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SCHRÖDINGER HANDBAG MODEL

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A B S T R A C T

The excitation of a non-relativistic composite system, by a weakly interacting probe, is considered in the style of the properly covariant parton model. It is hoped in particular thereby to illuminate the handbag approximation. It is found that even for a confining potential such an approximation can sensibly be defined, and then describes correctly the poorly resolved deep inelastic scattering. The approximation is significantly improved (subasymptotically) by the suppression of the displacement off-mass-shell of the initial parton.

It can be instructive to approach a simple problem in a complicated way. Here the complicated way is that of the properly covariant parton model ¹⁾. The simple problem is that of the excitation of a non-relativistic composite system by a weakly interacting probe. In particular the adequacy of the handbag diagram will be studied. It has been realized ²⁾ that for partons which are deeply bound, or completely confined, the handbag diagram is quite irrelevant, or non-existent. However, it has also been realized that even then there may be a handbag diagram which is relevant, in which the "free" parton of the handle is free at short rather than long times. It is indeed so in the non-relativistic model. It will be seen that the handbag results are in fair agreement with accurate results available in that case. But a particular feature of the handbag model, that partons are initially and finally off-shell to different degrees in a kinematically determined way, is not borne out.

Lorentzian Kinematics

We begin by recalling the main features of the properly covariant handbag model. For simplicity, let both probing field A and parton field ϕ be scalar, and let the interaction be

$$g A \phi^2$$

Then we have to evaluate

$$W(q) = \sum_F |(F|\phi^2(0)|I)|^2 (2\pi)^4 \delta^4(P_F - P - q) \quad (1)$$

where q is the probing 4-momentum, P that of the initial hadron, and P_F that of the final hadronic system. In a first approximation this is supposed to be given by the absorptive part of the handbag diagram

or by the summed squared modulus of the half handbag

The final states F of (1) are here approximated by states of a single free parton, with 4-momentum p_1 and mass m_1 , and a residual group of particles with total 4-momentum p_2 . We further simplify now by taking the residual group to be just a single particle of spin zero and mass m_2 . Then

$$W = \iint \frac{d^4 p_1}{(2\pi)^4} \frac{d^4 p_2}{(2\pi)^4} 2\pi \delta_0(\Delta_1) 2\pi \delta_0(\Delta_2) (2\pi)^4 \delta^4(p_1 + p_2 - P - q) \left| \frac{\Sigma(\Delta)}{\Delta} \right|^2 \quad (2)$$

where δ_0 is the usual δ function multiplied by a step function restricting energy to be positive, and

$$\left. \begin{aligned} \Delta_1 &= m_1^2 + p_1^2 \\ \Delta_2 &= m_2^2 + p_2^2 \\ \Delta &= m^2 + p^2 \end{aligned} \right\} \quad (3)$$

with

$$\begin{aligned} p &= P - p_2 \\ m &= m_1 \end{aligned}$$

We have taken the initial hadron to have zero spin, and Σ is an invariant vertex function.

Changing from p_2 to p as integration variable in (2), it is seen that the target can be replaced by a 4-dimensional distribution of incoherent free partons

$$\varphi(p, P) d^4 p / (2\pi)^4$$

with

$$\varphi = 2\pi \delta_0(\Delta_2) \left| \frac{\Sigma(\Delta)}{\Delta} \right|^2 \quad (4)$$

and

$$p_2 = P - p$$

Note that because of the covariant norms implicit in this calculation so far, the distribution of parton number per target particle is given by

$$\bar{p} = (2p_0 / 2P_0) \varphi \quad (5)$$

Taking $\vec{P}$ in the 3-direction, denoting by $\vec{p}_\perp$ the part of $\vec{p}$ perpendicular to this, and with

$$x = (p_3 + p_0)/(P_3 + P_0)$$

the expression (4) can be rewritten

$$\begin{aligned} \varphi &= \theta(1-x) \frac{x}{1-x} 2\pi S(\Delta - d) \left| \frac{\Sigma(\Delta)}{\Delta} \right|^2 \\ d &= (1-x)^{-1} (p_\perp^2 + m_2^2 x - M^2 x(1-x)) + m^2 \end{aligned} \quad (6)$$

It is on the basis of such an expression that Landshoff³⁾ has recently proposed a certain dependence of the $p_\perp$ distribution on x .

Galilean Kinematics

Instead of

$$\Delta = m^2 + p^2 = \vec{p}^2 + (m + p_0)(m - p_0), \text{ etc}$$

now take

$$\Delta = \vec{p}^2 + 2m(\varepsilon - p_0) \quad (7)$$

$$\Delta_1 = \vec{p}_1^2 + 2m_1(\varepsilon_1 - p_{01}) \quad (8)$$

$$\Delta_2 = \vec{p}_2^2 + 2m_2(\varepsilon_2 - p_{02}) \quad (9)$$

Note the distinction here between rest energy ε and inertial mass m . We take $\varepsilon = \varepsilon_1$ as well as $m = m_1$.

