UNSTABLE PARTICLES IN AN ELECTRIC FIELD

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ABSTRACT

When a particle decays in an electric field, and parity is violated, the mean angular momentum of the products is tilted relative to the angular momentum of the parent. It is in this way that the Zeldovich electric dipole moment manifests itself; there is no linear Stark effect, and no precession of the spin of undecayed particles.

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Ref. TH. 2324-CERN

16 May 1977
A stable particle cannot have an electric dipole moment if time
reversibility holds, even if parity is violated \(^1\). It was observed by
Zel’Dovich \(^2\) that the familiar reasoning does not apply immediately to an
unstable particle. The question was further discussed by Bell \(^3\) and by
Perelomov \(^4,5\). It has come up again \(^6,7\), in the context of the search
for parity violation in atomic physics, in a way that shows that the earlier
literature has not been fully digested. This is an attempt to clarify the
matter by tracing more explicitly than before the flow of angular momentum
when an unstable particle decays in an electric field. We consider always
a time reversible theory, but assume parity is violated.

It was shown rather generally by Bell \(^3\), and in a particular
model by Perelomov \(^4\) that the electric field induces no first order splitting
of the complex energy levels appropriate to the various projections of the
spin along the field. Correspondingly there is no precession of the spin of
the particle so long as it remains undecayed. As defined by these phenomena
the electric dipole moment of even an unstable particle is zero. But, in the
unstable case, as observed by Zel’Dovich the expectation value of the electric
dipole moment operator is not in general zero. The electric field then
exerts a torque on the system. The angular momentum so delivered must go
entirely to the decay products \(^3\). We will check explicitly that the mean
angular momentum of the decay products is at a fixed angle to the angular
momentum of the parent. There is then a change of mean angular momentum,
as the decay of an ensemble of unstable particles proceeds, the parent con-
tribution diminishing and the child contribution increasing. It is in this
way that the torque exerted by the electric field on the Zel’Dovich dipole
moment manifests itself.

Consider first a very simple model. Let \(N\) and \(V\) be spin \(\frac{1}{2}\)
particles, fixed in position, and let \(\Theta\) be a spin-zero "meson". Consider
the S wave decay:

\[
V \rightarrow N \Theta
\]

Suppose that close in energy to \(V\) is a particle \(U\) that can be mixed
into \(V\) by an electric field. Let the electric field be in the \(x\) direction, and let arrows denote projection of spin in the \(z\) direction. Then
the state

\[
|V \uparrow\rangle
\]

(1)
in the absence of the field becomes

$$|\Uparrow\uparrow\rangle + i g E_x |\Uparrow\downarrow\rangle \quad \text{(2)}$$

to first order in the weak field $E_x$. The corresponding state of $N$, generated by the decay, is:

$$|N\uparrow\rangle + i \alpha g E_x |N\downarrow\rangle \quad \text{(3)}$$

where $\alpha$ is the ratio of the decay matrix elements from $V$ and $U$, respectively. In the state (3) the spin is rotated away from the $z$ direction by a small angle $2|\alpha g| E_x$. But in (2) (because $V$ and $U$ are not mixed by $J$) any such deviation is of order $E_x^2$. So, the angular momentum of the children (carried entirely by $N$ in this case) is indeed rotated relative to that of the parent by the electric field. The angle of rotation is independent of the strength of the (weak) decay coupling constant. The resulting rate of change of angular momentum is proportional to the decay rate - as indeed is the Zel'dovich dipole moment.

Time reversibility requires $\alpha g$ to be real. Parity conservation would require $\alpha g$ to be zero.

For application to atomic physics the meson $\Phi$ has to be replaced by a photon, which is vector rather than scalar. The objects $U$ and $V$ would then be a pair of close excited atomic levels (with $j = \frac{1}{2}$) and $N$ the atomic ground state (with $j = \frac{1}{2}$). But the photon, unlike the scalar meson, can carry away angular momentum. Indeed in the present case, and in the dipole approximation for the radiation, the expectation value of the radiation angular momentum is four-thirds of the total final angular momentum, and the expectation value of the final-state atomic angular momentum is minus-one-third of the total. In the dipole approximation one finds that as far as the final state is concerned the effect of a weak electric field $\vec{E}$ is equivalent to a small rotation about $\vec{E}$ on the initial state (and therefore on the final state). So the angular momenta of photon and atomic ground state are separately rotated through the same angle.

