Ref.TH.2202-CERN

OPERATOR FORMULATION OF THE COULOMB INTERFERENCE PROBLEM

AT SPS AND ISR ENERGIES

L. Matsson
CERN -- Geneva

A B S T R A C T

An operator formulation of the Coulomb interference effect at high energies is presented. This provides a gauge and translation invariant solution to the indeterminacy problem of the Bethe phase.

Given the shape of electromagnetic form factors this technique permits extraction of electromagnetic effects with a theoretical error less than 1.5% in the hadronic phase of the forward elastic amplitude.

Ref.TH.2202-CERN

2 August 1976

INTRODUCTION

The optical theorem combined with dispersion relation techniques provides a possibility to test self-consistency in hadronic theories if the pure hadronic phase for zero momentum transfer in the elastic amplitude can be determined.

We recall that electromagnetic (EM) and strong forces are of the same order of magnitude around $t_0 \sim -0.003 (\text{GeV}/c)^2$ ¹⁾. For $t < t_0$ the EM interaction dominates, and a careful separation of all EM effects is needed before the pure hadronic phase can be obtained.

Interest in these studies has been greatly stimulated by the discovery of rising total cross-sections at ISR and NAL. It is clear that experiments must be done with very high precision ²⁾ in order to distinguish between $\sigma_{\text{tot}} \sim (\log s)^2$, $\log s$ or something similar. However, before these data can be used, the same degree of precision is required in the separation of EM effects. Such a separation has not yet been possible because of strongly model, energy and gauge dependent photon recoil effects.

This ambiguity is perhaps best understood by observing the big discrepancy between the result from potential scattering theory ³⁾ and the relativistic result derived from QED with pointlike particles and no photon recoil in the basic process ⁴⁾. At typical SPS energies it corresponds to $\sim 5\%$ difference in the hadronic phase.

In some sense the sources of this ambiguity were indicated by including photon recoil in the basic process ^{5),6)} (Fig.1a) or by including EM form factors ⁷⁾. To understand nature we should of course include both. Further, the recoil treatment of Refs. 5),6) is incomplete since only planar graphs (Fig.1b) have been investigated in this respect. Such graphs do have a gauge invariant photon recoil in the t behaviour of the basic process and the corresponding s recoil is zero.

The situation for t recoil in criss-crossed graphs (Fig.1c) is the same but the s recoil is then non-zero and non-gauge invariant. In passing we also notice that criss-crossed graphs have no correspondence in potential theory.

The strong forward peak is in most cases s dependent ^{8),9)} and we must therefore retain all photon recoil effects in order to have four-momentum conservation, i.e., to obtain a translation invariant result. Since we believe

in conservation of both charge and four-momentum, a certain "leakage" current must be induced to restore current conservation. Such a leakage obviously corresponds to the case where the EM current couples directly to the strong kernel (e.g., Fig.1f), a situation not included in pointlike QED.

On the basis of the results of Refs. 5)-7), which do not consider such contributions, it is therefore hard to say if the true result is closer to one or the other of the results of Refs. 3) or 4). At least, the no-recoil approximation in Ref. 4) is consequent and the result is certainly gauge invariant. Thus, without a translation and gauge invariant solution which includes all recoil effects, it seems that we still do not understand the big discrepancy between the results of Refs. 3) and 4), which corresponds to a $\sim 5\%$ discrepancy in the hadronic phase ¹⁰⁾.

Needless to say, a theoretical indeterminacy of that order is unacceptable and directly incompatible with the experimental situation with an expected error of the order of 1% to 2.5% at SPS and ISR, respectively. Before going into the solution, it is in order to mention that the sources to these leakage effects have been listed and in some cases estimated ¹¹⁾⁻¹³⁾. Before starting, notice also that our separation of strong and EM effects in the form factors, i.e., the blobs in the EM vertices in Fig. 1 and Fig. 2, is compatible with the separation of the strong blob in Fig. 1a from the rest.

