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THE ABSORPTIVE LINE SHAPE OF HADRONIC ATOMS

T.E.O. Ericson and L. Hambro

CERN - Geneva

ABSTRACT

The exact line shape for all energies is derived analytically in the limit of negligible nuclear dimensions. The shape deviates from the Breit-Wigner form. The high energy tail of the line has a universal shape, independent of the absorptive strength. The tails are different for different initial states, however. The integrated line strength diverges logarithmically. Renormalization of the hadron wave function by strong interactions leads to the usual shape near resonance and restores convergence for very large energies. The results resolve a logical inconsistency in the normal analysis of hadronic atoms and are of practical importance. It is shown that bound hadronic states (e.g., Y_0^*) give natural contributions in the high energy region. Numerical illustrations are given.

1. INTRODUCTION

Hadronic atoms differ fundamentally from other atoms by the strong absorption process. The atomic $1s$ state has a short lifetime with widths which frequently are a few per cent of the $2p - 1s$ transition energy. In exceptional cases the widths are as large as 5-10% of the atomic binding energy -- this is expected for the protonium ($\bar{p}p$) $1s$ state and is observed for pionic $1s$ states in the region $A = 20$. In all these cases the atomic binding energy is extremely small compared to the total energy liberated in the absorption process; in protonium it is 12 keV as compared to 2×10^6 keV, in pionic atoms it is 0.4 MeV as compared to 140 MeV. It is also characteristic that the nuclear absorption region is very small compared to atomic sizes.

The standard procedure for analyzing the line shape is to assume the Breit-Wigner form for electric dipole radiation:

$$I(\omega) \propto \frac{\omega^3}{(\omega - \omega_0)^2 + (\Gamma/2)^2} \quad (1)$$

For $\omega - \omega_0 \gg \Gamma$ this formula cannot apply since it diverges linearly for large ω , with a minimum at $\omega \approx 3\omega_0$. For $\Gamma/\omega_0 \approx 0.1$ the value at the minimum is over 1% of the peak value at the resonance. The total intensity contained in the $2p - 1s$ transition must not exceed the amount fed into the $2p$ level (probability conservation!), so the shape (1) is often physically impossible when normalized to the easily measured rate near the maximum. A damping mechanism must be at work for large ω . Since all the characteristic dimensions of the problem are large compared to nuclear dimensions, and since the total energy liberated in the ultimate nuclear absorption hardly changes on the nuclear scale even for vast changes in ω , we expect that the basic absorption mechanism remains unchanged: the damping mechanism must be primarily of atomic and not of nuclear origin. This effect is the subject of this paper.

2. MODEL FOR THE HADRONIC ATOM

The atom is accurately approximated as follows. Both the charge Ze and the strong interaction are concentrated in a massive point in the centre of the atom. The assumption is therefore that the nuclear radius $R \ll a_0$, the Bohr radius. Consequently, only s waves will have strong interactions. These can be described by a pseudopotential

$$V(\vec{x}) = V_0 \delta(\vec{x}) \quad (2)$$

The complex strength V_0 is proportional to the complex s wave scattering length A_0 ¹⁾

$$V_0 = - \frac{4\pi\hbar^2}{2m} A_0 \quad (3)$$

For the hadronic atom $|A_0| \ll a_0$. While the absorptive part of V_0 introduces new physics, the real part only shifts the resonance peak slightly. Higher partial waves are only absorbed when the effective incident wavelength is short enough to penetrate the centrifugal barrier. In our case this means strong suppression, so it is natural to neglect them.