We proceed with a more general discussion. Let the Hamiltonian of the system be $^*$:

$^*$ Here, and in the simple model, we assume the decay interaction to be $E$ independent. Note that $H_{\Phi\Phi}$ was called $H_\phi$ in Ref. 3).
\[ H = H_{oo} + \mathbf{E} \cdot \mathbf{D} + W \]  

\[ = H_{o} + W \]  

where \( W \) is the destabilizing perturbation, connecting parent and decay sectors of the state space, \( \mathbf{E} \) is the applied electric field, and \( \mathbf{D} \) the electric dipole moment operator. Working in the rest system, let \( |1> \) denote the relevant eigenstate of \( H_{oo} \), with eigenvalue \( E_1 \):

\[ H_{oo} |1> = E_1 |1> \]  

Switching on the electric field we have the corresponding eigenstate, to first order in \( \mathbf{E} \):

\[ |1_w> = |1> + \frac{Q}{E_1 - H_{oo}} \mathbf{E} \cdot \mathbf{D} |1> \]  

with

\[ Q = 1 - |1><1| \]  

There is no first order energy shift, and we can use non-degenerate perturbation theory, because \( \mathbf{D} \) has zero matrix elements in the degenerate subspace formed from \( |1> \) by rotation (see Appendix).

With (7) as initial state, but with the full Hamiltonian (5), we have after a time \( t \) the state:

\[ e^{-i (H_{o} + W) t} |1_w> \]

In the decay sector, to first order in \( W \) this is:

\[ e^{-i (E_1 + H_{o}) t/2} \cdot \frac{\sin [(E_1 - H_{o}) t/2]}{(E_1 - H_{o})/2} (\cdot i W) |1_w> \]  

Let us assume for simplicity that the decay products are electrically inert, so that in the decay sector \( H_{o} = H_{oo} \), and rewrite (9) as:
\[-i \epsilon e^{-i(E_1 + H_{oo})t/2} SWl_1\rangle \tag{10}\]

with
\[S = \frac{\sin[(E_1 - H_{oo})t]/2}{(E_1 - H_{oo})/2} \tag{11}\]

The mean angular momentum in the decay sector is then
\[
\langle l_1 WS \hat{J} SWl_1\rangle/\langle l_1 WSSWl_1\rangle \tag{12}\]

In the absence of the electric field (12) must and does reduce to the initial angular momentum (since \(H_{oo}\) and \(W\) commute with \(\hat{J}\) ) :
\[
\langle 11 \hat{J} 1 \rangle \tag{13}\]

According to (7) the application of a weak field \(\epsilon\) induces increments in the numerator and denominator of (12) :

\[
\delta\langle l_1 WS \hat{J} SWl_1\rangle = \langle 11 (WS \frac{\hat{J} W}{E_j - H_{oo}} \hat{D} + \frac{\hat{D} W}{E_j - H_{oo}} WS \hat{J} W) l_1 \rangle \tag{14}\]

\[
\delta\langle l_1 WSSWl_1\rangle = \langle 11 (WSSW \frac{\hat{J} W}{E_j - H_{oo}} \hat{D} + \frac{\hat{D} W}{E_j - H_{oo}} WSSW) l_1 \rangle \tag{15}\]

The operator coefficient of \(\epsilon\) on the right-hand side of (15) is even under time reversal, but vector under rotation, so that the matrix element must vanish \(^*)\). The corresponding operator in (14) is odd, like \(\hat{J}\), under time reversal.

\(^*)\) This means that there is no change in the lifetime to first order in the electric field.
reversal, but has mixed character with respect to rotation. Using the commutation of \( \vec{J} \) with the other factors, \( \vec{J} \) and \( \vec{D} \) can be brought together and written as the sum of their commutator and anticommutator (e.g.: 
\[
J_i^* D_j - \frac{1}{2}[J_i^*, D_j] + \frac{1}{2}(J_i^*, D_j)
\]
The anticommutator is a combination of scalar and tensor under rotation, and so cannot contribute because of the clash with the requirement of time reversal (see Appendix). With
\[
[J_{\ell}, D_m] = i \varepsilon_{\ell m n} D_n
\]
the mean angular momentum of decay products is then:
\[
\langle 1 | \vec{J} | 1 \rangle + \frac{\text{Re} \langle 1 | \text{WSSW} \frac{i Q}{E_i - H_{oo}} \vec{D} | 1 \rangle \wedge \vec{E}}{\langle 1 | \text{WSSW} | 1 \rangle}
\]
(16)

The numerator, by rotational invariance, must be a multiple of \( \langle 1 | \vec{J} | 1 \rangle \). So (16) is indeed just a rotation of the parent angular momentum (13) through a small angle proportional to \( |\vec{E}| \) around the direction of \( \vec{E} \). This angle is independent of the decay coupling constant, which cancels out between numerator and denominator. It is also independent of time (after a short transient time) because
\[
S^2 \rightarrow 2\pi t \int \text{d}(E_i - H_{oo})
\]
and then \( t \) also cancels between numerator and denominator.