We construct here a general formalism which resolves this ambiguity. EM effects are exactly treated to order α in the strong amplitude (Fig.1), to order α^2 in the "non-strong" part (Fig.2), and to infinite order in the infra-red (IR) sense. In this problem we are forced to carry out the IR exponentiation to avoid finite order logarithmic singularities at vanishing momentum transfer. Further, the exponentiation must be carried out uniquely in order to eliminate an indeterminacy of the type $\exp S \rightarrow \exp(S+C)$; otherwise we cannot say whether C is of strong or electromagnetic origin. Both S and C are here of order α and S is the usual gauge invariant eikonal part ¹⁴⁾.

Also in the case of pointlike QED the rules should be general enough to allow for an evaluation of C rather than be introduced ad hoc. In fact, C appears here because exponentiation has been obtained by a formal manipulation of the integrated ordinary perturbation expansion (OPE), truncated at some finite order. By means of a correlation rearrangement of OPE with respect to photon momenta, this problem is resolved in a gauge and translation invariant way by requiring exponentiation of all factorizable parts before integration ^{15),16)}. With these rules it can be proved that S represents all totally factorizable α order effects of the full perturbation expansion.

In the present problem not only the question of uniqueness is essential, but also that our formulation is gauge invariant, because this seems to be the only available way to control the leakage terms (Figs. 1f-1g). In other words, charge must be conserved independently of the shape of the conductor, i.e., irrespective of the form of the bare strong amplitude.

The correlations in this rearrangement ^{15),16)} are defined with respect to internal photon momenta before integration. In principle, OPE lends itself to such a rearrangement to infinite correlation order c and infinite order in α without any approximation, whereas this can of course not be performed in practice. However, since all correlations are IR regular ^{15),16)}, and due to the fact that they are of the order of α^c , with $c \geq 2$ relative to the bare strong kernel (Fig.1a) and the one-photon photon exchange (Fig.2a), it follows that explicit knowledge of some lower order correlations is enough, e.g., in this case inclusion of single correlations ($c = 1$) suffices ¹⁷⁾.

For the purely strong amplitude near forward scattering we assume an amplitude of Pomeron-type. Apart from logarithmic corrections, we obtain

$$A_h(s, t) \sim \beta(t) (g(t) + i) \left(\frac{s}{s_0}\right)^{\alpha_0 + \alpha' t} \quad (1.1)$$

where t dependence in the hadronic phase $\rho(t)$ can be neglected at these small t values ¹⁸⁾

$$g(0) = \frac{\text{Re } A_h(s, 0)}{\text{Im } A_h(s, 0)} \simeq \frac{\text{Re } A_h(s, 0)}{A_h(s, 0)} \quad (1.2)$$

We further insert

$$\beta(t) = \beta(0) \exp(R_A^2 + R_B^2)t \quad (1.3)$$

and normalize such that

$$\frac{d\sigma}{dt} = \frac{\sigma_{tot}^2(s)}{16\pi} (1 + g^2) e^{2\eta_h t} \quad (1.4)$$

Then η_h is defined by

$$\eta_h = R_A^2 + R_B^2 + \alpha' \ln(s/s_0) \quad (1.5)$$

with $R_A^2 \sim R_B^2 \sim 2(\text{GeV}/c)^{-2}$ and $\alpha' \sim 1/4$ the slope parameter η_h is of the order of 5 or $6(\text{GeV}/c)^{-2}$ at typical SPS or ISR energies, i.e., $200(\text{GeV}/c)^2$ or $1000(\text{GeV}/c)^2$, respectively.

It is well known that this is an excellent parametrization in the small t region. In this paper we shall not worry about the exact s behaviour of the slope parameter which in general could be different from that of (1.5). In what follows, let us just recall the form of a Regge-like amplitude

$$A_h(s, t) = \mathcal{F}(s) \exp(\eta_h(s) t) \quad (1.6)$$

where $\mathcal{F}(s)$ is essentially linear in s , in order not to violate the Froissart bound.

