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THE AXIAL ANOMALY FOR SPIN $\frac{3}{2}$ FIELDS *)

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ABSTRACT

The axial anomaly for a Majorana spin $\frac{3}{2}$ loop with external gravitons in supergravity is

$$\partial_\mu J_5^\mu = (192\pi^2)^{-1} e^{\alpha\beta\rho\sigma} R_{\rho\sigma}{}^{\mu\nu} R_{\mu\nu\alpha\beta}$$

This is twice the value of the corresponding anomaly for a Dirac spin $\frac{1}{2}$ loop.

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Recently there has been renewed interest in axial anomalies because of their topological significance and their relation to instanton effects. In this context, the case of spin $\frac{1}{2}$ fermions interacting with Yang-Mills mesons was first discussed by 't Hooft ¹⁾ while the case of spin $\frac{1}{2}$ fermions interacting with gravitons has been discussed by Eguchi and Freund ²⁾ and by Hawking ³⁾. In this paper we determine the axial anomaly for the case of a real (Majorana) spin $\frac{3}{2}$ fermion interacting with gravitons. We find

$$\partial_\mu J_5^\mu = (192\pi^2)^{-1} \epsilon^{\alpha\beta\rho\sigma} R_{\mu\nu\alpha\beta} R_{\rho\sigma}{}^{\mu\nu} \quad (1)$$

which is twice the value for the Dirac (complex) spin $\frac{1}{2}$ fermion. This result may be of relevance for the study of supersymmetry breaking by instantons in supergravity ⁴⁾.

In computing the anomaly we have adapted to gravitation the method used by Rosenberg and Adler to obtain the axial anomaly in quantum electrodynamics ⁵⁾. The matrix element of the axial current between the vacuum and a two-graviton state is written as:

$$\langle gg | J_5^\mu | 0 \rangle = \epsilon^{\alpha\beta}(q^1) \epsilon^{\gamma\delta}(q^2) M_{\alpha\beta, \gamma\delta}^\mu(q^1, q^2) \quad (2)$$

The part of M which does not vanish on shell [defined by $(q^j)^2 = q_T^j \epsilon^{\tau\sigma}(q^j) = \epsilon^{\sigma\sigma}(q^j) = 0$] is given by symmetrizing in $\alpha\beta$ and $\gamma\delta$ and by Bose-symmetrization of

$$M = \epsilon(\mu\alpha\gamma q^1) [A \delta_{\beta\delta} + B q_\beta^1 q_\delta^2] + \epsilon(\mu\alpha q^1 q^2) [C q_\gamma^1 \delta_{\beta\delta} + D q_\beta^2 q_\gamma^1 q_\delta^1] \quad (3)$$

The notation used is $\epsilon(\mu\alpha\gamma q^1) = (q^1)^\sigma \epsilon_{\mu\alpha\gamma\sigma}$. Note that off-shell more tensors contribute to M .

Gravitational gauge invariance can be used to relate the four form factors in Eq. (3). Since we always work on shell where $(q^1)^2 = (q^2)^2 = 0$, none of the extra tensors which we omitted in Eq. (3) contribute to the conservation relation

$$[q^{1\alpha} \epsilon^\beta(q^1) + q^{1\beta} \epsilon^\alpha(q^1)] \epsilon^{\gamma\delta}(q^2) M_{\alpha\beta, \gamma\delta}^\mu(q^1, q^2) = 0 \quad (4)$$

We thus find

$$(q^1 \cdot q^2)^2 A = -q^1 \cdot q^2 B = q^1 \cdot q^2 C = -\frac{1}{2} D \quad (5)$$

The divergence of the axial current follows from Eqs. (2) and (3)

$$\begin{aligned} \text{Div.} &= \langle q q | \partial_\mu J_5^\mu | 0 \rangle \\ &= -i \varepsilon^{\alpha\beta}(q^1) \varepsilon^{\gamma\delta}(q^2) \varepsilon(q^2 \alpha \gamma q^1) [A \delta_{\beta\delta} + B q_\beta^2 q_\delta^1] \end{aligned} \quad (6)$$

where again one is supposed to symmetrize as in Eq. (3). As expected, using Eq. (5), one can express this divergence in terms of the two-graviton part of the topological pseudoscalar

$$\text{Div.} = i(q^1 \cdot q^2) D[\varepsilon^{\rho\sigma\alpha\beta} R_{\mu\nu\alpha\beta} R_{\rho\sigma}{}^{\mu\nu}]^{\text{quadr}} \quad (7)$$

Our task is to calculate D.