The angular momentum of the system as a whole is the sum of the angular momentum in the initial state, and the angular momentum in the decay sector:
\[
\langle \vec{J} \rangle_{\text{total}} = \langle 1 | \vec{J} | 1 \rangle (1 - \langle 1 | \text{WSSW} | 1 \rangle) \\
+ \langle 1 | \vec{J} | 1 \rangle \langle 1 | \text{WSSW} | 1 \rangle \\
+ \text{Re} \langle 1 | \text{WSSW} \frac{i Q}{E_i - H_{oo}} \vec{D} | 1 \rangle \wedge \vec{E}
\]

Because of the time dependence exhibited in (17) there is a constant rate of change of total angular momentum
\[
\text{Re} \langle 1 | \text{W} \frac{2\pi i}{\text{d}(E_i - H_{oo})} \frac{Q}{E_i - H_{oo}} \vec{D} | 1 \rangle \wedge \vec{E}
\]
(18)
This must match the external torque on the system. So it remains to confirm that the coefficient of $\mathbf{A}$ is indeed the Zel'dovich dipole moment.

The latter is defined as the expectation value of $\mathbf{D}$ in the quasistationary state describing the decay. To second order in $W$ this state is \( \langle 1 | \mathbf{D} | 1 \rangle \)

\[
\langle 1 | \mathbf{D} | 1 \rangle = \frac{Q}{i\varepsilon + E_1 - H_\infty} W \frac{Q}{i\varepsilon + E_1 - H_\infty} W \langle 1 | \mathbf{D} | 1 \rangle \tag{19}
\]

plus decay sector terms which are not needed since we assumed $\mathbf{D}$ to be zero in that sector. The expectation value of $\mathbf{D}$ (the zero order term vanishing) is in second order:

\[
2 \Re \langle 1 | \mathbf{D} | 1 \rangle = 2 \Re \langle 1 | \mathbf{D} | 1 \rangle = 2 \Re \left( \frac{1}{i\varepsilon + E_1 - H_\infty} W \frac{Q}{E_1 - H_\infty} \mathbf{D} | 1 \rangle \right)
\]

where we have noted that $Q$ is unity in the decay sector and that $i\varepsilon$ is inactive in the sector $\mathbf{D}|1\rangle$, where $E_1$ is discrete. Now

\[
\frac{1}{i\varepsilon + E_1 - H_\infty} = \frac{\mathcal{P}}{E_1 - H_\infty} + i\pi \mathcal{S}(E_1 - H_\infty)
\]

Time reversibility, in the usual way, forbids any contribution from the Hermitian part, leaving

\[
\Re \langle 1 | \mathbf{D} | 1 \rangle = 2 \pi i \mathcal{S}(E_1 - H_\infty) W \frac{Q}{E_1 - H_\infty} \mathbf{D} | 1 \rangle \tag{20}
\]
as in (18).

To conclude we emphasize once more the distinction between the angular momentum of those particles which have not yet decayed and the angular momentum of the decay products. The latter is tilted relative to the former, the angle between them depending linearly on the external
electric field. The angular momentum of those particles which have not yet decayed does not precess to first order in the external electric field and thus there is no linear Stark effect if time reversal holds.

ACKNOWLEDGEMENTS

One of us (J.S.B.) thanks P.K. Kabir and the other thanks P.G.H. Sandars for conversations on this topic.
APPENDIX

We recall the main features of time reversal relevant here. Let \( i' \) and \( f' \) be the time reverses of states \( i \) and \( f \). Then we define for any operator \( A \) a related operator \( A' \) by *

\[
\langle f' | A' | i \rangle = \langle i' | A | f \rangle
\]

(21)

for all \( i, f \). Note that

\[
(\alpha A + \beta B)' = \alpha A' + \beta B'
\]

(22)

where \( \alpha \) and \( \beta \) are complex numbers, and

\[
(AB)' = B'A'
\]

(23)

which follows for example from the assumption that the time reversed states are a complete set. We assume

\[
H_{00}' = H_{00} \quad \omega' = \omega \quad \mathcal{D}' = \mathcal{D} \quad \mathcal{J}' = -\mathcal{J}
\]

(24)

When \( A' = \pm A \) then we say that \( A \) is even or odd under time reversal;

\[
\langle i' | A | i' \rangle = \pm \langle i | A | i \rangle
\]

(25)

When \( i \) is a state of definite squared angular momentum \( \mathbf{j}^2 \) with definite projection \( m \), then \( i' \) is the corresponding state with projection minus \( m \). When \( A \) has a definite irreducible tensor character the sign in (25) is also dictated by rotational invariance. In case of clash all the \((m,m')\) matrix elements of all components of the tensor operator vanish. The theorem on vanishing matrix elements of the electric dipole moment operator (which is even under time reversal but vector under rotation), and the various generalizations used in the text, come about in this way.

*) In the case of a Hermitian operator this is the conventional definition of time reversal operator.
REFERENCES


6) V.G. Gorshkov and L.N. Labzovskii - Zh. EFT Pis.Red. 21, 19 (1975); JETP Letters 21, 8 (1975).