3. THE STEADY STATE

To be specific, we take the initial state to be a 2p state which decays by unretarded E1 emission of X-rays of energy ω . We assume that the radiative width is small enough to be irrelevant. In the absence of absorption all 2p transitions will go to a stable 1s state. If the X-ray transitions are allowed to start at a time $t=0$, the ground state will increase its population steadily in time. The corresponding intensity distribution is, of course, a δ function in energy; the transition rate is proportional to the usual dipole matrix element between the 2p and 1s state

$$I(\omega) \propto \omega^3 \left| \int \phi_{2p}^*(\vec{x}) (\hat{e} \cdot \vec{x}) \phi_{1s}(\vec{x}) d\vec{x} \right|^2 \delta(\omega - \epsilon_{2p} + \epsilon_{1s}) \quad (4)$$

where $\epsilon_{n\ell}$ is the binding energy of the hadron in the atom, and $\hat{e}$ is the X-ray polarization vector. When absorption is introduced, a steady state develops for which transition and absorption rates balance. In this steady state the hadron has a wave function $\phi_e(\vec{x})$ in the Coulomb field with energy $\epsilon = \epsilon_{2p} - \omega$. This wave function is unnormalized and provoked by the driving source

$$j(\vec{x}) = \omega (\hat{e} \cdot \vec{x}) \phi_{2p}(\vec{x}) \quad (5)$$

due to the dipole operator $\omega(\hat{e} \cdot \vec{x})$ acting on the initial 2p state. The boundary conditions on this wave function must correctly describe the region just outside the range of strong interactions. This wave function exists for any energy; its absorption will match the radiative transitions at every energy in the steady state. This condition gives the photon intensity distribution or line shape $I(\omega)$.

As a preliminary we now derive the steady state wave function and its properties near the origin.

4. THE STEADY STATE WAVE FUNCTION IN THE COULOMB FIELD

In the Coulomb field outside the nuclear centre, the wave function is given by

$$\Phi_E(\vec{x}) = \int G_E(\vec{x}, \vec{x}') j(\vec{x}') d\vec{x}' + K G_E(\vec{x}, 0) \quad (6)$$

where $G_E(\vec{x}, \vec{x}')$ is the point Coulomb Green's function for energy ϵ and K is a constant. The first term in Eq. (6) comes from the source function and is regular at the origin. The second term is partly singular at the origin; it is the scattered wave from the nuclear strong interaction there. The Green's function $G_E(\vec{x}, 0)$ is a Whittaker function ²⁾:

$$G_E(\vec{x}, 0) = -\frac{1}{4\pi x} T(1-\beta) W_{\beta, \frac{1}{2}}(2\kappa x) \quad (7)$$

where $\kappa = (-2m\epsilon)^{\frac{1}{2}}/\hbar$ and $\beta = 1/(\kappa a_0)$; m is the mass of the hadron.

The constant K follows from the short distance behaviour of the wave function. In the spirit of a pseudopotential approach the ratio of the regular (constant) wave at the origin and the coefficient of the singular part should be independent of energy. We therefore expand $G_E(\vec{x}, 0)$ near the origin ³⁾:

$$G_E(\vec{x}, 0) \xrightarrow{x \rightarrow 0} -\frac{1}{4\pi} \left[\frac{1}{x} - \kappa + \frac{2}{a_0} \left(-\ln(2\kappa x) - \psi(1-\beta) + \psi(1) + \psi(2) \right) \right] \quad (8)$$

where $\psi(x) = d \ln \Gamma(x)/dx$ is the digamma function which has poles for $x=0, -1, -2, \dots$. In the absence of the Coulomb field, a zero energy particle interacting with the pseudopotential (2) and (3) has a wave function

$$\Phi_0(\vec{x}) = 1 + \frac{A_0}{x} \quad (9)$$

In this case the ratio of the coefficient of the singular term to the regular one is the scattering length A_0 . In the Coulomb case we substitute $x^{-1} - 2a_0^{-1} \ln(x/a_0)$ for x^{-1} . The constant K in Eq. (6) is fixed by requiring the same ratio of coefficients, for small x , as in Eq. (9)

$$K = -4\pi A_0 \Phi_\epsilon^{(0)}(0) \frac{1}{1+r_\epsilon} \quad (10)$$

where $\Phi_\epsilon^{(0)}(\vec{x})$ denotes the first term in Eq. (6). The wave effectively exciting the system at the origin [i.e., the sum of terms in Eq. (6) which are regular for $x \rightarrow 0$] is thus renormalized by the strong interaction. The renormalization factor $(1+r_\epsilon)^{-1}$ which multiplies the unrenormalized incident wave $\Phi_\epsilon^{(0)}(0)$ at the origin is related to the scattering length by

$$r_\epsilon = A_0 \left[x + \frac{2}{a_0} (\ln(2xa_0) + \psi(1-\beta) - \psi(1) - \psi(2)) \right] \quad (11)$$

The limit $r_\epsilon = 0$ corresponds to neglecting the effect of strong interactions in the wave function.

