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RF FIELDS DUE TO SCHOTTKY NOISE IN A
COASTING PARTICLE BEAM

by

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Abstract

The RF fields inside a rectangular chamber excited by the Schottky noise current inherently present in a coasting particle beam are calculated, using a simple beam model. Vertical betatron oscillations are assumed. The power flow accompanying the beam is given as well as the resulting characteristic impedance. Numerical results are presented.

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up incoherently. The method to be applied therefore consists of calculating first the field of one particular slice carrying a current density

$$\vec{I}_{u,z}(x,y,z) = \begin{cases} \begin{pmatrix} \Omega_u \\ \Omega_z \end{pmatrix} \exp(-jkx) \cdot \frac{I_0 \sqrt{2}}{2} \cdot \frac{1}{4\sigma} \cdot \delta(y - (\frac{a}{2} + d)) \vec{e}_x \\ 0 \text{ else} \end{cases} \quad (3)$$

$$\begin{pmatrix} \Omega_u \\ \Omega_z \end{pmatrix} \text{ for } \begin{cases} z \in \left[\frac{b}{2}, \frac{b}{2} + \sigma \right] \\ z \in \left[\frac{b}{2} - \sigma, \frac{b}{2} \right] \end{cases}$$

where

$$\begin{pmatrix} \Omega_u \\ \Omega_z \end{pmatrix} = \exp(jn\omega t) \pm \frac{1}{2}(\exp(j\omega t(n+Q)) + \exp(j\omega t(n-Q))) \quad (4)$$

and then taking the rms value of all contributions within w , averaged over time. The propagation constant in (3) is taken to be $k = \omega/c_0$. The continuity equation requires a vertical component of the current density too (see for example ref. 10)). It can be shown, however, that for parameters such as those used in the ISR and for the frequency range considered, this component becomes negligibly small compared to (3).

2.2 Fields⁸⁾

For practical reasons a rectangular (Fig. 1) chamber will be assumed and it is supposed that no radiation penetrates from outside the volume V_1+V_2 of length L (Fig. 2). At x_0 ,

$$\vec{E}(x_0) = \vec{E}^+(x_0) + \vec{E}^-(x_0) \quad (5)$$

where $\vec{E}^{\pm}(x_0)$ are waves emerging from V_1 and V_2 respectively, and similarly for $\vec{H}(x_0)$.

with

$$r = e^{-|\delta| \frac{\pi}{b}} \quad (22)$$

Taking into account the modifications for the upper half in (8), a similar expression is obtained for E_{zu} . Adding E_{zu} and E_{zl} and distinguishing between harmonic $n\omega$ and sidebands $(n \pm Q)\omega$, the common mode component becomes

$$\begin{aligned} E_z^c(z, \delta) = & \exp(j(n\omega t - kx_0)) I_0 \sqrt{2} Z_0 \cdot \frac{1}{\sigma} \cdot \frac{1}{32\pi} \cdot \left(\ln\left(1 + r^2 - 2r\cos\pi\left(\frac{1}{2} + \frac{2\sigma}{b} - \frac{z}{b}\right)\right) + \right. \\ & + \ln\left(1 + r^2 - 2r\cos\pi\left(\frac{1}{2} + \frac{2\sigma}{b} + \frac{z}{b}\right)\right) - \ln\left(1 + r^2 - 2r\cos\pi\left(\frac{1}{2} - \frac{2\sigma}{b} - \frac{z}{b}\right)\right) - \\ & \left. - \ln\left(1 + r^2 - 2r\cos\pi\left(\frac{1}{2} - \frac{2\sigma}{b} + \frac{z}{b}\right)\right) \right) \end{aligned} \quad (23)$$

and the push-pull component is

$$\begin{aligned} E_z^p(z, \delta) = & \frac{1}{2} \left(\exp(j\omega t(n+Q)) + \exp(j\omega t(n-Q)) \right) \exp(-jkx_0) I_0 \sqrt{2} Z_0 \cdot \frac{1}{\sigma} \cdot \frac{1}{32\pi} \cdot \\ & \cdot \left(\ln\left(1 + r^2 - 2r\cos\pi\left(\frac{1}{2} + \frac{2\sigma}{b} - \frac{z}{b}\right)\right) + \ln\left(1 + r^2 - 2r\cos\pi\left(\frac{1}{2} + \frac{2\sigma}{b} + \frac{z}{b}\right)\right) + \right. \\ & + \ln\left(1 + r^2 - 2r\cos\pi\left(\frac{1}{2} - \frac{2\sigma}{b} - \frac{z}{b}\right)\right) + \ln\left(1 + r^2 - 2r\cos\pi\left(\frac{1}{2} - \frac{2\sigma}{b} + \frac{z}{b}\right)\right) - \\ & \left. - 2\ln\left(1 + r^2 - 2r\cos\pi\left(\frac{1}{2} - \frac{z}{b}\right)\right) - 2\ln\left(1 + r^2 - 2r\cos\pi\left(\frac{1}{2} + \frac{z}{b}\right)\right) \right) \end{aligned} \quad (24)$$

2.3 Effective field strengths⁸⁾

It is useful to evaluate the rms fields as a function of distance $\bar{y}$ from beam centre. The distance from this point to the nearest beam edge will be denoted by a_1 , the other by a_2 , hence $|a_2 - a_1| = w$. Thus, including a factor 1/2 due to time averaging,

$$E_{zrms}^{c,p}(z, \bar{y}) = \left(\frac{1}{w} \cdot \frac{1}{2} \int_{a_1}^{a_2} |E_z^{c,p}(z, \delta)|^2 d\delta \right)^{\frac{1}{2}} \quad (25)$$

There was a practical interest in numerical values for $z = 0, b$.

Fig. 3a

$(x^c(\infty) - x^c(\bar{y}))$ versus $\bar{y}$; $z = 0, b$

Fig. 4 a

$E_{zrms}^c / I_0 Z_0$ versus $\bar{y}$; $z = 0, b$.

Fig. 7

$Z^P = P^P / I_0^2$ versus h (push-pull mode characteristic impedance) .