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THE DIFFRACTION PATTERN OF pp ELASTIC SCATTERING
AND THE COMPOSITE STRUCTURE OF THE PROTON

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A B S T R A C T

The consequences of the quark-gluon structure of hadrons for pp elastic scattering are examined in a simple model where the gluon is taken to be the active constituent in normal (low p_T) hadronic collisions. Earlier work based on the study of pp inelastic diffraction led to the conclusion that the gluon-gluon elastic amplitude t_g is approximately gaussian in the gluon-gluon impact parameter b_g , with maximum opacity at $b_g = 0$ ⁴⁾. A slight refinement of this approximation, in which t_g is almost maximally opaque over a small b_g interval and is gaussian at larger b_g , is now shown to account for the remarkable diffraction structure observed in pp elastic scattering at ISR energies. A very simple form is obtained for the form factor describing the impact parameter distribution of the gluon inside a proton relative to the proton itself.

1. INTRODUCTION

Much converging evidence has been obtained in recent years for a composite structure of hadrons with quarks as the quantum-number-carrying constituents which interact with leptons, the rest of the hadron being usually pictured as some form of binding agent or glue. Less clarity exists on how this internal hadron structure manifests itself in strong interaction collisions - we are here concerned with low p_T collisions - and many different possibilities are under discussion. A very simple one has been introduced some time ago under the name of quark-gluon model of hadronic collisions¹⁾⁻⁴⁾. The aim of the present paper is to show that it has interesting consequences for proton-proton elastic scattering.

In the quark-gluon model of Refs. 1)-4), hadrons are regarded as built up of valence quarks and gluons, and the gluons are taken to be the active element in non-diffractive hadronic collisions at high energy. The model for such collisions is that the valence quarks fly through to give rise to the leading particles or leading clusters, and the gluons interact and emit the bulk of secondaries in the central region of rapidity. Since diffractive collisions are the shadow of the non-diffractive ones, the model has consequences for elastic and inelastic diffraction.

The consequences for inelastic pp diffraction were studied in Ref. 4). In the model, the existence of a large cross section for inelastic diffraction is directly related to the fact that the gluons have inside the proton a distribution in longitudinal momentum and in impact parameter. In order to obtain a large inelastic diffractive cross section as measured experimentally, it was found that the gluon-gluon interaction had to be maximally absorptive (opacity 1) at zero impact parameter. This in turn provided a neat explanation for a remarkable property of pp elastic scattering, that in the centre-of-mass energy range 10-60 GeV the proton opacity at zero impact parameter remains practically constant at a value well below the unitarity limit although the cross section increases by as much as 10%.

Within the same model, we here discuss the shape of the elastic cross section $d\sigma_{el}/dt$ and, in particular, its diffraction minimum. The discussion

is based on eq. (18) of Ref. 4), which reads

$$t_{el}(|\vec{b}|) = \int t_g(|\vec{b}_g|) \Gamma(|\vec{b}-\vec{b}_g|) d^2\vec{b}_g \quad (1)$$

This equation expresses the elastic pp amplitude $t_{el}(b)$ as the convolution of the glue-glue amplitude $t_g(b_g)$, averaged over longitudinal glue variables, and the probability distribution $\Gamma(|\vec{b}-\vec{b}_g|)$, where $\vec{b}$ and $\vec{b}_g$ are the pp and glue-glue impact parameters respectively. The function Γ , which is positive, is determined by the internal proton structure, more precisely the glue distribution inside the proton, and is therefore independent of the collision energy.

By Fourier-Bessel transformation, eq. (1) becomes

$$\tau_{el}(t) = \tau_g(t) \gamma(t) \quad (2)$$

where τ_{el} and τ_g , the transforms of t_{el} and t_g , are the pp and glue-glue elastic amplitudes at momentum transfer $t^{1/2}$, and the form factor γ is the transform of $2\pi\Gamma$. The normalization of Γ is such that

$$\int \Gamma(\beta) d^2\beta = \gamma(0) = 1$$

In Ref. 4), t_{el} and t_g were approximated by gaussians, corresponding to simple exponentials for τ_{el} and τ_g . Our aim is now to go beyond this approximation and to discuss the shape of $d\sigma_{el}/dt$ for t up to $\sim 8^*$ on the basis of the high energy data from the CERN Intersecting Storage Rings. The modifications to be introduced are small b corrections to the gaussian approximation and affect only slightly the discussion of inelastic diffraction in Ref. 4). They have an important effect on $d\sigma_{el}/dt$ at larger t , however, the main point being that the observed diffraction minimum is readily explained by making $t_g(b_g)$ almost fully opaque not only at $b_g = 0$ but also in a small but finite interval around this point, retaining the gaussian shape at larger b_g . It then turns out that a surprisingly good description of the observed $d\sigma_{el}/dt$ for $0 < t \lesssim 8$ is obtained when taking a dipole form for $\gamma(t)$.

