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WHAT IS THE WEAK CHARGE OF THE ELECTRON?

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ABSTRACT

The experimental results on $\bar{\nu}_\mu$ electron scattering, when interpreted within $SU_2 \times U_1$ theories, put severe constraints on the weak charge of the right-handed electron. Under very general assumptions it is shown that the cases $I_3^{\text{weak}}(e_R^-) = 0$ or $-\frac{1}{2}$ are possible. The case $I_3^{\text{weak}}(e_R^-) = +\frac{1}{2}$ is found to be unlikely. All other assignments for e_R^- are excluded. Consequences for parity violation in atomic physics are discussed.

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The conventional way to interpret the observed universality of leptons and quarks with respect to the weak interaction is to place the left-handed leptons and quarks into doublets of the weak isotopic spin SU_2^{weak}

$$\begin{pmatrix} \nu_e & \nu_\mu \\ e^- & \mu^- \end{pmatrix}_L, \quad \begin{pmatrix} u & c \\ d \cos \theta_c + s \sin \theta_c & -d \sin \theta_c + s \cos \theta_c \end{pmatrix}_L$$

(L : left-handed, R : right-handed).

In the minimal $SU_2 \times U_1$ theory¹⁾, the right-handed fermions are assumed to be SU_2^{weak} singlets. However, in many other theories, in particular the vector-like theories, at least some of the right-handed leptons and quarks do transform non-trivially under SU_2^{weak} and are weak partners of new heavy leptons and quarks. [For a review, see Ref. 2)]. If the latter are chosen to be sufficiently heavy, the new charged currents, generated by the new SU_2^{weak} representations, will not be active at low energies, below the corresponding thresholds. However, in $SU_2 \times U_1$ theories the algebraic form of the neutral current depends critically on what kind of new weak multiplets exist. Thus the various neutral current cross-sections measured recently in neutrino scattering, when interpreted within $SU_2 \times U_1$ theories, will pose constraints on the representation content of the right-handed valence quarks u_R, d_R and of the right-handed electron. In this note we investigate those constraints within a very general approach, especially with respect to the lepton system. We shall use only the data of the Gargamelle experiments³⁾ on $\nu_\mu (\bar{\nu}_\mu) + \text{nucleus} \rightarrow \text{anything}$ and $\bar{\nu}_\mu + e^- \rightarrow \nu_\mu + e^-$:

$$R^{\nu N} = 0.26 \pm 0.04,$$

$$R^{\bar{\nu} N} = 0.39 \pm 0.06, \quad (N: I=0 \text{ nucleus})$$

$$\left(\frac{\sigma^{\bar{\nu}\bar{\nu}}}{\sigma^{\nu\nu}} \right)_N = 0.59 \pm 0.14;$$

(1)

$$\sigma^{\bar{\nu} e^-} = 1_{-0.9}^{+2.1} \cdot 10^{-42} E_{\bar{\nu}} [\text{GeV}] \text{ cm}^2 / \text{electron} \\ (\text{90\% c.l.}).$$

All other experimental results (on inclusive neutrino production, elastic neutrino-proton scattering, single pion production by neutrinos) are either associated with relatively large errors and are, therefore, less useful for providing constraints on the neutral current structure, or are carried out at very high energies, where it is difficult to estimate possible scaling violations or new thresholds.

We shall assume that all quarks have either charge $2/3$ or $-1/3$, and that

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W}$$

(θ_W : $SU_2 \times U_1$ mixing angle) is a free parameter, unlike the situation in the minimal theory where $\rho = 1$. The effective weak Hamiltonian can be written as

$$H^W = \frac{4G}{\sqrt{2}} (j_\mu^+ j_\mu^- + \rho j_\mu^n j_\mu^n) \quad (2)$$

where j_μ^+, j_μ^n are the charged (neutral) weak currents, respectively, and

$$j_\mu^n = j_\mu^3 - x_W j_\mu^e; \quad x_W = \sin^2 \theta_W.$$

The most general structure of the quark doublets is (neglecting the Cabibbo angle):

$$\begin{pmatrix} u \\ d \end{pmatrix}_L; \begin{pmatrix} u \\ b' \end{pmatrix}_R \cdot \alpha; \begin{pmatrix} t' \\ d \end{pmatrix}_R \cdot \beta + \text{doublets involving non-valence quarks only} \quad (3)$$

