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THE PHYSICAL STRUCTURE OF NON-SCALING PARTON MODELS

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ABSTRACT

We discuss the nature of the constituent system seen by probe of momentum Q in non-scaling parton models. The dependence at fixed energy of the parton transverse momentum upon x , the longitudinal momentum fraction, is also investigated. On rather general grounds it is shown that it will increase with x at large x and in a particular model it is found to decrease with x at small x .

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1. INTRODUCTION

A possible fundamental basis for understanding the breakdown of Bjorken scaling is provided by renormalizable field theory, particularly in the popular form of asymptotically free gauge theories¹⁾. However, it is also valuable to develop a more intuitive understanding of the phenomenon such as can be provided by parton models suitably modified to take into account interactions harder than those which yield pure Bjorken scaling. Two attempts have been made to create such models.

The first is due to Kogut and Susskind²⁾. It is based on the momentum slice ideas of Wilson³⁾. These envisage a hierarchy of systems. Those at level n appear as composed of those at level $n + 1$ which behave as partons or point-like objects, when viewed by a probe of momentum Q , where

$$\frac{1}{2} \Lambda^n \xi < Q < \Lambda^n \xi. \quad (1.1)$$

Here ξ is a scale variable and Λ a large number. Kogut and Susskind show that when this picture is combined with a scale-invariant relation between structure functions at neighbouring levels, it leads to a description of deep inelastic scattering in which the moments of νW_2 have power law scale breaking in Q^2 . They also obtain a logarithmic scale breaking, expected in asymptotically free theories, by retaining the same hierarchy of structures corresponding to (1.1) but making a different assumption about the relation between successive levels.

Recently a modification of the covariant parton model has been proposed⁴⁾, called the asymptotically free parton model (AFPM), which is also capable of reproducing power law or logarithmic scale breaking for the moments. In this note we attempt to clarify the physical basis of AFPM. In Section 3 we show that it leads to a picture of the system seen by the probe which is somewhat different from that suggested by KS, particularly for the case of logarithmic scale breaking.

All non-scaling parton models must give an average parton transverse momentum, $\langle p_T^2 \rangle$, which asymptotically increases like Q^2 . This follows from the fact that scaling violations arise from divergent integrals over transverse momentum. In Section 4 we discuss how $\langle p_T^2 \rangle$ may be expected to depend upon x , the fraction of parent longitudinal momentum carried by the parton. A simple model is used which is the analogue for the non-scaling case of the core model used by Landshoff⁵⁾ to discuss x dependence in scaling parton models. In AFPM we find that on rather general grounds $\langle p_T^2 \rangle$ will increase with increasing x at large

values of x , with v fixed in the asymptotic regime. The specific model considered also gives $\langle p_T^2 \rangle$ decreasing with x at small x . Subasymptotically the situation is more complicated, as discussed in Section 4.

Both the KS model and AFPM have the merit that they can be applied in regimes which are not light-cone dominated. They have both been used to discuss the Drell-Yan process of heavy μ pair production⁶⁾ and AFPM has also been used to investigate Regge behaviour⁷⁾ and large p_T hadronic processes⁸⁾.

2. THE ASYMPTOTICALLY FREE PARTON MODEL

A parton picture of deep inelastic scattering corresponds to the generalized "handbag" diagram of Fig. 1. The lower bubble $\tilde{T}$ corresponds to the emission of a constituent from the incident hadron while the upper bubble A represents its interaction with the photon probe. The division between A and $\tilde{T}$ is such that the integral of $\tilde{T}$ over the momentum k' is finite and all momentum integrals which become divergent in a naïve evaluation of the Bjorken limit are contained in A . Thus A contains all the mechanism responsible for the breaking of Bjorken scaling. This division between A and $\tilde{T}$ is discussed in greater detail in I.