It is instructive to derive the strong shift for the $1s$ state from Eq. (8). The unperturbed energy is $\epsilon_{1s} = -Ze^2/2a_0 = -Ry$, the Rydberg for the hadronic atom, so that $\epsilon = -Ry$ gives $\beta = 1$. For β close to unity, the regular part of Eq. (8) is dominated by the pole in $\psi(1-\beta) = \psi(2-\beta) - (1-\beta)^{-1}$. With the ratio of the singular-to-regular part in Eq. (8) proportional to A_0 , we obtain (ϵ is negative):

$$\epsilon = \epsilon_{1s} + \delta\epsilon \approx -\left(1 + 4\frac{A_0}{a_0}\right) Ry \quad (12)$$

where $\delta\epsilon = 4A_0/a_0 Ry$ is the shift. This agrees with the result obtained directly from the pseudopotential (2) and (3):

$$\delta\epsilon = \int V(\vec{x}) \phi_{1s}^2(x) d\vec{x} = -4\pi \hbar^2 A_0 \phi_{1s}^2(0) / 2m \quad (13)$$

since $\phi_{1s}^2(0) = (\pi a_0^3)^{-1}$ and $Ry = \hbar^2 / (2ma_0^2)$. The result (12) is valid to terms of order $(A_0/a_0)^2$.

5. THE EXPRESSION FOR THE LINE SHAPE

The absorption rate for the hadrons is proportional to the square of the renormalized value of $\Phi_e^{(0)}(\vec{x})$ at the origin and to the imaginary part of the pseudopotential. This depends only on photon energy ω , and not on the direction of photon emission. In addition, the line shape $I(\omega)$ is proportional to the density of photon states which is proportional to ω^2 and to the squared amplitude of the electromagnetic vector potential which is proportional to ω^{-1} . Thus

$$I(\omega) \propto \text{Im} A_0 \omega |\Phi_e^{(0)}(0)/(1+r_e)|^2 \quad (14)$$

6. LINE SHAPE FOR AN INITIAL 2p STATE

The Whittaker function in Eq. (7) is rather complicated ³⁾, but we need it only in the integral

$$\Phi_e^{(0)}(0) = \omega \int G_e(\vec{x}, 0) (\hat{e} \cdot \vec{x}) \phi_{2p}(\vec{x}) d\vec{x} \quad (15)$$

where $\epsilon = \epsilon_{2p} - \omega$. This is the unrenormalized hadron wave function at the origin for the case of an initial 2p state. The radial hydrogenic 2p wave function is proportional to $x \exp(-x/2a_0)$. The integral can then be done analytically ⁴⁾:

$$\begin{aligned} & \omega \Gamma(1-\beta) \int_0^\infty x^3 \exp(-x/2a_0) W_{\beta, \frac{1}{2}}(zx/\beta a_0) dx = \\ & = 144 a_0^4 Ry \frac{\beta^2}{(1+\beta/2)^4 (4-\beta)(3-\beta)(1-\beta)} {}_2F_1(5, 1-\beta, 5-\beta; \frac{\beta-2}{\beta+2}) \end{aligned} \quad (16)$$

Here ${}_2F_1(a, b, c; z)$ is the hypergeometric function, and

$$\beta = \frac{\hbar}{a_0 (2m|e|)^{\frac{1}{2}}} = \left(\frac{1}{4} + \frac{\omega}{Ry} \right)^{-\frac{1}{2}} \quad (17)$$

