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UNSTABLE PARTICLES IN AN ELECTRIC FIELD

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A B S T R A C T

When a particle decays in an electric field, and parity is violated, the mean angular momentum of the products is tilted relative to the angular momentum of the parent. It is in this way that the Zeldovich electric dipole moment manifests itself ; there is no linear Stark effect, and no precession of the spin of undecayed particles.

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A stable particle cannot have an electric dipole moment if time reversibility holds, even if parity is violated ¹⁾. It was observed by Zeldovich ²⁾ that the familiar reasoning does not apply immediately to an unstable particle. The question was further discussed by Bell ³⁾ and by Perelomov ^{4),5)}. It has come up again ^{6),7)}, in the context of the search for parity violation in atomic physics, in a way that shows that the earlier literature has not been fully digested. This is an attempt to clarify the matter by tracing more explicitly than before the flow of angular momentum when an unstable particle decays in an electric field. We consider always a time reversible theory, but assume parity is violated.

It was shown rather generally by Bell ³⁾, and in a particular model by Perelomov ⁴⁾ that the electric field induces no first order splitting of the complex energy levels appropriate to the various projections of the spin along the field. Correspondingly there is no precession of the spin of the particle so long as it remains undecayed. As defined by these phenomena the electric dipole moment of even an unstable particle is zero. But, in the unstable case, as observed by Zeldovich the expectation value of the electric dipole moment operator is not in general zero. The electric field then exerts a torque on the system. The angular momentum so delivered must go entirely to the decay products ³⁾. We will check explicitly that the mean angular momentum of the decay products is at a fixed angle to the angular momentum of the parent. There is then a change of mean angular momentum, as the decay of an ensemble of unstable particles proceeds, the parent contribution diminishing and the child contribution increasing. It is in this way that the torque exerted by the electric field on the Zeldovich dipole moment manifests itself.

Consider first a very simple model. Let N and V be spin $\frac{1}{2}$ particles, fixed in position, and let θ be a spin-zero "meson". Consider the S wave decay :

$$V \rightarrow N\theta$$

Suppose that close in energy to V is a particle U that can be mixed into V by an electric field. Let the electric field be in the x direction, and let arrows denote projection of spin in the z direction. Then the state

$$|V\uparrow\rangle$$

(1)

in the absence of the field becomes

$$|V\uparrow\rangle + igE_x|U\downarrow\rangle \quad (2)$$

to first order in the weak field E_x . The corresponding state of N, generated by the decay, is :

$$|N\uparrow\rangle + i\alpha gE_x|N\downarrow\rangle \quad (3)$$

where α is the ratio of the decay matrix elements from V and U, respectively. In the state (3) the spin is rotated away from the z direction by a small angle $2|\alpha g|E_x$. But in (2) (because V and U are not mixed by $\vec{J}$) any such deviation is of order E_x^2 . So, the angular momentum of the children (carried entirely by N in this case) is indeed rotated relative to that of the parent by the electric field. The angle of rotation is independent of the strength of the (weak) decay coupling constant. The resulting rate of change of angular momentum is proportional to the decay rate - as indeed is the Zel'dovich dipole moment.

Time reversibility requires αg to be real. Parity conservation would require αg to be zero

For application to atomic physics the meson Θ has to be replaced by a photon, which is vector rather than scalar. The objects U and V would then be a pair of close excited atomic levels (with $j = \frac{1}{2}$) and N the atomic ground state (with $j = \frac{1}{2}$). But the photon, unlike the scalar meson, can carry away angular momentum. Indeed in the present case, and in the dipole approximation for the radiation, the expectation value of the radiation angular momentum is four-thirds of the total final angular momentum, and the expectation value of the final-state atomic angular momentum is minus-one-third of the total. In the dipole approximation one finds that as far as the final state is concerned the effect of a weak electric field $\vec{e}$ is equivalent to a small rotation about $\vec{e}$ on the initial state (and therefore on the final state). So the angular momenta of photon and atomic ground state are separately rotated through the same angle.