*) t will always be measured in GeV^2

These are the main results of this paper and they are presented in section 2 for $t_g(b)$ and section 3 for $\gamma(t)$. The last section comments on the energy dependence of $d\sigma_{el}/dt$, the possible existence of further diffraction minima, and the significance of the dipole form found for $\gamma(t)$. It ends with some concluding remarks.

2. THE IMPROVED GAUSSIAN APPROXIMATION FOR THE GLUE AMPLITUDE

The CERN-Hamburg-Orsay-Vienna Collaboration have measured elastic pp scattering at the CERN Intersecting Storage Rings up to $t = 3.6$ at centre-of-mass energy $s^{1/2} = 23$ and 62 GeV ⁵⁾, and up to 9 at $s^{1/2} = 53$ GeV ⁶⁾. The observed shape of $d\sigma_{el}/dt$ has remarkable properties. After the fast but rather complicated drop for $t \lesssim 1$ (with a logarithmic slope which decreases from ~ 12 to ~ 10 ^{*}) for t going from 0.05 to 0.3, and which slowly increases with energy), the cross section has a pronounced minimum at a value $t_0 \approx 1.3$ of t , which decreases slowly for increasing energy. Beyond this minimum, $d\sigma_{el}/dt$ rapidly reaches its second maximum at $t \approx 1.8$ and then shows a slow, almost exponential decrease. The latter has been measured up to $t \sim 8$ at $s^{1/2} = 53$ GeV. Its logarithmic slope is ~ 1.8 . There is no sign of second diffraction minimum up to $t \sim 8$.

In Ref. 4), we adopted the following purely absorptive, gaussian approximations for t_{el} and t_g

$$t_{el}(b) = 0.75 \exp(-b^2/R^2) \quad (3)$$

$$t_g(b_g) = \exp(-b_g^2/0.75 R^2) \quad (4)$$

$$R^2 = 0.9 \text{ fm}^2 = 23.2 \text{ GeV}^{-2} \quad (5)$$

^{*}) Logarithmic slopes will always be measured in GeV^{-2} .

For the elastic amplitudes

$$\tau_{el}(t) = \int_0^{\infty} t_{el}(b) J_0(b t^{\frac{1}{2}}) b db \quad (6)$$

$$\tau_g(t) = \int_0^{\infty} t_g(b) J_0(b t^{\frac{1}{2}}) b db \quad (7)$$

these approximations give

$$\tau_{el}(t) = (0.75 R^2/2) \exp(-R^2 t/4) \quad (8)$$

$$\tau_g(t) = (0.75 R^2/2) \exp(-0.75 R^2 t/4) \quad (9)$$

The optical theorem for purely absorptive amplitudes reads

$$\sigma_{tot} = 4\pi \tau_{el}(0) = 4\pi \tau_g(0) \quad (10)$$

The above approximations correspond to the experimental value of the total cross section at $s^{\frac{1}{2}} = 53$ GeV as given in Ref. 7)

$$\sigma_{tot} = 42.5 \text{ mb} \quad (11)$$

To allow a more satisfactory description of the elastic cross section

$$d\sigma_{el}/dt = \pi [\tau_{el}(t)]^2 \quad (12)$$

we must go beyond the approximation (3). Continuing to treat the amplitude as purely absorptive (t_{el} real) and neglecting the large b improvements which would be needed to reproduce the sharpening of the elastic peak at $t \lesssim 0.1$, we concentrate on the small b region in order to reproduce $d\sigma_{el}/dt$ in the region of the diffraction minimum and beyond. The small b improvements are

introduced very naturally in the model. Instead of taking for $t_g(b_g)$ an exact gaussian reaching the unitarity limit at $b_g = 0$ as was done in eq. (4), all we have to do is to assume that t_g is very close to the unitarity limit for a small but finite interval of b_g and only becomes gaussian at larger b_g . This will automatically produce a diffraction zero in $\tau_g(t)$.