As discussed above, a direct combination of a strong model and point-like QED is not in general compatible with gauge invariance without leakage terms (Figs. 1f-1g). Here we show that it is possible to derive these leakage terms by a generalization^{19),20)} of Low's theorem²¹⁾, going via our correlation rearrangement^{15),16)}.

The paper is organized as follows. In Section 2, we describe the correlation rearrangement of OPE in terms of formulas obtained in Ref. 15) and Ref. 16), here modified to account for the presence of (1.6) and form factors. In Section 3 we prove this generalized form and in Section 4 we sum up the important points and discuss consequences.

2. ELECTROMAGNETIC EFFECTS

The second order EM amplitude (Fig. 3) can be written in the form

$$\begin{aligned} A_c(s, t) &= ie \frac{1}{q^2} F_A(q^2) F_B(q^2) = \\ &= ie \int d^4x \mathcal{F}_{AB}(x) e^{iqx} V_\mu^A V^{B\mu} \end{aligned} \quad (2.1)$$

For this tree graph, it is suitable to use exponential form factors near $q^2 = 0$

$$\exp \eta_c t = F_A(t) F_B(t) ; t = q^2 \quad (2.2)$$

$$F_Q(t) = \exp(R_Q^2 t); Q = A \text{ or } B ; \eta_c = R_A^2 + R_B^2 \quad (2.3)$$

whereas on the loop level the result is strongly dependent on the choice of $F(t)$ ⁷⁾.

We have here assumed that the charge radius is the same as the strong radius $\eta_c \sim 4(\text{GeV}/c)^{-2}$ given by (1.3) and (2.3). The index sets A and B in (2.1) to (2.3) are given by $\{a, a'\}$ and $\{b, b'\}$ respectively, where these indices label the momenta of external prongs in Fig. 3. The vertices V_μ^Q are defined by

$$V_\mu^Q = \begin{cases} \bar{u}(p_i') \gamma_\mu u(p_i) & \text{if (i) is } \begin{cases} \text{spinor} \\ \text{scalar} \end{cases} \\ (p_i + p_i')_\mu & \end{cases} \quad (2.4)$$

Loop effects are included by means of the correlation rearranged OPE ^{15), 16)}, where pair and higher correlations can be neglected. The corresponding summed up amplitude is given by

$$M_c(s, t) = ie^2 \int d^4x e^{iqx} F_{AB}(x) \cdot \left\{ 1 + \frac{i}{1!} \left(S^e(x) \frac{\delta}{\delta(iu)} + \hat{S}^r \right) \right\} \frac{e^{iU(x)} - 1}{iU(x)} e^{i\hat{R}} V_\mu^A V^{B\mu} \quad (2.5)$$

The photon propagator $\Delta_F^1(x)$ is everywhere regularized and dressed to order α (Fig.4). The renormalized quantities $\hat{U}$, $\hat{R}$, $\hat{S}^e$ and $\hat{S}^r$ are here represented by $\mathcal{O}$ and defined by

$$\lim_{p_i^2 = m_i^2} \mathcal{O} = \mathcal{O}_0 - \mathcal{O}_0^{S.P.}; \mathcal{O}_0 = \hat{U}_0, \hat{R}_0, \hat{S}_0^e \text{ or } \hat{S}_0^r \quad (2.6a)$$

where S.P. is an arbitrary off-shell subtraction point

$$\hat{U}_0(x) = i \int \frac{d^4 k}{(2\pi)^4} \Delta'_F(k) e^{-ikx} \hat{J}_\mu^A(+k) \hat{J}^{B\mu}(-k) F_A(k^2) F_B(k^2) \quad (2.7)$$