We consider a real spin $\frac{3}{2}$ field Ψ_μ coupled to the gravitational vierbein e_μ^a according to supergravity theory [which seems to be the only possible interaction ⁶⁾]. The relevant part of the action is

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} \bar{\Psi}_\mu \gamma_5 \gamma_\nu D_\rho \Psi_\sigma + \frac{1}{4} \bar{\Psi} \cdot \bar{\gamma} \not{\partial} \bar{\gamma} \cdot \Psi \\ &\quad - \sqrt{-g} \bar{\chi} \bar{\gamma}^\nu D_\nu \chi \end{aligned} \quad (8)$$

where χ is the supergravity ghost field, $D_\rho \Psi_\sigma = \partial_\rho \Psi_\sigma + \frac{1}{2} \omega_{\rho ab} \sigma^{ab} \Psi_\sigma$ and $\bar{\gamma}^\mu$ is a constant, vierbein independent gamma matrix. To facilitate the calculation we have chosen the gauge in which the spin $\frac{3}{2}$ propagator takes the simple form $P_{\mu\sigma} = (2p^2)^{-1} \gamma_\sigma \not{p} \gamma_\mu$ ⁷⁾. We expect that on-shell the anomaly is independent of the choice of gauge fixing term.

The one-loop contributions to Eq. (2) are cubically divergent and therefore ambiguous due to different possible routings of momenta ⁸⁾. However, the contributions to D have five external momentum factors and are thus finite and unambiguous. Furthermore, although several graphs contribute to Eq. (2) ⁹⁾, only the triangle graph in Fig. 1 contributes to D. In particular, the seagull graph in Fig. 1 need not be considered.

It is interesting to note that in the gauge chosen the ghost does not contribute to the triangle graph, because the one-graviton terms in $\sqrt{-g}$ and the spin connection $\omega_{\mu ab}$ vanish when the graviton is on-shell. [Similar features were found in the calculations showing that in supergravity the one-loop divergences cancel ¹⁰⁾.]

The calculation of D proceeds as follows: the axial current is obtained from the effective action in Eq. (8) by the Noether prescription with

$$\delta \Psi_\mu = -i \gamma_5 \Psi_\mu \quad (9)$$

Since the ghost does not contribute to the triangle graph, we omit its contribution to the axial current. We find

$$J_5^\mu = -\frac{i}{2} \varepsilon^{\rho\sigma\nu\mu} \bar{\Psi}_\rho \gamma_\nu \Psi_\sigma + \frac{i}{4} \bar{\Psi} \cdot \bar{\gamma} \gamma^\mu \gamma_5 \bar{\gamma} \cdot \Psi \quad (10)$$

For comparison we note that for a spin $\frac{1}{2}$ Dirac particle $J_5^\mu = -i\bar{\lambda} \gamma^\mu \gamma_5 \lambda$.

The matrix element for the triangle graph contains a trace over 14 gamma matrices:

$$M = \frac{1}{8} \int_0^1 dx \int_0^x dy \int d^4 k \left[(k + (1-x)q^1 - yq^2)^2 + 2q^1 \cdot q^2 (1-x)y \right]^{-3} \cdot \text{tr} \left[\gamma_\kappa \not{k} \gamma_\tau G_{\chi\lambda}(q^1) A_{\lambda\nu}^\mu G_{\nu\tau}(q^2) \right] \quad (11)$$

where the gravitational vertices are given by

$$G_{\chi\lambda}(q^1) = -\frac{1}{4} \left[\varepsilon_{\chi\beta\rho\lambda} \gamma_5 \gamma_\alpha i(2k + q^1)_\rho h_{\alpha\beta}(q^1) + \varepsilon_{\chi\nu\alpha\lambda} \varepsilon_{\nu\beta\sigma\delta} \gamma_\delta \partial_\sigma h_{\alpha\beta}(q^1) \right] \quad (12)$$