A simple way of achieving the above aim is to take for $t_g(b_g)$ a difference of gaussians. We adopt the parametrization

$$t_g(b_g) = (1-x)^{-1} \exp[-(1+x)b_g^2/b_o^2] - x(1-x)^{-1} \exp[-(1+x)b_g^2/xb_o^2] \quad (13)$$

It obeys the properties

$$t_g(0) = 1, \quad (dt_g/db_g^2)_{b_g=0} = 0 \quad (14)$$

and remains close to the unitarity limit for a small b_g interval. This is illustrated in Fig. 1 for $b_o^2 = 16 \text{ GeV}^{-2}$ and $x = 0.133$, the values to be used later.

The Fourier-Bessel transform of (13) is

$$\begin{aligned} \tau_g(t) = & [b_o^2/2(1-x^2)] \exp[-b_o^2 t/4(1+x)] \\ & - [x^2 b_o^2/2(1-x^2)] \exp[-xb_o^2 t/4(1+x)] \end{aligned} \quad (15)$$

and gives

$$\tau_g(0) = b_o^2/2$$

Since we shall not try to reproduce the sharpening of $d\sigma_{el}/dt$ at $t < 0.1$, we write instead of the optical theorem (10)

$$4\pi \tau_g(0) = 2\pi b_o^2 = \xi \sigma_{tot} \quad (16)$$

where $\xi^2 (d\sigma_{el}/dt)_{t=0}$ is the value of $(d\sigma_{el}/dt)_{t=0}$ which would be obtained by extrapolating $\log(d\sigma_{el}/dt)$ linearly from $t \geq 0.2$ to $t = 0$. Barbiellini et al. ⁸⁾ estimated ξ^{-2} to be 1.2 (see remark 2 on p. 666 of their paper). Using (11) and $\xi = 1$ in (16) would give

$$b_o^2 = 17.4 \text{ GeV}^{-2}$$

We shall adopt

$$b_o^2 = 16 \text{ GeV}^{-2} \quad (17)$$

corresponding to

$$\xi = 0.92, \quad \xi \sigma_{tot} = 39 \text{ mb.} \quad (18)$$

The value of x in (13) and (15) determines the location of the zero of $\tau_g(t)$, which must coincide with the observed diffraction minimum of $d\sigma_{el}/dt$. We adopt the value

$$x = 0.133 \quad (19)$$

For the value (17) of b_o^2 , it corresponds to the position $t = 1.32$ of the diffraction minimum, in agreement with the measurement of De Kerret et al. ⁶⁾ at $s^{\frac{1}{2}} = 53 \text{ GeV}$.

3. THE FUNCTION $\gamma(t)$

Having thus adopted a specific expression for $\tau_g(t)$ by using the (extrapolated) value of $d\sigma_{el}/dt$ at $t = 0$ and the position of the diffraction minimum, we must now see whether a physically reasonable $\gamma(t)$ will reproduce, through eqs. (2) and (12), the observed elastic cross section $d\sigma_{el}/dt$ over the whole range $0.2 \lesssim t \lesssim 8$. The function $\gamma(t)$, Fourier-Bessel transform of the probability distribution $\Gamma(|\vec{b}-\vec{b}_g|)$, has the physical nature of a form factor, or more exactly of the square of a form factor of each proton (see next section). It should be approximately exponential at small t but can be expected to decrease more slowly at larger t , presumably as a power of t . The important test of our approach, however, is that $\gamma(t)$ should be a smooth, reasonably shaped function of t throughout the range considered, and in particular should show no sign of the sharp diffractive structures of $d\sigma_{el}/dt$ in the range $1 \lesssim t \lesssim 2$.

The remarkable fact is now that a simple dipole form

$$\gamma(t) = (1 + yt)^{-2} \quad (20)$$

gives a very satisfactory description of $d\sigma_{el}/dt$ throughout the range $0.2 \lesssim t \lesssim 8$. Adopting

$$y = 0.9 \text{ GeV}^{-2} \quad (21)$$

we find for

$$d\sigma_{el}/dt = \pi [\tau_{el}(t) \gamma(t)]^2 \quad (22)$$

the curve of Fig. 2, where the ordinate is in mb/GeV^2 . The second maximum has the value $6 \times 10^{-5} \text{ mb/GeV}^2$ (the experimental value given in Ref. 6) is $5.9 \times 10^{-5} \text{ mb/GeV}^2$), which is reached at $t = 1.74$ (the experimental value given in Ref. 6) is 1.8). For $2 \lesssim t \lesssim 8$ the drop of $d\sigma_{el}/dt$ is approximately exponential; the logarithmic slope reaches a maximum of 1.9 at $t = 2.6$ and decreases to 1.4 at $t = 8$, as compared to the mean value of 1.8 found

experimentally ⁶⁾. A more detailed picture of the logarithmic slope can be read from Fig. 3 which shows the logarithmic derivatives $d\ln\tau_{e\ell}/dt$ and $d\ln\tau_g/dt$, the difference being the logarithmic derivative $d\ln\gamma/dt$.