where b' stands for an orthogonal linear combination of heavy quarks of charge $-1/3$, t' for an orthogonal linear combination of heavy quarks of charge $2/3$, and α, β are parameters such that $|\alpha| \leq 1, |\beta| \leq 1$. The doublet structure denoted above represents a continuum of weak-interaction models. For example, for $\alpha = 1, \beta = 1$ the pure vector model is obtained. The choice $\beta = 0, \alpha = 1$ reproduces the model discussed, e.g., in Ref. 4). If $\alpha \neq 0, 1$ or $\beta \neq 0, 1$, or both, the $u_R(d_R)$ quark

enters the weak interaction with a non-zero, but reduced strength, in which case flavour changing neutral currents involving the u(d) quark will arise [see, for example, Ref. 5)].

In the valence quark approximation ($\bar{u} = \bar{d} = \bar{s} = s = 0$) one finds

$$\begin{aligned} R^{vN} &= g^2 \left[\frac{1}{2} + \frac{1}{12} (\alpha^4 + \beta^4) - x_W \left(1 + \frac{2}{9} \alpha^2 + \frac{1}{9} \beta^2 \right) + \frac{20}{27} x_W^2 \right] \\ R^{\bar{v}N} &= g^2 \left[\frac{1}{2} + \frac{3}{4} (\alpha^4 + \beta^4) - x_W (1 + 2\alpha^2 + \beta^2) + \frac{20}{9} x_W^2 \right]. \end{aligned} \quad (4)$$

The weak charge of the left-handed electron is fixed to be $I_3^{\text{weak}}(e_L^-) = -\frac{1}{2}$ by universality. The main purpose of our investigation is to present constraints for $I_3^{\text{weak}}(e_R^-)$ within $SU_2 \times U_1$ theories. The neutral leptonic current is, in general,

$$j_r^{n,l} = \frac{1}{2} (\bar{\nu}_e \nu_e - \bar{e}^- e^-)_L + I_3^{\text{weak}}(e_R^-) (\bar{e}^- e^-)_R + \quad (5)$$

+ muonic contributions + heavy lepton contributions.

The neutral current cross-sections for $\nu_\mu^{(-)}$ electron scattering can be written as

$$\begin{aligned} \sigma^{\nu_e}(e^-) &= \frac{2G^2 m_e}{\pi} E_\nu g^2 \left[\left(-\frac{1}{2} + x_W \right)^2 + \frac{1}{3} \left(I_3^{\text{weak}}(e_R^-) + x_W \right)^2 \right] \\ \sigma^{\bar{\nu}_e}(e^-) &= \frac{2G^2 m_e}{\pi} E_{\bar{\nu}} g^2 \left[\frac{1}{3} \left(-\frac{1}{2} + x_W \right)^2 + \left(I_3^{\text{weak}}(e_R^-) + x_W \right)^2 \right]. \end{aligned} \quad (6)$$

In this paper we restrict ourselves to unmixed cases, i.e.,

$$I_3^{\text{weak}}(e_R^-) = \pm \frac{n}{2} \quad (n = 0, 1, \dots).$$

We consider the following ratios which are independent of the strength parameter ρ [see also Ref. 2)]:

$$R = \left(\frac{\sigma_{\bar{\nu}\nu}}{\sigma_{\nu\nu}} \right)_N = \frac{\frac{1}{6} + \frac{1}{4}(\alpha^4 + \beta^4) - x_W \left(\frac{1}{3} + \frac{2}{3}\alpha^2 + \frac{1}{3}\beta^2 \right) + \frac{20}{27}x_W^2}{\frac{1}{2} + \frac{1}{12}(\alpha^4 + \beta^4) - x_W \left(1 + \frac{2}{3}\alpha^2 + \frac{1}{3}\beta^2 \right) + \frac{20}{27}x_W^2} \quad (7)$$

$$r = \frac{\sigma_{\bar{\nu}\nu}(e^-)}{E_{\bar{\nu}} \cdot \frac{2G^2 m_e}{\pi}} \bigg/ R_{\nu N} = \frac{\left(-\frac{1}{2} + x_W \right)^2 \cdot \frac{1}{3} + \left(I_3^{\text{weak}}(e_R^-) + x_W \right)^2}{\frac{1}{2} + \frac{1}{12}(\alpha^4 + \beta^4) - x_W \left(1 + \frac{2}{3}\alpha^2 + \frac{1}{3}\beta^2 \right) + \frac{20}{27}x_W^2} \quad (8)$$

Let us first concentrate on the hadronic ratio R [Eq. (7)]. In Table 1 we denote the range of values of x_W for the various choices of α and β , which is allowed on the basis of the data (1). Almost all values of α and β can be admitted [see also Ref. 6)]. The pure vector case ($\alpha = \beta = 1$) is excluded. It is curious to note the sensitivity of R to α and β . For example, the choice $\alpha = 1$, $\beta = 0.9$, which is very close to the pure vector case, is still allowed.