$$\begin{aligned} \hat{R}_0 = i/2 \int \frac{d^4 k}{(2\pi)^4} \Delta'_F(k) \{ & \hat{J}_\mu^A(+k) \hat{J}^{A\mu}(-k) [F_A(k^2)]^2 + \\ & + \hat{J}_\mu^B(+k) \hat{J}^{B\mu}(-k) [F_B(k^2)]^2 \} \end{aligned} \quad (2.8)$$

$$\begin{aligned} \hat{S}_0^e(x) = i \int \frac{d^4 k}{(2\pi)^4} \Delta'_F(k) e^{-ikx} \{ & \hat{S}_\mu^A(+k) \hat{J}^{B\mu}(-k) + \\ & + \hat{J}_\mu^A(+k) \hat{S}^{B\mu}(-k) + \hat{S}_\mu^A(+k) \hat{S}^{B\mu}(-k) \} F_A(k^2) F_B(k^2) \end{aligned} \quad (2.9)$$

$$\begin{aligned} \hat{S}_0^r = i/2 \int \frac{d^4 k}{(2\pi)^4} \Delta'_F(k) \{ & [\hat{S}_\mu^A(+k) \hat{J}^{A\mu}(-k) + \hat{J}_\mu^A(+k) \hat{S}^{A\mu}(-k) \\ & + \hat{S}_\mu^A(+k) \hat{S}^{A\mu}(-k)] [F_A(k^2)]^2 + (A \rightarrow B) \} \end{aligned} \quad (2.10)$$

In the presence of EM form factors all photons are essentially soft except for certain self-energies of sea-gull type (fig.2e) in the case of scalar electrodynamics. But because the rearrangement is gauge invariant, all such effects are absorbed by a mass renormalization [see, e.g., Section 3 Ref. 15]. We can then make the approximation

$$\mathcal{O} \simeq \mathcal{O}_0 \quad (2.6b)$$

to which we make error estimates in the summary.

The currents are defined by

$$\hat{J}_\mu^A = \hat{f}_\mu^q + \hat{f}_\mu^{q'}; \quad \hat{S}_\mu^A = \hat{S}_\mu^q + \hat{S}_\mu^{q'} \quad (A \rightarrow B) \quad (2.11)$$

$$\hat{f}_\mu^i = f_\mu^i + f_\mu^i \hat{k}^i - ie \frac{\partial}{\partial p_i^i} \quad ; \quad f_\mu^i = \frac{2p_\mu^i + \varepsilon^i k_\mu}{2\varepsilon^i p_i \cdot k + k^2 + p_i^2 - m_i^2} \quad (2.12)$$

$$\hat{S}_\mu^i = ie \frac{\varepsilon^i/2 [\gamma_\mu, \hat{k}] - (\hat{p}_i - m_i) \gamma_\mu}{2\varepsilon^i p_i \cdot k + k^2 + p_i^2 - m_i^2} (1 + \hat{k}^i), \quad \hat{k}^i = \varepsilon^i k \cdot \frac{\partial}{\partial p_i} \quad (2.13)$$

and are obtained in the same way as in Refs. 15) and 16). In Section 3 we show that these currents also take care of the photon recoil in the strong kernel. The operator notation on the spin current refers to the derivative operators and also implies that all components $\hat{S}_\mu^i$ should be inserted in the spinor part of (2.1) in the same order as in OPE. The label ε_i is +1 (-1) for out- (in-) going particle p_i . It should be noticed that the spin current (2.13) is conserved on-shell without addition of any leakage, whereas the convection current (2.12) had to be corrected in this respect.