For positive ω the dimensionless variable β is between 0 and 2. The only pole in Eq. (16) is then for $\beta = 1$ or $\omega = 3/4$ Ry. The line shape (14) for an initial $2p$ state follows from

$$I_{2p}(\omega) \propto \frac{(2-\beta)\beta^2 {}_2F_1^2(5, 1-\beta, 5-\beta; \frac{\beta-2}{\beta+2})}{(2+\beta)^7(4-\beta)^2(3-\beta)^2} \frac{\text{Im } A_0}{(1-\beta)^2|1+r_\epsilon|^2} \quad (18)$$

No constant factors are given in Eq. (18) since over-all normalization of $I_{2p}(\omega)$ is not essential. We note that in the limit $\omega \rightarrow 0$ ($\beta \rightarrow 2$), the line shape (18) is linear in ω and not cubic as Eq. (1), except for very small energies, $\omega < (|A_0|/a_0)\text{Ry}$, where the pole in r_ϵ near $\beta = 2$ [from $\psi(1-\beta)$] in Eq. (11) again gives a ω^3 dependence.

7. LINE SHAPE IN THE RESONANCE REGION

Near the resonance at $\beta = 1$ ($\omega = 3/4$ Ry) the renormalization constant (11) is dominated by the pole at $\beta = 1$ in $\psi(1-\beta)$, so that

$$\begin{aligned} |(1-\beta)(1+r_\epsilon)|^2 &\approx |1-\beta - 2 \frac{A_0}{a_0}|^2 \\ &\approx \frac{1}{4} \left[\left(\frac{\omega}{\text{Ry}} - \frac{3}{4} - 4 \frac{\text{Re } A_0}{a_0} \right)^2 + \left(4 \frac{\text{Im } A_0}{a_0} \right)^2 \right] \end{aligned} \quad (19)$$

Terms of the order $(|A_0|/a_0)^2$ and higher have been neglected, and $\beta = 1$ has been used everywhere except in pole terms. The limiting form of Eq. (18) for β close to unity is then

$$I_{2p}(\omega) \xrightarrow{\omega \rightarrow \frac{3}{4} \text{ Ry}} \frac{1}{3^9} \frac{\text{Im } A_0}{\left(\frac{\omega}{\text{Ry}} - \left(\frac{3}{4} + 4 \frac{\text{Re } A_0}{a_0} \right) \right)^2 + \left(\frac{4}{2} \right)^2} \quad (20)$$

This is the usual Breit-Wigner form, with $\Gamma = 8 \text{ Im } A_0/a_0$. The result agrees with the resonance shape for a complex energy eigenvalue shifted according to Eq. (12). The resonance shape is a direct consequence of the driving source (5), acting on the physical system in the presence of strong interactions.

8. LINE SHAPE FOR LARGE ENERGIES

When the photon energy is outside the resonance region, $|\omega - \omega_0| < |\delta\epsilon|$, the shape is independent of strong interactions. This region has two parts. In the first, $|\omega - \omega_0| \lesssim 1 \text{ Ry}$, Coulomb effects still dominate the generated

wave function. The second part is from here on up to energies where the wave length κ^{-1} shrinks to the scale of strong interactions, $\omega - \omega_0 \approx (a_0/|A_0|)^2 \text{ Ry}$. In these regions the line shape (18) is independent of the absorption strength, although it is proportional to $\text{Im } A_0$. The shape remains unchanged even in the limit $\text{Im } A_0 \rightarrow 0$ when the absorption becomes negligible. This reflects the fact that the virtual transitions always force the wave function $\Phi_e^{(0)}(\vec{x})$ to be present. It shows up physically only in the absorption process since energy conservation normally prevents one from observing it. As long as the absorption is sufficiently weak, this wave function can be treated in the Born approximation. The line shape is then universal, independent of the absorptive strength. It is only near resonance, and for very large energies that the shape depends on the absorption strength.