We now include the leptonic data in our analysis. According to the data Eq. (1) we have, using the relation $2G^2 m_e E_V / \pi \cong 1.7 \times 10^{-41} E_V [\text{GeV}] \text{ cm}^2$:

$$r = 0.23^{+0.49}_{-0.22} \quad (90\% \text{ confidence level}).$$

In Table 2 we describe the allowed range of α and β for the case $I_3(e_R^-) = 0$. The forbidden region in the (α, β) space increases slightly. In Table 3 the same analysis is carried through for $I_3(e_R^-) = -\frac{1}{2}$. Again the forbidden region increases only slightly. Note, however, that in both cases the inclusion of the leptonic data provides stronger bounds for x_W .

In Table 4 we continue our analysis for the case $I_3(e_R^-) = +\frac{1}{2}$. The forbidden region increases strongly, only the region $\alpha \geq 0.7$ is still allowed, and only very small values of x_W are possible. If we require that the leptonic neutral current

cross-section is within $1\frac{1}{2}$ standard deviation of the data Eq. (1), all values of α and β are forbidden. Thus the case $I_3(e_R^-) = +\frac{1}{2}$ is rather unlikely.

Finally, we investigate the cases $|I_3(e_R^-)| \geq 1$. It is easy to see that such cases are very unlikely [see also Ref. 7)]. For example, the case $I_3^R = +1$ requires $r > 1.6$ for all choices of α and β . This is excluded on a high confidence level. The case $I_3^R = -1$ requires $r > 1.0$ for all choices of α and β and is also excluded on a relatively high confidence level. In all cases with $|I_3(e_R^-)| > 1$ one obtains $r \gtrsim 2$; such assignments are excluded.

Summarizing our conclusions we can say that under the very general assumptions stated above, the right-handed electron can either be a SU_2^{weak} singlet, or the $I_3 = -\frac{1}{2}$ member of a SU_2^{weak} doublet. It is unlikely that e_R^- in the $I_3 = +\frac{1}{2}$ member of an SU_2^{weak} doublet [such a theory was recently discussed in Ref. 8)]. All other assignments of e_R^- are excluded.

Our conclusion rules out many lepton theories discussed recently, in particular theories in which the right-handed electron belongs to an SU_2 triplet^{2,7)}.

Recently the ratio $R^{\text{lept.}} = \sigma^{\nu\mu}(e^-)/\sigma^{\nu\mu}(e^-)$ has been measured in a counter experiment [see, for example, Ref. 9)], with the result $R^{\text{lept.}} \approx 1.3 \pm 80\%$. Both the vector case $I_3(e_R^-) = -\frac{1}{2}$ and the case $I_3(e_R^-) = 0$ are allowed by this data.

Clearly better data for $(\bar{\nu}_\mu^-)$ electron scattering are needed in order to distinguish between the vector case and the "standard" case $I_3(e_R^-) = 0$. In the "standard" case parity violation in atomic physics should exist, and the effects would add coherently in case of heavy atoms. [For a discussion of the experimental situation, see Ref. 10).] The coherent effects would be absent in the vector case.

If new, refined experiments designed to search for parity violation in atomic physics should fail to show an effect, and if the new $(\bar{\nu}_\mu^-)$ electron scattering experiments should rule out the vector case, all $SU_2 \times U_1$ theories would be ruled out, and one would be forced by experiment to discuss theories involving several neutral intermediate bosons. As discussed in Ref. 11) [see also Ref. 12)], theories with at least two Z bosons can describe a situation in which parity is conserved by the neutral current Hamiltonian, and yet the cross-sections for ν_μ and $\bar{\nu}_\mu$ neutral current scattering are not equal.