Clearly if A is a spinor and B is a scalar particle, all $\hat{S}_\mu^B$ currents drop out in (2.9) and (2.10) and of course if both are scalars, then both $\hat{S}_\mu^A$ and $\hat{S}_\mu^B$ disappear. In an analysis of experimental data ²²⁾, it has been shown that spin dependent effects in $\text{Re } A_h$ are less than 1% of A_h . We therefore neglect all spin currents (2.13) and obtain

$$M_e(s,t) = ie^2 \int d^4x e^{iqx} \mathcal{F}_{AB}(x) \cdot \int_0^1 d\theta e^{i\theta \hat{U}(x)} e^{iR} V_\mu^A V_\mu^B \quad (2.14)$$

At higher t values one of the photons should be treated as different in the symmetrization leading to the replacement $(\exp i\hat{U} - 1)(i\hat{U})^{-1} \rightarrow \exp i\hat{U}$ in (2.5) or insertion of a $\delta(\theta - 1)$ function in (2.14).

The corresponding strong amplitude with EM corrections is, if we first include spin currents, given by [see section 3]

$$M_h(s,t) = e^{i\hat{U}(0)} e^{iR} \left\{ 1 + \frac{i}{1!} (\hat{S}_e(0) \frac{\delta}{\delta(iu)} + \hat{S}_r) \right\} A_h(s,t) \quad (2.15)$$

Neglecting all spin currents, we obtain

$$M_h(s, t) = e^{i\hat{U}(0)} e^{i\hat{R}} A_h(s, t) \quad (2.16)$$

The operators $\hat{R}$, $\hat{U}(x)$, $\hat{S}_r$ and $\hat{S}_e(x)$ are also in this case defined by (2.6) to (2.10) and the graphical interpretations of M_h and M_c are illustrated in Fig. 5 and Fig. 6, respectively. The subclass of generalized ladders (Fig. 7) is generated by the $\hat{U}$ operator (2.7) and the remaining vertex and self-energy corrections (Fig. 8) are generated by the $\hat{R}$ operator (2.8). The last term of $\hat{f}_u^i$ in (2.12) generates graphs in which emission and absorption occur directly from the hadronic kernel (Fig. 5). The full amplitude is given by the sum of (2.14) and (2.16)

$$M = M_c + M_h$$

3. PHOTON RECOIL EFFECTS

In order to study the photon recoil from an attached virtual photon with momentum k (Fig. 9) in an arbitrary hadronic kernel $A_h(\{p_i\})$, we assume that the off-shell A_{hp_i} is sufficiently well-behaved to make a Taylor expansion in k meaningful. The corresponding recoiling amplitude is then proportional to

$$\begin{aligned} i e \frac{2p'_\mu + \varepsilon'_\mu k_\mu}{2\varepsilon i p_i \cdot k + k^2} A_h(\{p_i + \varepsilon i k\}) &= \\ &= f_\mu^i(k) e^{\varepsilon i k \cdot \frac{\partial}{\partial p_i}} A_h(\{p_i\}) \end{aligned} \quad (3.1)$$

which, however, is not gauge invariant for all functions A_h . In passing, we first notice that this form permits factorization of all photon recoil effects, which is necessary for summability into the closed expressions (2.5) and (2.15).

Clearly, the no-recoil approximation, i.e., the eikonal model, is gauge invariant since

$$\left(\sum_{i \in Q} k \cdot f^i \right) A_h = \left(\sum_{i \in Q} \varepsilon_i \right) A_h = 0; \quad Q = A \text{ or } B \quad (3.2)$$

This corresponds to the result of Ref. 4). In the case when A_h is a function only of t (3.1) is gauge invariant without leakage because

$$\varepsilon^i k \cdot \frac{\partial}{\partial p_i} = \varepsilon^i k \cdot \frac{dt}{dp_i} \frac{\partial}{\partial t} = -k \cdot q \frac{\partial}{\partial t} \quad (3.3)$$

is independent of i , in which case current conservation is demonstrated as in (3.2).

In general, however, A_h is also a function of s ⁸⁾, e.g., like in (1.6), in which case (3.1) is not gauge invariant. This easily follows from a derivation in the s channel similar to (3.3). Thus without calculating the leakage terms of the type illustrated in Figs. 1f-1g, required to restore gauge invariance, we cannot say anything about the status of the results of Refs. 5), 6) and 7).

