ABSOLUTE CALIBRATION OF THE NMR MAGNETOMETERS
AND OF THE MAGNETIC FIELD MEASURING
SYSTEM USED IN THE CERN MUON STORAGE RING

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ABSTRACT

The nuclear magnetic resonance (NMR) magnetometers designed for the measurement of the magnetic field of the Muon Storage Ring have been described in a preceding paper. Here we report results on the measuring accuracy of these units. The tests were extended over the range of local gradients which exist in the storage region from better than 1 ppm/cm to the worst case of some 300 ppm/cm. The magnetic field is 14.7 kG.

All NMR probes used for the mapping and monitoring of the magnetic field have been absolutely calibrated with calibration probes especially constructed for this purpose. All corrections applied to the measured NMR frequencies in order to get the spin precession frequency of a free proton in the same magnetic field are summarized.

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1. INTRODUCTION

The muon g-factor anomaly $a \equiv \frac{1}{2} (g-2)$ is measured by observing the precession of the spin relative to the velocity vector of muons circulating in the storage ring¹⁾. Apart from small corrections, the frequency of this precession ω_a is directly proportional to the magnetic field (SI units):

$$\omega_a = a \frac{e}{m} \bar{B}.$$

Here e/m is the charge-to-mass ratio of the muon; $\bar{B}$ is the mean field value resulting from a complete map of the magnetic storage field, properly weighted according to the muon orbit distribution²⁾. To extract the value of the anomaly a from the frequency ω_a , which is measured with a statistical accuracy somewhat better than 10 ppm, the quantity $(e/m) \bar{B}$ must be determined. It may be expressed in the following way:

$$\frac{e}{m} \bar{B} = (1 + a)^{-1} \frac{\omega_\mu}{\omega_p^0} \omega_p^0. \quad (1)$$

The ratio of the Larmor frequency ω_μ of free muons at rest relative to the resonance frequency ω_p^0 of free protons is known from other experiments^{3,4)}. The nuclear magnetic resonance (NMR) frequencies measured when mapping or monitoring the magnetic field will therefore finally have to be corrected to the frequency of free protons.

In Section 2 of this paper we report on the accuracy tests of the NMR magnetometer system, which has been described in detail in a preceding publication⁵⁾. In the following paragraphs we give details of the calibration method used to derive from the NMR frequencies measured with the mapping machine the associated free proton frequencies ω_p^0 .

2. ACCURACY TESTS OF THE NMR MAGNETOMETERS

The search and lock detection mode of the NMR magnetometers (also called for short the automatic mode) as well as the measuring heads used have been described in Ref. 5. Various tests of stability and reproducibility have been made. Variations of the modulation amplitude and of the RF amplitude by an order of magnitude as well as exchange of magnetometer NIM units result in the same NMR frequencies within ± 0.3 ppm. Exchange of measuring heads of identical construction, or the exchange of the NMR water samples doped with 0.1 molar NiSO_4 , gave somewhat larger variations within ± 0.6 ppm in spite of using only selected Cu-Be screws and taking care to avoid magnetic contamination.

As an independent check of the magnetometers and their accuracy, the NMR frequency measured in the automatic detection mode was compared with the result obtained by a slow stepwise frequency scan of the resonance line. A schematic diagram

of the set-up used for this detection mode is shown in Fig. 1. The RF frequency is supplied by a frequency synthesizer. The sinusoidal audio-frequency field modulation is derived from the reference signal of a lock-in amplifier. The parts of the magnetometer used in this detection mode are only the RF amplifier, the automatic tuning of the resonance circuit containing the water sample to the applied RF frequency, as well as the detection circuit and amplifier for the NMR signal. The NMR signal is fed into the input of the phase-sensitive detector.

The field modulation frequencies used lie within the range 10-90 Hz. The field modulation amplitude is set to a value which is small with respect to the line width, the latter being about 3 ppm peak-to-peak for the normal measuring heads. The output of the lock-in amplifier is the derivative of the NMR absorption resonance curve. With an integration time constant of 1 sec, the signal-to-noise ratio is typically better than 50 near the peak values of the measured resonance curves, and the line centre can easily be determined to ± 0.2 ppm. This is then compared with the NMR frequency obtained with the fast automatic mode where the absorption signal is ~ 18 ppm wide and asymmetric, and where the NMR frequency is essentially determined by symmetry requirements for upward- and downward-going field sweep. The agreement was found to be better than ± 0.3 ppm.