From Eq. (18) we obtain for $\omega \gg 1 \text{ Ry}$ ($\beta \ll 1$):

$$I_{2p}(1 \text{ Ry} \ll \omega < (\frac{a_0}{|A_0|})^2 \text{ Ry}) \propto \frac{\text{Im } A_0}{2^{1/2} 3^2} \frac{\text{Ry}}{\omega} \quad (21)$$

Here we have used ${}_2F_1(5,1,5,z) \rightarrow \frac{1}{2}$ for $z \rightarrow -1$. The damping mechanism for high-energy X-rays is now easily understood. The absorption rate depends on the probability of finding a hadron at the origin. In the high-energy limit Coulomb effects become negligible, so that the bound hadron propagates freely with a Green's function $g_e(\vec{x}, \vec{x}') = \exp(-\kappa |\vec{x} - \vec{x}'|) / (4\pi |\vec{x} - \vec{x}'|)$. The wave function at the origin depends then only on the source function inside a region $\kappa^{-1} \propto \omega^{-1/2}$ around the origin, and this region ($\ll a_0$) decreases with increasing photon energy. Consequently, the probability of finding a hadron at the origin goes down with increasing ω .

To be more precise:

$$\begin{aligned} \Phi_e^{(0)}(0) &\propto \omega \int g_e(0, \vec{x}) \chi^2 \exp(-x/2a_0) d\vec{x} \\ &= \frac{6\omega}{(\kappa + \frac{1}{2a_0})^4} \xrightarrow{\omega \rightarrow \infty} \frac{3\hbar^2}{2m} \frac{1}{\omega} \end{aligned} \quad (22)$$

The probability of finding the hadron at the origin therefore varies as ω^{-2} , and from Eq. (14) the asymptotic line shape varies as ω^{-1} . The decisive point is thus that only a decreasing fraction of the source function is effective in the transition strength.

The forms (18) (with $r_e = 0$) or (21) for the line shape are great improvements over Eq. (1) and show how the atomic physics of the problem damp the high-energy behaviour. But the ω^{-1} behaviour of Eq. (21), although valid over an enormous range in ω , is still unsatisfactory, since the integrated line shape diverges logarithmically. This is remedied by the strong interaction.

9. CONVERGENCE FOR VERY LARGE ENERGIES *)

For energies so large that the propagation distance κ^{-1} is comparable to the strong interaction scattering length, a new regime begins. The energy scale is now that of the strong interactions and not that of atomic physics, which is what we mean by "very large energies". In this regime [$\omega \gtrsim (a_0 |A_0|)^2 \text{ Ry}$ in atomic units] the renormalization factor (11) once more becomes crucial:

$$1 + r_e \rightarrow 1 + A_0 \kappa = 1 + \frac{A_0}{a_0} \left(\frac{\omega}{\text{Ry}} \right)^{\frac{1}{2}} \quad (23)$$

so that Eq. (21) modifies into

$$\begin{aligned} I_{2p}(\omega) &\propto \frac{1}{2^{1/2} 3^2} \frac{\text{Ry}}{\omega} \frac{\text{Im } A_0}{\left| 1 + \frac{A_0}{a_0} \left(\frac{\omega}{\text{Ry}} \right)^{\frac{1}{2}} \right|^2} \\ &= \frac{1}{2^{1/2} 3^2} \frac{\text{Ry}}{\omega} \text{Im } T(-\omega) \\ &\longrightarrow \frac{1}{2^{1/2} 3^2} \frac{\text{Im } A_0}{|A_0/a_0|^2} \left(\frac{\text{Ry}}{\omega} \right)^2 \end{aligned} \quad (24)$$

where the s wave T matrix below threshold, $T(\epsilon)$, has been introduced formally. Equation (24) gives a line shape whose integral converges. The strong interaction provides the necessary cut-off in Eq. (18) or Eq. (21). From Eq. (24) we also see that bound states below threshold, due to strong interactions, may give rise to high-energy resonances in the X-ray spectrum. In our model there is only one such state which can contribute, namely, an s state at $(-\epsilon/\text{Ry})^{\frac{1}{2}} = -(a_0/A_0)$, so it only occurs when $\text{Re } A_0 < 0$. The Y_0^* resonance at 27 MeV below threshold in the K^-p system is a state of this type. Such states can be discussed generally in the T matrix formulation, but it is outside the scope of this paper. It is of considerable interest in its own right, particularly in connection with the quasibound states of the $\bar{p}p$ system ⁵⁾.