In the covariant parton model A is simply the Born approximation tree diagram; there are no integrals and scaling is exact. In AFPM a more complicated structure is required, involving a ladder exchange of scalar gluons as in Fig. 2. It is the form of A which determines the nature of the system seen by the photon probe. Of course AFPM is somewhat crude when compared with the intricacy of gauge theories. It would be possible to move in the direction of greater realism by allowing gluons to couple to closed parton loops and by attempting to deal with the further complications which would arise from the presence of spin 1 gluons. The first of these changes would produce a rather straightforward modification of the physical picture we discuss. Gluons would be replaced by gluons plus parton-antiparton pairs. The second change is one that we comment on at the end of Section 3. We may summarise our position by saying that we regard AFPM as a useful source of intuition for non-scaling parton dynamics, playing in this respect a rôle which might be compared to that of the multi-peripheral model in relation to Regge dynamics.

The details of AFPM depend upon what is assumed for the gluons exchanged in Fig. 2. There are two cases to be considered.

(a) They are taken to be point-like-coupled particles so that for large momentum k carried by the gluon one has the propagator behaviour

$$\sim \frac{1}{k^2} . \quad (2.1a)$$

This gives logarithmically divergent loop integrations in A in the naïve evaluation of the Bjorken limit, corresponding to $(\ln Q^2)^n$ behaviour asymptotically, as explained in I. On summing over n , the number of gluons exchanged, the logarithms exponentiate for the moments, leading to power law scale breaking. [In case (a) one should also sum over crossed gluon exchanges but that does not essentially alter the situation.]

(b) It is assumed in this case that the gluon coupling is softened in a way that mimics the effect of asymptotic freedom. That is, (2.1a) is replaced by

$$\sim \frac{1}{k^2 \ln k^2} . \quad (2.1b)$$

The loop integrals in A are now only loglog divergent in the Bjorken limit, giving an asymptotic behaviour $(\ln \ln Q^2)^n$. Again, on summing over n , an exponentiation is found for the moments which leads to a logarithmic breaking of scaling. [In case (b) the ladders give exactly the leading behaviour, all other contributions being at least $(\ln Q^2)^{-1}$ smaller.]

In the following section we discuss the physical picture of what is seen by the probe in these two cases.

3. THE PHYSICAL PICTURE

We consider first case (a). The contribution of the n gluon ladder to the r^{th} moment of the structure function takes the form⁴⁾

$$M_r \frac{(g_r)^n (\ln Q^2)^n}{n!} (Q^2)^{-\alpha} . \quad (3.1)$$

Power law scale breaking is obtained formally by summing (3.1) over n to give $M_r(Q^2) g_r^{-\alpha}$. However there is another way of thinking about it which is more physically illuminating.

For a large fixed value of Q^2 the terms in the series (3.1) first increase with n , due to increasing powers of $\ln Q^2$, and then decrease as this effect is damped out by the growth of $n!$. The maximum is readily found to occur at values of n such that

$$n \sim g_r \ln Q^2, \quad (3.2)$$

at which the value of (3.1) is approximately proportional to $(Q^2)^{g_r - \alpha}$ with logarithmic modifications. Thus the sum of the series comes, for fixed large Q^2 , from just those terms in the immediate neighbourhood of (3.2). A similar phenomenon is familiar in the multi-peripheral model in Regge theory. The constant inelastic cross-section is the envelope of the rising and subsequently falling partial cross-sections, being predominantly given at energy s by the σ_n 's with $n \sim \ln s$.

We interpret this as saying that in AFPM a photon of momentum Q predominantly sees a constituent system consisting of a parton and n gluons, with n given by (3.2).

When KS estimate their hierarchy number n from (1.1) they get just such a logarithmic dependence on Q^2 . However the two pictures are not transparently the same for our n is a measure of the complexity of the observed system in terms of fixed types of constituents (that is changing n is simply altering the number of gluons associated with the parton) whilst KS's n is a label for a level in a nesting system of ever smaller constituent systems along the lines of Wilson's ideas^{*)}.

The two pictures diverge further when one considers the case (b) of logarithmic scale breaking. KS, of course, continue to define an n through (1.1). However in AFPM the contribution analogous to (3.1) is now

$$M_r = \frac{(g_r)^n (\ln \ln Q^2)^n}{n!} (\ln Q^2)^{-\alpha} \quad (3.3)$$

*) Clearly our picture does not correspond to anything resembling Fig. 1 of the first paper of Ref. 2).