While the simple parametrizations adopted for τ_g and γ are in no way unique and are not intended to give a perfect fit to the data, they reproduce remarkably well the experimental facts over a wide range of t and some 10 orders of magnitude of $d\sigma_{e\ell}/dt$. Moreover, the explanation proposed for the occurrence of the diffraction minimum is straightforward - the scattering agent (the glue) is almost black over a small but finite range of impact parameter. As to $\gamma(t)$, it turns out to have a very simple shape. All this makes our quark-glue model approach to proton-proton scattering rather attractive. Some further aspects are discussed in the next section.

4. FURTHER REMARKS

In the context of the parametrization here adopted, the energy dependence of the elastic cross section can stem from a variation of the two parameters x and b_0 . The experimental observation that geometrical scaling is approximately correct over the ISR range suggests that the main energy variation is in b_0 , while x stays more or less constant. It should be noted, however, that varying b_0 at constant x in our model only gives exact geometrical scaling for the glue amplitude $\tau_g(t)$, because $\gamma(t)$ is independent of energy. As seen in Fig. 3, for t below the diffraction minimum, the logarithmic slope of $d\sigma_{e\ell}/dt$ is mostly due to τ_g , so that a variation of b_0 at constant x would give geometrical scaling of $d\sigma_{e\ell}/dt$ to a good approximation in this t range. The situation is different beyond the diffraction minimum where the form factor contributes about half of the logarithmic slope of $d\sigma_{e\ell}/dt$, and only the other half scales with b_0^2 .

We next address the question of possible additional diffraction minima beyond the one observed at $t = 1.3$. None has been found experimentally, nor does our parametrization give any. This cannot be considered as a general consequence of our model, however, because additional minima would occur with other parametrizations of the glue amplitude τ_g . For example, the (rather

unphysical) form

$$t_g(b_g) = \begin{cases} 1 & \text{for } b_g \leq b_1 \\ \exp \left[-(b_g^2 - b_1^2)/b_2^2 \right] & \text{for } b_g > b_1 \end{cases} \quad (23)$$

would give an infinite number of diffraction minima of $d\sigma_{el}/dt$. Smoothing the kink of this amplitude at b_1 would push the higher minima to larger t values without affecting much the first one, which is the only one to survive for a very smooth amplitude like the difference of two gaussians we have adopted in eq. (13). It is a characteristic of our model that the existence and position of additional minima are not directly related to those of the first one, because the latter is mainly controlled by the flatness of $t_g(b_g)$ at small b_g and not by the smoothness of its behaviour where it turns into a gaussian. This is in contrast with those models of elastic scattering which account for diffraction minima in terms of multiple scattering (interference between successive terms in multiple scattering expansion). In such models, for given position of the first minimum, the second one cannot easily be constrained to fall at very large t , and it is even more difficult to eliminate it altogether (see also the Appendix).

It is quite interesting that our model leads to the dipole form (20) for $\gamma(t)$. As mentioned earlier, $\gamma(t)$ is in reality the square of a proton form factor $g(t)$. This results from the relation

$$\begin{aligned} \Gamma(\vec{b}-\vec{b}_g) &= \int G(\vec{\beta}-\vec{\beta}_g) G'(\vec{\beta}'-\vec{\beta}_g) \delta^2(\vec{b}-\vec{b}_g-\vec{\beta}+\vec{\beta}_g+\vec{\beta}'-\vec{\beta}_g) \\ &\times d^2(\vec{\beta}-\vec{\beta}_g) d^2(\vec{\beta}'-\vec{\beta}_g) = \int G(\vec{b}-\vec{b}_g+\vec{b}') G'(\vec{b}') d^2\vec{b}' \end{aligned} \quad (24)$$

where $G = G'$ describes the impact parameter distribution of the glue inside the proton relative to the proton itself. This is eq. (14) of Ref. 4), with a misprint corrected. It gives

$$\gamma(t) = [g(t)]^2 \quad (25)$$

where $g(t)$, the Fourier-Bessel transform of $2\pi G$, is the proton form factor mentioned above.