We come again to (4), in which we now write

$$\Delta_2 = (\vec{P} - \vec{p})^2 + 2m_2(\varepsilon_2 + \varepsilon - p_0 + (\vec{P}^2 - \Delta)/2m) \quad (10)$$

so that with

$$M = m + m_2 \quad P_0 = \varepsilon + \vec{P}^2/2M$$

we have

$$\varphi = 2\pi(m/m_2) \delta(\Delta - d) |\Sigma(d)/d|^2 \quad (11)$$

with

$$d = \frac{m^2 M}{m_2} \left(\frac{\vec{P}}{M} - \frac{\vec{P}}{m} \right)^2 + 2m(\varepsilon + \varepsilon_2 - \mathcal{E}) \quad (12)$$

$$\Delta = \vec{P}^2 + 2m(\varepsilon - p_0) \quad (13)$$

Corresponding to (5), after eliminating "covariant" norms $2m$ and $2M$, the distribution of parton number per target particle is given by

$$\bar{\varphi} = (2m/2M) \varphi \quad (14)$$

Schrödinger Dynamics

Suppose now that the target is a bound state of just two partons, which are not identical, and only one of which is seen by the probe. Let the system be governed by a Schrödinger equation with Hamiltonian

$$H = \frac{\vec{P}^2}{2m} + \frac{\vec{P}_2^2}{2m_2} + \varepsilon + \varepsilon_2 + V(|\vec{P} - \vec{P}_2|)$$

We could set up perturbation diagrams in which V is regarded as interaction and the remainder of H defines propagators. They would include the handbag diagram. Such a diagram could be immediately relevant for a peripheral process [if $V(\omega) = 0$] but could be only remotely relevant for deep processes.

Let us instead first redefine V , by absorbing a constant into a redefined $\varepsilon_1 + \varepsilon_2$, so that then

$$V(a) = 0 \quad (15)$$

where a is some separation typical of the bound state. Note that with V so defined, an infinitely deep potential well $[V(\omega) = \omega, \text{confinement}]$ is not embarrassing at all. It is with such a definition of interaction, and residual free Hamiltonian

$$H_0 = \frac{\vec{p}^2}{2M} + \frac{\vec{p}_2^2}{2m_2} + \epsilon + \epsilon_2 \quad (16)$$

that we define perturbation diagrams, and in particular a handbag diagram.

Such a handbag is identified with the bound state pole contribution to the infinite sum of perturbation diagrams like

Then the vertex function Σ is given by the pole term in the sum of semi-infinite ladders

This can be written

$$2\pi \delta(p_0 + p_{02} - p_0) V \psi \quad (17)$$

in terms of the potential V and the Schrödinger bound state wave function $\psi(\vec{p}, \vec{p}_2)$. There is no dependence on relative energy $p_0 - p_{02}$ because of the instantaneous nature of the potential, with which the diagram ends.

Because of the Schrödinger equation

$$(p_0 - H_0 - V) \psi = 0$$

the factor V in (17) can be replaced by

$$\begin{aligned} \epsilon + \frac{\vec{p}^2}{2M} - \epsilon - \epsilon_2 - \frac{\vec{p}^2}{2M} - \frac{\vec{p}_2^2}{2m_2} \\ = \epsilon - \epsilon - \epsilon_2 - \frac{Mm}{2m_2} \left(\frac{\vec{p}}{M} - \frac{\vec{p}}{m} \right)^2 \\ = -d/2m \end{aligned} \quad (18)$$

with d defined in (12).

The wave function ψ consists of a momentum conserving delta function and an internal wave function depending only on relative velocity

$$\psi = (2\pi)^3 \delta^3(\vec{p} + \vec{p}_2 - \vec{p}) \chi(|\frac{\vec{p}}{M} - \frac{\vec{p}}{m}|^2) \quad (19)$$

Then to obtain Σ from (17) we have only to drop the delta functions and supply some "covariant" norm factors :

$$\Sigma(d) = (2m \ 2m_2 \ 2M)^{1/2} \frac{-d}{2m} \chi\left(\frac{m_2 d}{m^2 M} + \frac{2m_2}{mM}(\varepsilon - \varepsilon_1 - \varepsilon_2)\right) \quad (20)$$

From (11) and (14), the parton distribution is then $(d^4 p) \bar{p}$ with

$$\bar{p} = 2m \bar{u} \delta(\Delta - d) |\chi(|\frac{\vec{p}}{M} - \frac{\vec{p}}{m}|^2)|^2 \quad (21)$$

This is precisely the conventional distribution of momentum $\vec{p}$ associated with the wave function ψ - supplemented, however, by the off-shell prescription for initial parton energy

$$p_0 = \varepsilon + \frac{\vec{p}^2}{2m} - \frac{d}{2m} \quad (22)$$

Final State Interaction and Sum Rules

The handbag model ignores diagrams with final state interaction :

It is difficult to evaluate such effects in general. There are, however, standard sum rules ⁴⁾ which suggest that a significant part of the effect, for the simple model considered here, is just to remove the off-shell displacement $(d/2m)$ from (22).