*) We have consistently used atomic units. They are not the natural ones for strong interactions. Some readers may wish to use other units in the following Sections.

10. LINE SHAPE FOR INITIAL np STATES

This previous reasoning can be used directly for transitions from other initial states. The line shape will differ for different initial states because the initial driving source, Eq. (5), is not the same for different initial states. For large values of ω the energy dependence of the line shapes are the same for all initial np states (ω^{-1}), but with the additional proportionality factor $N_{n1}^2/N_{21}^2 = 32(n^2 - 1)/(3n^5)$. This is because the source function for an initial np state varies as $N_{n1}(\hat{e} \cdot \vec{x})x$ at short distances x . For large energies the driving source is only felt in this inner region, and it therefore scales with N_{n1} . The absorption rate and thus the line shape will therefore scale with N_{n1}^2 for large energies ω . On the other hand, the peak of the resonance is proportional to

$$M_n = (E_{n1} - E_{10})^3 \left| \int_0^\infty R_{n1}(x) R_{10}(x) x^3 dx \right|^2 \quad (25)$$

where $R_{n\ell}$ is the normalized $n\ell$ radial wave function. Thus ⁶⁾

$$\frac{M_n}{M_2} = \frac{3^{15}}{2^8} \frac{n^8(n-1)^{4n-7}}{(n+1)^{4n+7}} \quad (26)$$

Therefore, normalized to the 2p-1s resonance peak, more and more intensity is shifted into the high-energy tail as n increases. The ratio of tail to peak increases roughly as n^3 for large n . In addition, for $n \geq 3$, there are secondary maxima in the line shapes $I_{n1}(\omega)$ corresponding to $np \rightarrow 2s$, $np \rightarrow 3s$, etc., which occur for $\omega < 1$ Ry.

11. NUMERICAL ILLUSTRATIONS

The 2p intensity distribution is shown in Fig. 1, for a width of 6%, i.e., $\Gamma = 0.045$ Ry. The actual function plotted is

$$I_{2p}(\omega) = \frac{(2-\beta)(1+\beta)^2}{\beta^2(2+\beta)^7(4-\beta)^2(3-\beta)^2} {}_2F_1^2\left(5, 1-\beta, 5-\beta; \frac{\beta-2}{\beta+2}\right) \times \frac{1}{(\omega - \omega_0)^2 + (\Gamma/2)^2} \quad (27)$$

where $\omega_0 = 0.75$ Ry. (We have not included any shift.) The hypergeometric function is evaluated directly from its definition as a series. In Fig. 1 the standard form, Eq. (1), is also shown. The two curves are normalized to coincide at maximum. We note that the function

$$\text{const} \frac{\omega}{(\omega - \omega_0)^2 + (\Gamma/2)^2} \quad (28)$$

gives a good approximation to $I_{2p}(\omega)$ for $(|A_0|/a_0)\text{Ry} < \omega < 1$ Ry. It also has the correct ω^{-1} behaviour for large ω , but with a coefficient which is too big.