The axial current sandwiched between two propagators can be simplified considerably:

$$A_{\lambda\nu}^\mu = [\gamma_\rho (\not{k} + \not{q}^1) \gamma_\lambda] \left[\frac{1}{2} \gamma_\rho \gamma_\mu \gamma_\sigma - \delta_{\mu\sigma} \gamma_\rho - \delta_{\mu\rho} \gamma_\sigma + \delta_{\rho\sigma} \gamma_\mu \right] \cdot \gamma_5 [\gamma_\nu (\not{k} - \not{q}^2) \gamma_\sigma] = 2\gamma_\nu (\not{k} - \not{q}^2) \gamma_\mu (\not{k} + \not{q}^1) \gamma_\lambda \gamma_5 \quad (13)$$

By further amusing manipulations one reduces the matrix element to a trace over six gamma matrices. We then find for the terms in M with five external momentum factors

$$M = \frac{i}{(32\pi)^2} \int_0^1 dx \int_0^x dy \text{tr} \left[\frac{\sum S_j}{q^1 \cdot q^2 (1-x)y} \gamma_\rho (\not{k} - \not{q}^2) \gamma_\mu (\not{k} + \not{q}^1) \gamma_\nu \right] \quad (14)$$

where $P = (x-1) q^1 + \gamma q^2$

and

$$\begin{aligned}
 S_1 &= -(2P+q^1)_\rho (2K-q^2)_\sigma \delta_{\beta\nu} \delta_{\rho\tau} \delta_{\alpha\sigma} \delta_{\delta\varphi} \gamma_5 \\
 S_2 &= (2P+q^1)_\rho q^2_\sigma \varepsilon_{\alpha\tau\sigma\eta} \delta_{\varphi\delta} (\delta_{\beta\eta} \delta_{\rho\nu} - \delta_{\beta\nu} \delta_{\rho\eta}) \\
 S_3 &= q^1_\rho (2P-q^2)_\sigma \varepsilon_{\tau\beta\rho\eta} \delta_{\alpha\nu} (\delta_{\eta\delta} \delta_{\varphi\sigma} - \delta_{\eta\sigma} \delta_{\varphi\delta}) \quad (15) \\
 S_4 &= -q^1_\rho q^2_\sigma \varepsilon_{\lambda\beta\rho\tau} \varepsilon_{\varepsilon\tau\sigma\eta} (\delta_{\lambda\eta} \delta_{\nu\alpha} - \delta_{\lambda\nu} \delta_{\alpha\eta}) \\
 &\quad \cdot (\delta_{\tau\varepsilon} \delta_{\varphi\delta} - \delta_{\tau\delta} \delta_{\varphi\varepsilon}) \gamma_5
 \end{aligned}$$

After evaluating the traces (by hand) the invariant D is determined and one obtains the result in Eq. (2).

Some comments are in order:

- a) We have applied this method also to the spin $\frac{1}{2}$ case and reproduced the result of Ref. 2).
- b) The on-shell Ward identity in Eq. (4) can be derived from the properties of polarization tensors of physical gravitons and supergravity Ward identities ^{11),12)}.
- c) We have redone the whole calculation (on the Brookhaven computer using Schoonship) without the second term in Eq. (10). We found the same result. Thus, the extra term in Eq. (10), a gauge artifact, does not change the value of the anomaly [but its presence actually facilitates the calculation because of the identity in Eq. (13)].
- d) We have used the Rosenberg-Adler method, because it seems the shortest method and does not commit us to any particular regularization scheme. It is unknown whether a regularization scheme exists which preserves local supersymmetry. Among those that break it, massive regulator schemes have the disadvantage that the numerators of massless and massive spin $3/2$ fields are different. Dimensional regularization is inconvenient, because it requires first to express all ϵ symbols in terms of gamma matrices.

In principle our result could also be obtained by using point splitting methods ¹³⁾ and the Atiyah-Singer theorem for spin $\frac{3}{2}$ fields. A preliminary computation along these lines ¹⁴⁾ indeed reproduces our result for the anomaly.

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FIGURE CAPTION

Fig. 1: The triangle and seagull graphs.

FIG.1