We proceed with a more general discussion. Let the Hamiltonian of the system be ^{*)} :

*) Here, and in the simple model, we assume the decay interaction to be E independent. Note that H_{00} was called H_0 in Ref. 3).

$$H = H_{oo} + \vec{E} \cdot \vec{D} + W \quad (4)$$

$$= H_o + W \quad (5)$$

where W is the destabilizing perturbation, connecting parent and decay sectors of the state space, $\vec{E}$ is the applied electric field, and $\vec{D}$ the electric dipole moment operator. Working in the rest system, let $|1\rangle$ denote the relevant eigenstate of H_{oo} , with eigenvalue E_1 :

$$H_{oo} |1\rangle = E_1 |1\rangle \quad (6)$$

Switching on the electric field we have the corresponding eigenstate, to first order in $\vec{E}$:

$$|1_{\sim}\rangle = |1\rangle + \frac{Q}{E_1 - H_{oo}} \vec{E} \cdot \vec{D} |1\rangle \quad (7)$$

with

$$Q = 1 - |1\rangle\langle 1| \quad (8)$$

There is no first order energy shift, and we can use non-degenerate perturbation theory, because $\vec{D}$ has zero matrix elements in the degenerate subspace formed from $|1\rangle$ by rotation (see Appendix).

With (7) as initial state, but with the full Hamiltonian (5), we have after a time t the state :

$$e^{-i(H_o + W)t} |1_{\sim}\rangle$$

In the decay sector, to first order in W this is :

$$e^{-i(E_1 + H_o)t/2} \cdot \frac{\sin[(E_1 - H_o)t/2]}{(E_1 - H_o)/2} (-iW) |1_{\sim}\rangle \quad (9)$$

Let us assume for simplicity that the decay products are electrically inert, so that in the decay sector $H_o \equiv H_{oo}$, and rewrite (9) as :

$$-i e^{-i(E_1 + H_{00})t/2} S W |1\rangle \quad (10)$$

with

$$S = \frac{\sin[E(E_1 - H_{00})/2]}{(E_1 - H_{00})/2} \quad (11)$$

The mean angular momentum in the decay sector is then

$$\langle 1 | W S \vec{J} S W | 1 \rangle / \langle 1 | W S S W | 1 \rangle \quad (12)$$

In the absence of the electric field (12) must and does reduce to the initial angular momentum (since H_{00} and W commute with $\vec{J}$) :

$$\langle 1 | \vec{J} | 1 \rangle \quad (13)$$

According to (7) the application of a weak field $\vec{\epsilon}$ induces increments in the numerator and denominator of (12) :

$$\delta \langle 1 | W S \vec{J} S W | 1 \rangle = \langle 1 | (W S \vec{J} S W \frac{Q}{E_1 - H_{00}} \vec{\epsilon} \cdot \vec{D} + \vec{\epsilon} \cdot \vec{D} \frac{Q}{E_1 - H_{00}} W S \vec{J} S W) | 1 \rangle \quad (14)$$

$$\delta \langle 1 | W S S W | 1 \rangle = \langle 1 | (W S S W \frac{Q}{E_1 - H_{00}} \vec{\epsilon} \cdot \vec{D} + \vec{\epsilon} \cdot \vec{D} \frac{Q}{E_1 - H_{00}} W S S W) | 1 \rangle \quad (15)$$

The operator coefficient of $\vec{\epsilon}$ on the right-hand side of (15) is even under time reversal, but vector under rotation, so that the matrix element must vanish ^{*}). The corresponding operator in (14) is odd, like $\vec{J}$, under time

^{*}) This means that there is no change in the lifetime to first order in the electric field.

reversal, but has mixed character with respect to rotation. Using the commutation of $\vec{J}$ with the other factors, $\vec{J}$ and $\vec{D}$ can be brought together and written as the sum of their commutator and anticommutator (e.g. : $J_i D_j = \frac{1}{2} [J_i, D_j] + \frac{1}{2} \{J_i, D_j\}$). The anticommutator is a combination of scalar and tensor under rotation, and so cannot contribute because of the clash with the requirement of time reversal (see Appendix). With

$$[J_e, D_m] = i \epsilon_{lmn} D_n$$

the mean angular momentum of decay products is then :

$$\langle 1 | \vec{J} | 1 \rangle + \frac{\text{Re} \langle 1 | W S S W \frac{iQ}{E_i - H_{00}} \vec{D} | 1 \rangle \wedge \vec{E}}{\langle 1 | W S S W | 1 \rangle} \quad (16)$$

The numerator, by rotational invariance, must be a multiple of $\langle 1 | \vec{J} | 1 \rangle$. So (16) is indeed just a rotation of the parent angular momentum (13) through a small angle proportional to $|\vec{E}|$ around the direction of $\vec{e}$. This angle is independent of the decay coupling constant, which cancels out between numerator and denominator. It is also independent of time (after a short transient time) because

$$S^2 \rightarrow 2\pi t \delta(E_i - H_{00}) \quad (17)$$

and then t also cancels between numerator and denominator.