As an example of the line shapes for higher np initial states, we have performed a similar calculation for an initial $3p$ state. If the strong interaction contributions are kept only in the pole terms, we obtain

$$I_{3p}(\omega) = \frac{(3-\beta)(1+\beta)^2(2+\beta)^2}{\beta^6(3+\beta)^7(4-\beta)^2} \left[{}_2F_1\left(5, 1-\beta, 5-\beta; \frac{\beta-3}{\beta+3}\right) - \frac{10}{(5-\beta)(3+\beta)} {}_2F_1\left(6, 1-\beta, 6-\beta; \frac{\beta-3}{\beta+3}\right) \right]^2 \times \frac{1}{\left[(\omega - \omega_1)^2 + (\Gamma/2)^2 \right] \left[(\omega - \omega_2)^2 + (\Gamma/16)^2 \right]} \quad (29)$$

Here $\beta = (1/9 + \omega/\text{Ry})^{-1/2}$ and $\omega_1 = 8/9$ Ry, $\omega_2 = 5/36$ Ry, while Γ is the $1s$ width as in Eq. (27). The width (and the shift) of the $2s$ line is exactly $1/8$ that of the $1s$ line. This follows from an expansion similar to Eq. (19) near the pole for $\beta = 2$ in the renormalization constant (1). This is of course well known¹⁾. The function (29) is shown in Fig. 2. For comparison we have also plotted $I_{2p}(\omega)$ with its maximum drawn to coincide with the main peak of the $I_{3p}(\omega)$ curve. When the $3p-1s$ and $2p-1s$ maxima coincide, the $3p$ tail lies above the $2p$ tail as discussed in the previous Section.

As a specific example of the damping mechanism by strong interactions, we take a hypothetical " K^-p " system with scattering lengths $A_0 = (-1.65 + 0.75i)\text{fm}$ and $(1.65 + 0.75i)\text{fm}$. The first case gives a negative resonance energy at -27 MeV with a 40 MeV width. This corresponds to the Y_0^* resonance in the isospin 0 system with the same scattering length

(in a realistic case the statistical weight of this channel is $\frac{1}{2}$ and not 1). In the line shape this produces a very broad structure centered on the resonance. Integrating the line shape from $2\omega_0 = 1.5 \text{ Ry}$ to ∞ gives, respectively, a contribution of 25% and 7% of the total integrated line shape for the two cases. Thus, there is a substantial contribution from the high-energy tail, particularly when we have the rather strong transition to Y_0^* .

12. RETARDATION EFFECTS

We have completely neglected the effect of the photon wavelength, although it could easily have been incorporated in our approach. The wavelength becomes smaller than the $2p$ orbit for $\omega \gtrsim (Z\alpha)^{-1} \text{ Ry}$, but retardation effects are not governed by the size of the initial $2p$ state. They are determined by the range κ^{-1} of the Yukawa term in the Green's function (i.e., the free Green's function) which only allows contributions to $\Phi_e^{(0)}(0)$ to come from a region of size κ^{-1} around the origin. The range to wavelength ratio is $(\omega/2mc^2)^{\frac{1}{2}} \ll 1$ for all reasonable ω . Retardation effects are thus negligible.

13. SIZE EFFECTS

For a point-like absorbing nucleus we obtain an exact solution. What new effects result from the finite nuclear size? There are at least two consequences. First, waves with non-zero angular momentum may penetrate the centrifugal barrier and be absorbed by the nucleus. This is only important for energies $\epsilon \gg \frac{1}{2} \text{ Ry}$. The electric dipole transition from the initial $2p$ state generates both (and only) s and d waves. In the "almost point-like" approximation for the nucleus, we can express the d wave interaction in terms of the d wave generalized scattering length A_2 . The only change in Eq. (24) is then to replace the hadron-nuclear scattering amplitude $T(-\omega)$ by

$$T(-\omega) = \frac{A_0}{1 + \frac{A_0}{a_0} \left(\frac{\omega}{\text{Ry}}\right)^{\frac{1}{2}}} + \frac{35}{2} \frac{\left(\frac{\omega}{\text{Ry}}\right)^2 \frac{A_2}{a_0^4}}{1 + \frac{A_2}{a_0^5} \left(\frac{\omega}{\text{Ry}}\right)^{\frac{5}{2}}} \quad (30)$$

Roughly speaking, the d wave is negligible as long as

$$\text{Im } A_0 > \frac{35}{2} \left(\frac{\omega}{\text{Ry}}\right)^2 \frac{\text{Im } A_2}{a_0^4} \quad (31)$$

Contributions from other multipole transitions enter only by retardation. They are of order $(\omega/Ry)^2 A_1/a_0^3$ or smaller, where A_1 is the generalized p wave scattering length.