Similar arguments may be applied but now the significant value of n is given by

$$n \sim g_r \ln \ln Q^2, \quad (3.4)$$

which is very different from what (1.1) would give. Again n is to be interpreted in AFPM as the average number of gluons accompanying the parton, as seen by the probe of momentum Q . The extremely slow growth of n with Q^2 that is given by (3.4) can be expected to have as a consequence the persistence to quite high Q^2 of a predominantly two-jet structure in e^+e^- annihilation.

In AFPM g_r is positive, as is necessary for the consistency of the arguments we have used. g_r also tends to zero as r becomes large. Since large r moments are dominated by values of x near 1 this implies the reasonable conclusion that partons carrying a large fraction of the parent hadron momentum are unlikely to be accompanied by many gluons.

When AFPM is compared with an asymptotically free gauge theory an important difference is that in the latter $g_r \sim -\ln r$ as $r \rightarrow \infty$, due to the presence of the so-called logarithmic term¹⁾. Clearly the arguments given above cannot be used in the presence of such a term since it makes g_r negative for r sufficiently large. The term is a direct consequence of the presence of vector gluons in the field theory and KS²⁾ have given an argument which shows how it arises from soft vector gluons accompanying the parton. This is certainly consistent with the diagrammatics, since the logarithmic term is associated with vector gluons coupled to the parton line joining the two current vertices in the handbag. Once the logarithmic term is subtracted out the remaining terms are susceptible to the discussion already given. Thus in an asymptotically free gauge theory one would expect a similar physical picture to be valid as in AFPM provided account is taken of soft vector gluons accompanying the hard particles.

Finally one should emphasize that in Fig. 2 one is dealing with clothed partons and gluons. The logarithmic factors in (2.1b) and (3.3) summarize the effects of renormalization on the partons and gluons.

4. PARTON DISTRIBUTION FUNCTIONS AND PARTON TRANSVERSE MOMENTA

The distribution functions which correspond to the probability of finding partons with an assigned momentum are, of course, functions of Q , the momentum of the probe, in any non-scaling parton model. The Q dependence arises from divergent integrals, as discussed in Section 2, and these are effectively

integrals over transverse momenta. In all AFPM calculations of cross-sections these integrals are either logarithmically divergent [case (a)] or loglog divergent [case (b)]. Such slowly divergent integrals receive their principal contributions from regions where the p_T are of order Q^α ($0 < \alpha < 1$). This mathematical point is illustrated by the trivial example:

$$\frac{1}{n} Q^2 = \int_1^{Q^2} \frac{dp_T^2}{p_T^2} = \int_{1/Q^2}^1 \frac{d\xi}{\xi}, \quad p_T^2 = \xi Q^2. \quad (4.1)$$

The logarithm arises from the lower end-point of the integral over the scaled variable ξ in accordance with the statement that values of $p_T^2 \ll Q^2$ are the most important.

All cross-section calculations contain a δ function constraint of some form which arises from evaluating the imaginary part of a diagram. For example, in calculating the Drell-Yan process by which a parton of longitudinal fraction x_1 and transverse momentum $\underline{p}_{T1}$ annihilates with an antiparton of longitudinal fraction x_2 and transverse momentum $\underline{p}_{T2}$ to form a heavy virtual photon of mass Q^2 , there is a δ function which enforces the condition

$$Q^2 = x_1 x_2 s - (\underline{p}_{T1} + \underline{p}_{T2})^2. \quad (4.2)$$

To get an expression resembling the Drell-Yan formula modified to take account of scale breaking it is necessary to be able to neglect the second term on the right of (4.2) in relation to the first. The argument given above assures us that in a cross-section calculation in AFPM this is legitimate. This is confirmed by explicit calculations, the most general rules for which have been given by Stirling⁸⁾. Apparently this fact is less clear in the KS picture. It is concluded⁶⁾ that one can only neglect the transverse momenta in (4.2), in that picture, when Q^2/s is small.