These tests were extended to the highest gradients, up to some 300 ppm/cm, found at a few points of the storage region; this is also the limit at which the NMR signals become too small for the automatic mode to lock. Figure 2 shows the derivative of the absorption curve measured in a field gradient of 175 ppm/cm. The shape and the symmetry of the resonance curve result from the constancy of the gradient across the 2 mm diameter of the tube containing the NMR water sample. The result of the automatic mode NMR frequency lies inside the uncertainty of about ± 1 ppm with which the centre frequency of the resonance line can be determined.

3. CALIBRATION PROCEDURE

A complete map of the magnetic field was made both before and after a physics run, with the vacuum chamber removed but all the other components (e.g. photomultiplier shielding and vacuum pumps) in place. Figure 3 shows part of a block of the storage ring with the mapping machine. It consists of a movable arm with the support for eight mapping NMR probes which is driven in radial and vertical directions by motors incorporated in a device rolling on the inner edge of the magnet gap and centred in the middle of the ring by an 'I'-shaped aluminium bar. One block is measured at a time by driving the machine back and forth in the azimuthal direction, the vertical and radial positioning of the arm with the eight NMR samples being done at the turning points. The positioning and read-out of the NMR probes is done by computer control.

The NMR probes of the mapping machine have to be absolutely calibrated against an NMR probe, called calibration probe in the following, with a well-defined geometry and susceptibility correction, in order to obtain the free proton resonance frequency ω_p^0 . As will be explained below, this calibration was done by introducing as an intermediate standard an NMR probe with a measuring head identical to the one used for the mapping machine. This "standard probe" has an aluminium holder for the measuring head allowing easy and reproducible mounting of this probe in the different steps of the calibration.

Although the materials used for the mapping machine are all of low permeability, it does -- at the level of a few ppm -- change the field at the position (r, z, θ) of the measuring head but also at the position of the stabilization probe just outside of the storage region in the centre of each magnet block. This latter field change will be restored by an equivalent change of the current in the compensating coils wound around each pole piece, leading to an additional field change at the point (r, z, θ) in the storage region.

The corrections which must be applied to the NMR frequency $\omega(r, z, \theta)$ measured with a probe of the mapping device at any point (r, z, θ) in order to get the free proton resonance frequency $\omega_p^0(r, z, \theta)$ are summarized in the following equation:

$$\omega_p^0(r, z, \theta) = \omega(r, z, \theta) \left[1 + \delta^{\text{cal}} + \delta^{\text{map}}(r, z, \theta) + \delta^{\text{stab}}(r, z, \theta) \right]. \quad (2)$$

Here δ^{cal} is the correction to be applied to the NMR frequency measured by the calibration probe to obtain the free proton resonance frequency ω_p^0 . The next term $\delta^{\text{map}}(r, z, \theta)$ is the relative frequency change composed of the possibly coordinate-dependent influence of the mapping machine on the local field and of the deviation of the mapping probes relative to the calibration probe. Finally, $\delta^{\text{stab}}(r, z, \theta)$ is the additional frequency change caused by the influence of the mapping machine on the field stabilization.

The NMR probe support is moved in radial and vertical directions over only 3 cm and 4 cm, respectively, when mapping the field, and the amount of material is small. In the azimuthal direction there is only a slight deviation from symmetry owing to the fact that the Muon Storage Ring is really a polygon of 40 C-shaped magnets. It is therefore not surprising that no detectable coordinate dependence of $\delta^{\text{map}}(r, z, \theta)$ was found at the level of a few tenths of a ppm and $\delta^{\text{map}}(r, z, \theta)$ is equal to δ^{map} . The same holds for the radial and vertical dependence of $\delta^{\text{stab}}(r, z, \theta)$, but here the influence depends on the azimuthal position of the mapping machine with respect to the stabilization probe, i.e. $\delta^{\text{stab}}(r, z, \theta)$ becomes $\delta^{\text{stab}}(\theta)$.

3.1 Calibration probe and corrections

The calibration probe is mounted on a special support and is shown in Fig. 4. It can be positioned in the magnetic field in a reproducible way. The probe consists of a pair of Helmholtz coils and a cylindrical sample filled with $\text{H}_2\text{O} + 0.1$ molar NiSO_4 . With a length of 50 mm and 2 mm inner diameter the length-to-diameter ratio of the NMR solution is sufficiently large to approximate well the demagnetization factor of an infinitely long transverse cylinder⁶⁾.

