-}) = -\frac{1}{2}$.

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Table 4

Allowed range of $x^w$ as a function of $\alpha$ and $\beta$ in case $I_z(e_R) = +\frac{1}{2}$.

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