The expression (1) can be rewritten

$$\int d^4x \ e^{iqx} (I | \phi^2(0) \phi^2(x) | I) \quad (23)$$

Going over to first quantization language

$$\phi^2(x) \Rightarrow \delta(\vec{x} - \vec{r}(t)) \quad (24)$$

where $\vec{r}(t)$ is the Heisenberg position operator for the particle seen by the probe. Dropping also the "covariant" norm in favour of a normal norm

$$W/2M = \int dt \ e^{-iq_0 t} \langle \delta^3(\vec{r}(0)) e^{i\vec{q} \cdot \vec{r}(t)} \rangle \quad (25)$$

with the expectation value taken in the target state (19).

Recall first the result of the free particle approximation

$$\vec{r}(t) = \vec{r} + t \vec{p}/m \quad (26)$$

where operators without time argument are for $t=0$. By the Campbell-Baker-Hausdorf theorem

$$e^{i\vec{q} \cdot (\vec{r} + t \vec{p}/m)} = e^{i\vec{q} \cdot \vec{r}} e^{it(\frac{\vec{q} \cdot \vec{p}}{m} + \frac{q^2}{2m})} \quad (27)$$

So

$$W/2M = \langle \delta^3(\vec{r}) 2\pi \delta(q_0 + \frac{\vec{p}^2}{2m} - \frac{(\vec{p} + \vec{q})^2}{2m}) \rangle \quad (28)$$

$$= \int \frac{d^3\vec{p}}{(2\pi)^3} 2\pi \delta(q_0 + \frac{\vec{p}^2}{2m} - \frac{(\vec{p} + \vec{q})^2}{2m}) |\chi(|\frac{\vec{p}}{M} - \frac{\vec{p}}{m}|^2)|^2 \quad (29)$$

This is just the excitability of the conventional momentum distribution $|\chi|^2$ of free on-shell incoherent particles.

Recall too that instead of considering W itself we might consider $\bar{W}$, obtained from W by allowing for poor resolution (deliberate or accidental) in q_0 . If the resolution function is a Gaussian with root mean square width δq_0 , then (25) is replaced by

$$\bar{W}/2M = \int dt e^{-\frac{1}{2}(tsq_0)^2} e^{-iq_0 t} \langle s^3(\vec{r}(t)) e^{i\vec{q} \cdot \vec{r}(t)} \rangle \quad (30)$$

So for poor energy resolution, it is plausible that the validity of (29) (with the appropriately smeared delta function) depends on the validity of (26) at small t .

Now (26) is clearly right (for velocity independent potential) to first order in t . But the result is actually a little better than that. Adding to (26) the next term

$$\frac{1}{2} t^2 \vec{F} \quad (31)$$

where

$$\vec{F} = -\partial V / \partial \vec{r} \quad (32)$$

the extra t^2 terms in the Fourier transform of $(W/2M)$ are found proportional to

$$\langle s^3(\vec{r}) \vec{F} \rangle \quad (33)$$

which is clearly zero. So we have the first three powers of t correct, and therefore (29) is accurate as regards the first three moments of the q_0 distribution at fixed $\vec{q}$:

$$M_n(\vec{q}) = \int \frac{dq_0}{2\pi} (q_0)^n \frac{W}{2M}, \quad n=0,1,2 \quad (34)$$

To this extent the $d/2m$ off-shell displacement in (22), the final parton having been put on mass shell, is just an error due to neglect of final state interaction.

Optimum Handbag

The use of (22) does not give any error in M_0 . But in the mean excitation energy for given $\vec{q}$ there is an error

$$\left\langle \frac{d}{2m} \right\rangle = \left\langle \frac{m_1 m_2}{2 m_1 m_2} \left(\frac{\vec{p}}{m_1} - \frac{\vec{p}}{m_2} \right)^2 + \varepsilon + \varepsilon_2 - \varepsilon \right\rangle$$

With $\varepsilon = P_0 - \vec{P}^2/2M$ and $\vec{P} = \vec{p} + \vec{p}_2$, and using finally the bound state Schrödinger equation, this becomes

$$\left\langle \frac{\vec{p}_2^2}{2m_2} + \frac{\vec{p}^2}{2m} + \varepsilon + \varepsilon_2 - P_0 \right\rangle = - \langle V \rangle$$

So in transferring a constant between V and $\varepsilon_1 + \varepsilon_2$ we could replace the vague prescription (15) by the precise

$$\langle V \rangle = 0$$

The handbag would then, in the simple case considered, give the mean excitation energy correctly. However, the mean square deviation from the mean would still be wrong.

Note finally that if some relativistic theory were to resemble in this matter the above velocity independent potential model, (6) would be replaced by

$$\zeta = 2\pi \delta_0(\Delta) F \left(x + \frac{m^2 + p_\perp^2}{M^2 x} \right) \quad (35)$$

Such a distribution of on-shell partons, symmetric in the target rest frame, has indeed been considered in the context of attempts to estimate mass effects in scaling violation ⁵⁾.

I have profited from remarks (some of them quite uncomplimentary) by M. Chanowitz, J. Ellis, M.K. Gaillard, P. Landshoff, and G. Preparata.

R E F E R E N C E S

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