The resonance curves were measured by a stepwise scan of the frequency and the use of a lock-in amplifier, as described above in Section 2 (see also Fig. 1). Several samples were made, either of plexiglas or teflon and with different ways of fixing the RF coil of six turns of 0.2 mm copper wire wound over a length of 4 mm on the outside. The central frequencies measured with five different calibration samples in the same magnetic field gave identical results within ± 0.1 ppm. Figure 5 shows a typical example of a resonance curve. The comparison with the derivative of a Lorentzian line of 1 ppm peak-to-peak line width (1.65 ppm FWHM) shows good agreement.

The measured frequency has to be corrected for the effect of the paramagnetic Ni^{++} ions of the 0.1 molar NiSO_4 solution, for the bulk diamagnetic susceptibility of the water, and for the diamagnetic screening σ produced by the orbital electrons of a water molecule. The paramagnetic shift was experimentally determined by varying the NiSO_4 concentration. Defining $\Delta\omega$ as the frequency shift relative to pure water, our result is

$$\frac{\Delta\omega}{\omega} = (-0.88 \pm 0.09) \text{ ppm}/0.1 \text{ molar } \text{NiSO}_4 ,$$

in good agreement with Dickinson⁶⁾.

In ellipsoids and their limiting cases (e.g. sphere, long cylinder), the water molecule finds itself in a field B (SI units):

$$B = \left[1 + \left(\frac{1}{3} - N \right) \chi \right] B_0 = (1 + 1.51 \times 10^{-6}) B_0 ,$$

where N is the demagnetization factor depending on the shape of the ellipsoid ($N = \frac{1}{2}$ for a long circular cylinder transverse to the applied field B_0), and χ is the susceptibility of water [$\chi_{\text{H}_2\text{O}} = -9.050 \times 10^{-6}$ ⁷⁾]. The diamagnetic shielding correction σ to obtain the free proton resonance frequency gives the largest contribution⁸⁾:

$$\sigma = 25.67 (2) \times 10^{-6} \text{ for } T = 25^\circ\text{C} .$$

The quantity δ^{cal} , as defined in Eq. (2), applying to the calibration probe filled with $\text{H}_2\text{O} + 0.1$ molar NiSO_4 , is therefore

$$\delta^{\text{cal}} = (+0.88 - 1.51 + 25.67) \times 10^{-6}$$

with an uncertainty of 0.1 ppm.

If the process of interchanging the probes leads to a field change at the point of comparison caused by differing distributions of materials (probe supports, etc.), this must be taken properly into account. Specifically, to measure the field at some point with the calibration probe and then with one of the probes mounted on the mapping machine, the whole calibration probe support has to be moved away, and consequently the field changes at the point of comparison by a few ppm. To avoid this difficulty we introduced the "standard probe", which was calibrated relative to the calibration probe including the effect of the aluminium holder and without moving the support. The cylindrical sample of the calibration probe (Fig. 4) can easily be interchanged with the standard probe, the aluminium sample holder of the latter allowing a precise positioning to better than 0.5 mm in all coordinates.

The only material removed when calibrating the standard probe is the NMR sample of the calibration probe. The materials used for the cylindrical sample container have a permeability μ such that $(\mu - 1) < 10^{-4}$. The resulting relative change of the magnetic field in the water sample is of the order of $(\mu - 1)^2$ and therefore negligible. This means at the same time that the error introduced by removing the calibration sample when interchanging it with the standard probe can be completely neglected.

3.2 Calibration of the mapping machine

The combined effect of the influence of the mapping machine on the field map and the calibration factor for the NMR probes used, leads to the relative frequency shift $\delta^{\text{map}}(r, z, \theta)$. This quantity has been defined in Eq. (2) and was found, with sufficient accuracy, to be constant throughout the storage volume. It is therefore straightforward to determine the relative frequency shift δ^{map} by comparing the frequency of every one of the eight probes of the mapping machine with the frequency measured by the standard probe, which merely serves as an intermediate standard, necessary in this step of the calibration but dropping out in the final result. This comparison measurement can be done conveniently in a homogeneous part of the field where the requirement for precise positioning of the NMR probes to be compared is relaxed. The magnet block in which this measurement is done is stabilized in series with a remote block in order to have no additional effect of the mapping device on the stabilization (see Section 3.3).