In order to derive the leakage parts, with a certain care in the choice of integration contour, we can make use of the EM form factors which strongly suppress all non-soft photon momenta. Therefore the photon recoil current in (3.1) can be approximated by a first order expansion in k

$$f_\mu^i(k) e^{\varepsilon^i k \cdot \frac{\partial}{\partial p_i}} \simeq f_\mu^i(k) \left(1 + \varepsilon^i k \cdot \frac{\partial}{\partial p_i} \right) \quad (3.4)$$

Moreover, a discussion of higher order contributions, i.e., the details of the non-soft contributions, would clearly be highly fictitious since then we would not know the relevance of separating strong and EM interactions. This is intimately connected with the question of renormalizability of the joint theory, i.e., subtractions of the same order of magnitude as next order recoil [see, e.g., Section 3 in Ref. 15].

Thus the problem with non-conserved hard virtual photons is reduced to a corresponding approximately soft one. Further, since we can always choose to renormalize on the mass shell of particle (i) ^{15), 16)} after leakage compensation, all currents must in fact be conserved. In other words, the problem is reduced to one which can be solved by the generalized Low theorem ¹⁹⁾.

In order to restore gauge invariance in (3.4), we add a leakage current ℓ_μ^i and require

$$k_\mu \cdot \left(f_\mu^i \varepsilon^i k \cdot \frac{\partial}{\partial p_i} + \ell_\mu^i \right) = 0 \quad (3.5)$$

By use of $k \cdot f^i = \varepsilon^i$ we then obtain

$$k \cdot \ell^i = - k \cdot \frac{\partial}{\partial p_i} \Rightarrow \ell_\mu^i = - \frac{\partial}{\partial p_{i,\mu}} \quad (3.6)$$

which demonstrates that (2.12) defines the complete current. Here we neglect undetermined terms of order k_μ in (3.6) for the same reason as we approximated (3.4). The graphical meaning of ℓ_μ^i is illustrated in Fig. 10.

However, with the full current $\hat{f}_\mu^i$ (2.12) acting on both s and t in A_h , this corresponds to the fact that a leakage has been added also to the t recoil! As we discussed before, this should not be necessary. In fact, this problem is quite harmless because

$$\frac{\partial}{\partial p_{i,\mu}^t} = \frac{\partial t}{\partial p_{i,\mu}^t} \cdot \frac{\partial}{\partial t} = -2 \varepsilon^i q_\mu \frac{\partial}{\partial t} \quad (3.7)$$

Summing over in- and outgoing particles (3.7) adds up to zero. It is also useful to keep this symmetric form of the radiative corrections, i.e., to retain these terms despite the fact that they are zero, if we want to use crossing invariance in the strong model.

Further, we could also expect off-mass-shell effects (Figs. 11a-11b). However, the essentially soft nature of the virtual photons implies that

$$\begin{aligned} \frac{p_\mu^i}{p_{i,k}} k \cdot \frac{\partial}{\partial p_i} - \frac{\partial}{\partial p_\mu^i} &= \left(\frac{p_\mu^i}{p_{i,k}} k \cdot \frac{\partial p_i^2}{\partial p_i} - \right. \\ &\left. - \frac{\partial p_i^2}{\partial p_\mu^i} \right) \frac{\partial}{\partial p_i^2} = (2 p_\mu^i - 2 p_\mu^i) \frac{\partial}{\partial p_i^2} = 0 \end{aligned} \quad (3.8)$$

This cancellation was observed already in Ref. 23).

We should also think of a situation where the large momentum, going through the strong kernel (1.6), drastically changes the basic process, e.g., into some Regge exchange. This, however, should be accounted for already in the bare strong amplitude where we can consider interference effects of Reggeons and a Pomeron exchange. The basic process is then a sum of terms of the type (1.6) and again the same EM machinery applies. Triple Reggeon effects and higher orders can be neglected at these high energies for what concerns EM corrections, whereas in the purely strong part all these neglected effects, spin dependence, etc., must, of course, be carefully examined.