When AFPM is used to calculate physical quantities which correspond to linearly divergent integrals over transverse momenta the situation is completely different. In such integrals it is indeed the momenta of order Q which make the significant contribution, a point which is illustrated by the analogue of (4.1):

$$Q^2 = \int_0^{Q^2} dp_T^2 = Q^2 \int_0^1 d\xi, \quad p_T^2 = \xi Q^2, \quad (4.3)$$

where the non-zero values of ξ now correspond to the important region of integration. The simplest such physical quantity is the average parton transverse momentum, $\langle p_T^2 \rangle$, perceived by the probe of momentum Q . In all non-scaling parton models the appropriate integral is necessarily linearly divergent so that all these models predict that asymptotically

$$\langle p_T^2 \rangle \sim Q^2, \quad (4.4)$$

modulo possible logarithmic modifications. [Even in covariant scaling parton models $\langle p_T^2 \rangle$ can grow with Q^2 if they are of "stretched" type⁹⁾, though the rate of growth cannot be as fast as (4.4).]

A question of some interest, which we shall endeavour to answer, is how $\langle p_T^2 \rangle$ varies with the longitudinal fraction x . It will be necessary to make use of a specific model, for which we choose that illustrated by Fig. 3. Here s_0 will be the "mass" of the lower core state and there will be a parameter μ^2 representing the parton "mass". The rate of decrease of the vertical parton propagators with the associated momentum will be chosen to make the loop integral logarithmically divergent. Of course this is a simpler model than AFPM but it incorporates what we believe to be the most important feature: logarithmically divergent integrals for the cross-section and linearly divergent integrals for $\langle p_T^2 \rangle$. Figure 3 may be thought of as the analogue for non-scaling parton models of the simple core picture used by Landshoff⁵⁾ to discuss the dependence of parton p_T on x in scaling models.

Such models are most conveniently discussed using Sudakov parameters¹⁰⁾, so we write the loop momentum k as

$$k = \alpha p + \frac{\beta}{2\nu} q + \kappa, \quad K \cdot p = K \cdot q = 0. \quad (4.5)$$

Then

$$s_0 = (\alpha-1)\beta + (\alpha-1)^2 p^2 - \kappa^2, \quad (4.6)$$

$$k^2 = \alpha\beta + \alpha^2 p^2 - \kappa^2. \quad (4.7)$$

In the integral (4.11) n , the power of α , is a parameter. The values $n = 1$ or 2 arise most naturally in parton models¹⁰⁾ but $n = 0$ gives the more physically acceptable behaviour of an expression for νW_2 which does not vanish at $x = 0$.

The numerator integral necessary to evaluate $\langle p_T^2 \rangle$ is proportional to (4.11) with an additional factor of $2\xi\nu$ in the integrand. The leading contribution to this integral is proportional to 2ν with a coefficient given, as expected, by an integral over ξ . This may be evaluated by elementary but laborious means for $n = 0, 1, 2$. In each case one verifies that the integral does not vanish at $x = 1$ so that $\langle p_T^2 \rangle$ increases with x at large x . In each case we also find that $\langle p_T^2 \rangle$ decreases with increasing x at small x . In the cases of $n = 1$ and $n = 2$ this is trivially so because the α smearing prevents the numerator from vanishing at $x = 0$ whilst νW_2 vanishes there. In the case $n = 0$, which corresponds to the more realistic behaviour of non-vanishing νW_2 at $x = 0$, this decrease is still found to hold. Clearly the more detailed behaviour exhibited by this simple model cannot be expected to be of particular significance.

So far our discussion has applied to the strictly asymptotic regime in which (4.4) holds. A $\langle p_T^2 \rangle$ which grows like Q^2 or 2ν (modulo logarithms) can only derive from the dynamics represented by the divergent loop integrations in A. Subasymptotically, however, it is possible to draw significant but bounded p_T from the lower bubble $\tilde{T}$ of Fig. 1, via the loop momentum k' . The calculation of this effect is exactly similar to Landshoff's treatment⁵⁾ of the covariant parton model. Such contributions may well still be important in the Drell-Yan regimes where x dependent parton p_T effects are thought to have been seen experimentally¹¹⁾.

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FIGURE CAPTIONS

- Fig. 1 : The generalized "handbag" diagram.
- Fig. 2 : The form of A in AFPM.
- Fig. 3 : The simple core model considered.

FIG.1 The generalised
"hand bag" diagram

FIG.2 The form of A
in AFPM.

FIG.3 The simple core
model considered