The second consequence of the finite nuclear size occurs for energies such that $\kappa R = (\omega/Ry)^{1/2} (R/a_0) \gtrsim 1$ when the range of the Green's function is smaller than nuclear dimensions. The wave function $\phi_c^{(0)}(\vec{x})$ inside the nuclear region then has contributions mainly from that part of the source $j(\vec{x})$ which is inside the nucleus. One must then abandon the assumption that an external wave is incident on the absorbing nucleus which was implicitly assumed in the approximation $R=0$. Although the description will then be model-dependent, the relevant quantity for the case of a nucleus is in any case the absorption rate in nuclear matter multiplied by the probability of presence inside the nuclear volume. If the energy ω is large compared to the interaction energy, the line shape in this region is once more proportional to ω^{-1} . Finally, for $(\omega/mc^2)(R mc/\hbar) > 1$ [or in atomic units $\omega > (a_0/R)(2/\alpha) Ry$], the X-ray wavelength becomes similar or smaller than nuclear dimensions. Further contributions are then rapidly quenched by the nuclear form factor. We conclude that a host of connected, but model-dependent phenomena, occur for energies $\omega \gtrsim (a_0/R)^2 Ry$. These are outside the scope of the exactly soluble model of this paper.

14. CONCLUSIONS

Our exact solution for the atomic absorptive line shape demonstrates that the Breit-Wigner form is inadequate for broad states. The high-energy side of the resonance develops a very long, but well-defined, tail decreasing as ω^{-1} . This shape is a direct consequence of the decreasing probability of finding the hadron at the origin, since the initial wave packet can be sampled only within a distance $\kappa^{-1} \propto \omega^{-1/2}$ around the origin. The integrated line intensity is then still logarithmically divergent, but at very high energies the unitarity condition on the strong interaction restores convergence by giving a line shape decreasing with ω^{-2} . These two results completely remove the inconsistency of the usual approximations which lead to divergences.

Our line shape is model-independent over an enormous energy range on the atomic scale. In the case of protonium ($\bar{p}p$) it is valid over about $10^5 \Gamma$ or about $10^4 Ry$. We also find that the line shape is universal in the entire region from $\omega - \omega_0 \gtrsim \Gamma$ to the domain where strong interactions impose convergence. The line shape $I(\omega)$ contains the absorptive strength only as a multiplicative factor, so that the actual shape does not depend

on the absorptive strength. The reason for this is that the hadron wave function (6) is entirely dominated by the source term in this energy region. Consequently, the Born approximation can be used. Universality is broken at very high energies by strong interaction effects resulting from unitarity.

While the line shape is universal outside the resonance region for any given initial state, it depends critically on the initial state itself. The reason is that the hadron wave function depends on the driving source which is related to the initial wave function as in Eq. (5). Therefore, even the Born approximation will give different line shapes for different initial states.

The numerical illustrations show that a substantial part of the X-ray transitions for broad lines will go to much higher energies than previously expected. This has several experimental consequences. In particular, we note that the direct photon transitions to bound states enter on an equal footing with the usual atomic transitions, and may even be of comparable strength as for the case of Y_0^* in the K^-p system. Furthermore, the results imply that there is an important physical background of high-energy X-rays. Since one often derives the absorption rate in some initial state from the competition between radiation and absorption, the existence of a substantial high-energy branch means that the 2p absorption rate must be revised for broad lines.

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FIGURE CAPTIONS

Fig. 1 : Line shape for an initial 2p state with a 6% width. The full line is the exact result (27), and the dotted curve is the usual Breit-Wigner form (1).

Fig. 2 : Full line: the line shape for an initial 3p state with a 6% 1s line width according to Eq. (29). The dotted curve is the 2p line shape.

FIG.1

FIG.2