The angular momentum of the system as a whole is the sum of the angular momentum in the initial state, and the angular momentum in the decay sector :

$$\begin{aligned} \langle \vec{J} \rangle_{\text{total}} &= \langle 1 | \vec{J} | 1 \rangle (1 - \langle 1 | W S S W | 1 \rangle) \\ &+ \langle 1 | \vec{J} | 1 \rangle \langle 1 | W S S W | 1 \rangle \\ &+ \text{Re} \langle 1 | W S S W \frac{iQ}{E_i - H_{00}} \vec{D} | 1 \rangle \wedge \vec{E} \end{aligned}$$

Because of the time dependence exhibited in (17) there is a constant rate of change of total angular momentum

$$\text{Re} \langle 1 | W 2\pi i \delta(E_i - H_{00}) W \frac{Q}{E_i - H_{00}} \vec{D} | 1 \rangle \wedge \vec{E} \quad (18)$$

This must match the external torque on the system. So it remains to confirm that the coefficient of $\vec{\Lambda} \vec{\epsilon}$ is indeed the Zel'dovich dipole moment.

The latter is defined as the expectation value of $\vec{D}$ in the quasistationary state describing the decay. To second order in W this state is ⁸⁾

$$|1\rangle + \frac{Q}{i\epsilon + E_1 - H_{00}} W \frac{Q}{i\epsilon + E_1 - H_{00}} W |1\rangle \quad (19)$$

plus decay sector terms which are not needed since we assumed $\vec{D}$ to be zero in that sector. The expectation value of $\vec{D}$ (the zero order term vanishing) is in second order :

$$\begin{aligned} & 2 \operatorname{Re} \langle 1 | W \frac{Q}{-i\epsilon + E_1 - H_{00}} W \frac{Q}{-i\epsilon + E_1 - H_{00}} \vec{D} | 1 \rangle \\ &= 2 \operatorname{Re} \langle 1 | W \frac{1}{-i\epsilon + E_1 - H_{00}} W \frac{Q}{E_1 - H_{00}} \vec{D} | 1 \rangle \end{aligned}$$

where we have noted that Q is unity in the decay sector and that $i\epsilon$ is inoperative in the sector $\vec{D}|1\rangle$, where E_1 is discrete. Now

$$\frac{1}{-i\epsilon + E_1 - H_{00}} = \frac{P}{E_1 - H_{00}} + i\pi \delta(E_1 - H_{00})$$

Time reversibility, in the usual way, forbids any contribution from the Hermitian part, leaving

$$\operatorname{Re} \langle 1 | W \cdot 2\pi i \delta(E_1 - H_{00}) W \frac{Q}{E_1 - H_{00}} \vec{D} | 1 \rangle \quad (20)$$

as in (18).

To conclude we emphasize once more the distinction between the angular momentum of those particles which have not yet decayed and the angular momentum of the decay products. The latter is tilted relative to the former, the angle between them depending linearly on the external

electric field. The angular momentum of those particles which have not yet decayed does not precess to first order in the external electric field and thus there is no linear Stark effect if time reversal holds.

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APPENDIX

We recall the main features of time reversal relevant here. Let i' and f' be the time reverses of states i and f . Then we define for any operator A a related operator A' by ^{*)}

$$\langle f | A' | i \rangle = \langle i' | A | f' \rangle \quad (21)$$

for all i, f . Note that

$$(\alpha A + \beta B)' = \alpha A' + \beta B' \quad (22)$$

where α and β are complex c numbers, and

$$(AB)' = B' A' \quad (23)$$

which follows for example from the assumption that the time reversed states are a complete set. We assume

$$H'_{00} = H_{00} \quad W' = W \quad \vec{D}' = \vec{D} \quad \vec{J}' = -\vec{J} \quad (24)$$

When $A' = \pm A$ then we say that A is even or odd under time reversal ;

$$\langle i' | A | i' \rangle = \pm \langle i | A | i \rangle \quad (25)$$

When i is a state of definite squared angular momentum $\vec{J}^2$ with definite projection m , then i' is the corresponding state with projection minus m . When A has a definite irreducible tensor character the sign in (25) is also dictated by rotational invariance. In case of clash all the (m, m') matrix elements of all components of the tensor operator vanish. The theorem on vanishing matrix elements of the electric dipole moment operator (which is even under time reversal but vector under rotation), and the various generalizations used in the text, come about in this way.

^{*)} In the case of a Hermitian operator this is the conventional definition of time reversal operator.

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