4. SUMMARY AND OUTLOOK

A gauge and translation invariant solution to the indeterminacy problem of the so-called Bethe phase is obtained. The summed up result is given in terms of two closed expressions for the summed up amplitudes, i.e., for the hadronic amplitude with EM corrections (Fig.1)

$$M_h(s,t) = e^{i\hat{U}(0)} e^{i\hat{R}} A_h(s,t) \quad (4.1)$$

and for the corresponding Coulomb part (Fig.2)

$$M_c(s,t) = ie^2 \int d^4x e^{iq \cdot x} F_{AB}(x) \cdot \int_0^1 d\theta e^{i\theta \hat{U}(x)} e^{i\hat{R}} V_\mu^\dagger V^{\beta\mu} \quad (4.2a)$$

The quantities $\hat{U}(x)$, $\hat{R}$, $F_{AB}(x)$ and V_μ^i are defined in Section 2, and A_h by (1.6). At higher t values we might also consider the case with one photon different ²⁴⁾⁻²⁷⁾, and obtain

$$M_c(s,t) = ie^2 \int d^4x e^{iq \cdot x} F_{AB}(x) e^{i\hat{U}(x)} e^{i\hat{R}} V_\mu^\dagger V^{\beta\mu} \quad (4.2b)$$

After charge renormalization, we could then also drop the α order correction to the photon propagator in (4.1) and (4.2), (Fig.4). Working with essentially soft photons, we know that the corresponding error is quadratic in k^2 . As a correction to the single photon exchange (Fig.2f), it therefore gives an error of the same order as terms neglected in (2.6b), (3.4) and (3.6). In all other graphs it is an α^2 order correction which is totally negligible.

In order to systematize and estimate the errors, we observe that our rearrangement in presence of EM form factors is essentially a power expansion in the "soft" photon momentum k . Each completely factorizable integrand, apart from the form factors, is of the form

$$\int dk \{ C_{-1} k^{-1} + C_0 k^0 + C_1 k^1 + \dots \} \quad (4.3)$$

where the powers of k indicate essential powers; e.g., $C_0 k^0$ could be of the form $p^i \cdot k / p^j \cdot k$. However, clearly the ultra-violet behaviour is much better than this already without form factors. It is important to notice here that the full OPE contains more contributions of order $C_0 k^0$. However, the structure of our rearrangement shows that such additional effects are at least pair correlated and therefore α^2 order corrections or higher.

With an EM form factor squared $(1+R^2 k^2)^{-2n}$, $n=1$ or 2 ⁷⁾, in each phase space it is clear that only some few of the first terms in (4.3) make sense. Here we have used only the two first terms, i.e., the usual logarithmic term $\sim C_{-1}$ and the first order recoil and leakage $\sim C_0$, whereas we neglect C_1 and higher order terms.

Estimating the contributions to $\rho(0)$ from C_1 by collecting the integrated contributions from the first order corrections to (2.6b), (3.4), (3.6), and adding the vacuum polarization correction, we arrive at the number 0.7% ¹⁰⁾. The corresponding error from the neglect of EM spin currents (2.9), (2.10) and anomalous magnetic moments is estimated to 0.8% . This gives a total theoretical error 1.5% in $\rho(0)$, provided that the shape of the EM form factor is given.

Although it has been shown that $\rho(0)$ depends considerably on the choice of EM form factors⁷⁾, our theoretical error in extracting EM effects is roughly independent of such variations. Also notice that our definition of form factors does not include any photons, in order to avoid double counting, i.e., EM effects are in fact separated.

Thus, provided we know the shape of form factors, our theoretical indeterminacy is of the same order as the experimental error and we can get almost purely hadronic information about $\rho(s,0)$, its zeros²⁸⁾, EM renormalization of particle slopes and intercepts, energy dependence of the forward peak slope parameter (1.6), etc. Having calculated EM corrections to both real and absorptive parts of A_h (2.14), (2.16), we could then in principle test hadronic theories via the optical theorem by dispersion relation techniques.