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References

- 1) S.L. Glashow, Nuclear Phys. 22, 569 (1961).
A. Salam and J.C. Ward, Phys. Letters 13, 168 (1964).
S. Weinberg, Phys. Rev. Letters 19, 1264 (1967).
- 2) H. Fritzsch, Proc. of the Int. Neutrino Conference, Aachen (1976).
- 3) J. Blietschau et al., to be published in Nuclear Phys. B, (CERN preprint 76-55).
J. Blietschau et al., Nuclear Phys. B114, 189 (1976).
- 4) P. Fayet, Nuclear Phys. B78, 14 (1974).
Y. Achiman, K. Koller and T.F. Walsh, Phys. Letters 59B, 261 (1975).
R.M. Barnett, Phys. Rev. D13, 671 (1976).
P. Ramond, Nuclear Phys. B110, 214 (1976).
F. Gürsey and P. Sikivie, Phys. Rev. Letters 36, 775 (1976).
- 5) Y. Achiman and T.F. Walsh, DESY preprint 76/46 (1976).
H. Fritzsch, Phys. Letters 66B, 42 (1977).
S.L. Glashow and S. Weinberg, to be published in Phys. Rev.
- 6) Y. Achiman and T.F. Walsh, DESY preprint 76/46 (1976).
- 7) J. Bernabeu and C. Jarlskog, CERN preprint TH.2207 (to be published in Nuovo Cimento).
H. Fritzsch, Caltech preprint CALT-68-584 (to be published in: Proc. of the Orbis Scientiae 1977, Univ. of Miami, Florida) and references quoted therein.
- 8) F. Wilczek and A. Zee, Princeton University preprint (1977).
- 9) A. Faissner, Talk given at the Tbilissi Conference (1976).
- 10) P. Soreide et al., Seattle-Oxford preprint (1976).
- 11) H. Fritzsch and P. Minkowski, Nuclear Phys. B103, 61 (1976);
see also Ann. Phys. 93, 193 (1974).
- 12) J.J. Sakurai, Proceedings of the Int. Neutrino Conference (Aachen 1976).

Table 1

Allowed range of $x_W = \sin^2 \theta_W$ as a function of α and β .

$\begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.26- 0.41	0.27- 0.42	0.26- 0.43	0.27- 0.45	0.33- 0.48	0.35- 0.50	0.39- 0.54	0.42- 0.58	<div>0.. 0.24 0.41- 0.61</div>	0- 0.64	0- 0.61
0.1	0.26- 0.41	0.26- 0.42	0.26- 0.43	0.29- 0.45	0.32- 0.47	0.35- 0.50	0.4- 0.54	0.42- 0.58	<div>0- 0.23 0.42- 0.62</div>	0- 0.64	0- 0.62
0.2	0.27- 0.42	0.27- 0.42	0.27- 0.44	0.29- 0.46	0.32- 0.48	0.36- 0.51	0.40- 0.55	0.44- 0.59	<div>0- 0.25 0.45- 0.63</div>	0- 0.67	0- 0.66
0.3	0.28- 0.43	0.26- 0.43	0.28- 0.45	0.3- 0.46	0.34- 0.43	0.38- 0.52	0.43- 0.56	<div>0- 0.04 0.42- 0.61</div>	<div>0- 0.20 0.51 0.66</div>	0- 0.70	0- 0.71
0.4	0.28- 0.44	0.29- 0.44	0.28- 0.45	0.31- 0.47	0.35- 0.50	0.39- 0.54	0.44- 0.58	<div>0- 0.05 0.50- 0.63</div>	<div>0- 0.20 0.55- 0.68</div>	0- 0.73	0- 0.77
0.5	0.28- 0.44	0.29- 0.45	0.28- 0.46	0.31- 0.48	0.35- 0.51	0.41- 0.55	0.46- 0.59	<div>0- 0.08 0.53- 0.65</div>	<div>0- 0.20 0.59- 0.71</div>	0- 0.77	0- 0.84
0.6	0.26- 0.44	0.27- 0.45	0.26- 0.46	0.29- 0.48	0.34- 0.52	0.40- 0.56	0- 0.61	<div>0- 0.15 0.54- 0.67</div>	<div>0- 0.25 0.62- 0.74</div>	0- 0.82	0.07- 0.59
0.7	0.17- 0.42	0.18- 0.43	0- 0.45	0.18- 0.47	0- 0.46	0.32- 0.55	0- 0.60	<div>0- 0.25 0.52- 0.67</div>	<div>0 0.35 0.62- 0.75</div>	0- 0.83	0.18- 0.61
0.8	0- 0.39	0- 0.39	0- 0.41	0- 0.43	0- 0.46	0- 0.51	0- 0.57	0- 0.64	0- 0.73	0- 0.85	0.34- 0.70
0.9	0- 0.30	0- 0.30	0- 0.32	0- 0.34	0- 0.32	0- 0.40	0- 0.47	0- 0.54	0 0.63	<div></div>	0.60- 0.95
1	0- 0.05	0- 0.02	0- 0.08	0- 0.08	0- 0.08	<div></div>	<div></div>	<div></div>	<div></div>	<div></div>	<div></div>