From eqs. (20) and (21) we have the very simple result

$$g(t) = (1+yt)^{-1}, \quad y = 0.9 \text{ GeV}^{-2} \quad (26)$$

This "monopole" form of $g(t)$ implies that G has the value

$$G(\vec{\beta}-\vec{\beta}_g) = (2\pi y)^{-1} K_0(y^{-1/2} |\vec{\beta}-\vec{\beta}_g|) \quad (27)$$

$$K_0(b) = \frac{1}{2} \int_0^\infty \exp(-b^2/4u - u) du/u \quad (28)$$

where $K_0(b)$, the Fourier-Bessel transform of $(1+t)^{-1}$, is the Macdonald function of order 0. This function has a logarithmic singularity at $b = 0$, corresponding to the slow decrease of $g(t)$ at large t . It means that the glue in a proton is rather tightly bound to the position of the centre of the whole proton. One notes from eq. (28) that G is positive, as a probability distribution should be.

In conclusion, we would like to come back to the essential point of our work: by making only part of the proton constituents - in our specific model the glue - responsible for strong interaction collisions, one can achieve that the collisions of these active constituents are fully absorptive (opacity 1) at zero impact parameter. As shown in Ref. 4), this increases the amount of diffraction dissociation and accounts for the fact that the pp opacity at zero impact parameter remains practically constant at a value well below the maximum despite an increasing cross section. The present paper has shown that also the diffractive structure found in pp elastic scattering at ISR energies is easily accounted for in this approach: it follows from the natural assumption that the collisions of the active constituents (the glue in our model) have an opacity close to the maximum over a small but finite range of impact parameters. As a by-product, we obtain the simple "monopole" from (26) - (27) for the proton form factor describing the impact parameter of the glue with respect to the proton itself.

It is clear that for the results just summarized, the particular identification of the hadronically active constituents with the glue is not an essential feature of the argument, all what is needed being that only part of the proton constituents are active in the hadronic collisions considered. The identification with the glue was based on other arguments, especially those related to the leading particle effect in non-diffractive collisions²⁾. It should also be recalled that the hadronic collisions relevant to our study are the normal ones of the so-called low p_T type. The high p_T collisions, which may very well be effects associated with quarks, have so small a cross section that their shadow is unimportant for a discussion of elastic and inelastic diffraction.

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APPENDIX

To illustrate the general comments of section 4 on the question of additional diffraction minima, we mention here a few properties of the parametrization (23) for the glue amplitude. The conditions (16) and (17) fixing the value of the forward scattering amplitude imply for the parametrization (23)

$$b_1^2 + b_2^2 = 16 \text{ GeV}^{-2}$$

The values of b_1 and b_2 are then obtained by constraining the first diffraction minimum, i.e. the first zero of the Fourier-Bessel transform of (23), to fall at $t = 1.32$. One finds $b_1 = 1.6 \text{ GeV}^{-1}$. The further zeroes determine the additional diffraction minima. The second and third ones are found to fall at $t = 10.6$ and 28.0 , confirming that they are very high compared to what is normally obtained in multiple scattering models.

If one adopts for $\gamma(t)$ the dipole form (20) with $y = 1.2 \text{ GeV}^{-2}$, one obtains a differential cross section $d\sigma_{el}/dt$ which is very close to experiment and to the parametrization of sections 2 and 3 in the range $t \lesssim 2$, i.e. up to

and including the second diffraction maximum. For $2 \lesssim t \lesssim 8$ the resulting $d\sigma_{el}/dt$ decreases approximately as an exponential, but the logarithmic slope is ~ 1.4 , definitely lower than the experimental value of 1.8. This could be improved by going over to more complicated forms of $\gamma(t)$.

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FIGURE CAPTIONS

Figure 1 The glue amplitude $t_g(b_g)$ of eq. (13) for the values $x = 0.133$ and $b_o = 16 \text{ GeV}^{-2}$ used in our description of pp elastic scattering. The broken line represents the gaussian in the first term of eq. (13).

Figure 2 The differential elastic cross section with τ_g and γ given by eqs. (15) and (20) for $x = 0.133$, $b_o^2 = 16 \text{ GeV}^{-2}$, $y = 0.9 \text{ GeV}^{-2}$.

Figure 3a The logarithmic derivatives $-\tau_{el}^{-1} d\tau_{el}/dt$ (full line) and $-\tau_g^{-1} d\tau_g/dt$ (broken line) for the same parameters as in Fig. 2.

Figure 3b Same as Fig. 3a with vertical scale enlarged.

FIG. 1

FIG. 2

FIG. 3a

FIG. 3b