It was found that the eight NMR probes of the mapping machine agree within ± 0.6 ppm with each other. The field at the point of one of these probes was then measured, after the mapping machine has been driven far away, with the standard

probe, which resulted in a 2 ppm higher frequency value. Since the measuring heads of the mapping probes and the standard probe are of identical construction, this difference is mainly caused by the material of the mapping machine. The standard probe reads about 2 ppm higher than the calibration probe, which accidentally roughly cancels the other effect: the result is $\delta^{\text{map}} = (-0.1 \pm 0.5) \text{ ppm}$.

3.3 Measurement of the influence of the mapping machine on the stabilization

The $\delta^{\text{stab}}(\theta)$ frequency shift as defined above was determined by taking a full map of a block in the normal configuration, i.e. with its own stabilization probe controlling the current in its compensating coils. This measurement takes about 10 minutes. Immediately after this, the block was mapped again, but this time the compensating coils were connected in series with those of another block sufficiently far away for the stabilization probe of this remote block to remain unaffected by the mapping machine. The reference frequency for the stabilizing probe of the remote block was now chosen with a frequency synthesizer such that the current flowing in the compensating coils of the block to be measured produced a field value at the position of its stabilization probe equal to the appropriate value, i.e. the stabilization probe is used as a monitor probe to ensure that the frequency measured is equal to the frequency of the reference crystal when the mapping machine is removed. The difference of the two field maps is $\delta^{\text{stab}}(\theta)$. The result is a $1.5 \pm 0.2 \text{ ppm}$ increase of the mean field value due to the influence of the mapping device on the stabilization.

4. CONCLUSIONS

For the determination of the anomaly factor a of the muon, the magnetic field of the Muon Storage Ring is expressed by the equivalent spin resonance frequency ω_p^0 of free protons. The NMR probes used had therefore to be absolutely calibrated including the effect of the mapping device on the measurement.

When the field has been mapped, the following equation relates the average field value expressed via the measured NMR frequency ω with the associated free proton resonance frequency ω_p^0 .

$$\omega_p^0 = \omega [1 + (+0.88 - 1.51 + 25.67 - 0.1 - 1.5 \pm 0.6) \times 10^{-6}] .$$

The first three correction terms apply to the calibration probe and are in the above order: the relative frequency shift caused by doping the water sample with 0.1 molar NiSO_4 ; the bulk diamagnetic effect of the cylindrical water sample; and the diamagnetic screening by the orbital electrons of a water molecule. The next terms are the calibration factor of the NMR probes mounted on the mapping machine, the effect of the latter on the field via the influence on the stabilization probes, and finally the over-all uncertainty of $\pm 0.6 \text{ ppm}$.

As noted above, the mapping machine is driven back and forth over the length of a block when the field is mapped. The speed was about 6 cm/sec. It was verified experimentally that this has a negligible effect on the field map after averaging in the azimuthal direction. The comparison of the azimuthally averaged field maps obtained with the normal driving motor, and one which was slower by more than a factor of 10, gave identical mean field values within ± 0.2 ppm.

In addition to these corrections, the accuracy and reproducibility of the magnetometer system was tested under various conditions. The NMR frequency resulting from the fast automatic mode agrees within ± 0.3 ppm with the frequency derived from the narrow and symmetric resonance line which is obtained by a stepwise frequency scan using a lock-in amplifier.

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Figure captions

- Fig. 1 : Block diagram of the NMR detection mode with lock-in amplifier and synthesized signal generator.
- Fig. 2 : Derivative of absorption curve measured with a mapping probe in a field gradient of 175 ppm/cm. The modulation amplitude is 9 mG, the modulation frequency is 90 Hz. The result of the automatic search and lock mode is indicated by the arrow.
- Fig. 3 : The mapping machine rolls on the edge of the lower pole piece. The movable arm with the eight NMR probes as well as the stabilization probe of one of the 40 magnet blocks can be seen.
- Fig. 4 : Helmholtz coils with the cylindrical sample of the calibration probe are mounted on a movable support. The NMR sample can easily be interchanged with the "standard probe", which serves as intermediate calibration standard.
- Fig. 5 : Example of the derivative of an NMR absorption curve measured with the calibration probe. The modulation amplitude is 4 mG, the modulation frequency 12 Hz.
X are measured points; the full line is the calculated derivative of a Lorentzian with 1 ppm peak-to-peak line width.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5