Table 2

Allowed range of x_W as a function of α and β in case $I_z(e_R^-) = 0$

[illegible]

Table 3

Allowed range of x_W as a function of α and β in case $I_z(e_R^-) = -\frac{1}{2}$.

0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.26- 0.40	0.27- 0.40	0.26- 0.40	0.27- 0.39	0.33- 0.40	0.35- 0.41	0.39- 0.42	0- 0.03	0- 0.24	0- 0.41	0- 0.41
0.1	0.26 0.40	0.26- 0.40	0.26 0.40	0.29- 0.39	0.32- 0.40	0.35 0.41	0.40 0.42	0- 0.03	0- 0.23	0- 0.42	0- 0.4
0.2	0.27- 0.40	0.27- 0.40	0.27- 0.40	0.29- 0.40	0.32- 0.40	0.36- 0.41	0.40- 0.42	0- 0.03	0- 0.25	0- 0.42	0- 0.41
0.3	0.28- 0.40	0.26- 0.39	0.28- 0.40	0.30- 0.40	0.34- 0.41	0.38- 0.41		0- 0.05	0- 0.20	0- 0.42	0- 0.41
0.4	0.28- 0.40	0.29- 0.39	0.28- 0.40	0.31- 0.40	0.35- 0.41	0.39- 0.41		0- 0.05	0- 0.20	0- 0.42	0- 0.41
0.5	0.27- 0.40	0.29- 0.39	0.27- 0.40	0.31- 0.40	0.35- 0.41	0.41- 0.42		0- 0.08	0- 0.20	0- 0.42	0- 0.41
0.6	0.26- 0.40	0.27- 0.39	0.26- 0.40	0.29- 0.40	0.34- 0.41	0.40- 0.42	0- 0.05	0- 0.15	0- 0.25	0- 0.42	0- 0.41
0.7	0.17- 0.40	0.18- 0.39	0- 0.39	0.18- 0.40	0- 0.41	0.32 0.41	0- 0.42	0- 0.25	0- 0.35	0- 0.41	0- 0.41
0.8	0- 0.39	0- 0.39	0- 0.39	0- 0.40	0- 0.41	0- 0.41	0- 0.41	0- 0.42	0- 0.42	0- 0.41	0.57- 0.70
0.9	0- 0.30	0- 0.30	0- 0.32	0- 0.34	0- 0.41	0- 0.41	0- 0.41	0- 0.41	0- 0.41	0- 0.41	0.60- 0.71
1	0- 0.05	0- 0.02	0- 0.08	0- 0.08	0- 0.08

Table 4

Allowed range of x_W as a function of α and β in case $I_z(e_R^-) = +\frac{1}{2}$.

$\begin{array}{c} \rightarrow \beta \\ \downarrow \alpha \end{array}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0								0- 0.03	0- 0.04	0- 0.05	0- 0.06
0.1								0- 0.03	0- 0.04	0- 0.05	0- 0.06
0.2								0- 0.03	0- 0.04	0- 0.05	0- 0.06
0.3								0- 0.04	0- 0.04	0- 0.05	0- 0.06
0.04								0- 0.04	0- 0.04	0- 0.05	0- 0.06
0.05								0- 0.04	0- 0.05	0- 0.05	0- 0.06
0.6								0- 0.04	0- 0.05	0- 0.06	0- 0.06
0.7								0- 0.04	0- 0.06	00- 0.06	0- 0.07
0.8								0- 0.05	0- 0.06	0- 0.06	0- 0.07
0.9								0- -0.06	0- 0.06	0- 0.06	0- 0.08
1											