Conversely, if we believe in existing hadronic models, recalling that different $F(k^2)$ could give a $\sim 5\%$ difference in $\rho(0)$ ⁷⁾ we might have a small chance to discriminate among various form factor candidates.

As we saw in Section 2, we could roughly ignore the effect of normalization (2.6b), with a corresponding error of the order of 0.2%. From a rigorous point of view, however, we might require renormalizability for the joint EM and strong theory, which could, in principle, help to discriminate among various strong models already at the present level of accuracy, where EM and strong effects are approximately separable in the above sense, i.e., what concerns the hadronic phase within the order of 1.5% error. In a subsequent paper, we present the integrated EM effects of (4.1) and (4.2) in a new Coulomb interference formula.

As a final remark, we repeat the observation made in Ref. 15) in the case of scalar electrodynamics, i.e., in the case of scattering of two scalar particles. Pair corrections contain IR divergences for $t=0$ so care must be taken when including such effects.

ACKNOWLEDGEMENTS

I would like to thank D. Amati, J.S. Bell, N. Craigie, A.M. Din, T.E.O. Ericson and M. Jacob for useful discussions, and R.J. Crewther for reading the manuscript. I also thank U. Amaldi, T. Ekelöf, S. Kullander and A.M. Wetherell for illuminating discussions on the experimental situations at ISR and SPS.

REFERENCES AND FOOTNOTES

- 1) For Kp^- , πp and pp scattering at $t \sim -0.0037$, $t \sim -0.0029$ and $t \sim -0.0018$ $(\text{GeV}/c)^2$, respectively, roughly energy independent above 10 GeV/c .
- 2) T. Ekelöf - Nuovo Cimento 31A, 505 (1976).
- 3) H. Bethe - Ann.Phys. 3, 190 (1958).
- 4) L.D. Soloviev - Soviet Phys. JETP 22, 205 (1966) ;
L.D. Soloviev and A.V. Shchelkachev - Soviet Phys.Nucle.Phys. 11, 240 (1970).
- 5) J. Rix and R.M. Thaler - Phys.Rev. 152, 1357 (1966).
- 6) G.B. West and D.R. Yennie - Phys.Rev. 172, 1413 (1968).
- 7) M.P. Locher - Nuclear Phys. B2, 525 (1967).
- 8) U. Amaldi, M. Jacob and G. Matthiae - Ann.Rev.Nucl.Sci., Vol. 26, (1976).
- 9) In the Coulomb interference region Kp^- , πp and pp scattering have been measured up to 14, 60 and 500 (GeV/c) , respectively.
- 10) Relative to the norm of the strong amplitude ; $\Delta \rho(0) = \Delta \text{Re } A_h / \text{Im } A_h \simeq \simeq \Delta \text{Re } A_h / A_h \simeq \alpha \Delta \phi_{\text{Bethe}}$.
- 11) V.G. Gorshkov, E.A. Kuraev, L.M. Lipatov and M.M. Nesterov - Soviet Phys.JETP 33, 655 (1971).
- 12) V.B. Kopeliovich - Yad.Fiz. 18, 406 (1973).
- 13) B. Tromborg and J. Hamilton - Nuclear Phys. B76, 483 (1974).
This paper also contains a more complete list of references on this subject.
- 14) D.R. Yennie, S.C. Frautschi and H. Suura - Ann.Phys.(N.Y.) 13, 379 (1961).
- 15) L. Matsson - CERN Preprint TH. 2168 (1976).
- 16) L. Matsson - CERN Preprint TH. 2177 (1976).
- 17) OPE is then exactly reproduced to order α^2 in the non-strong amplitude (Fig. 2) to order α in the strong amplitude (Fig. 1) and to infinite order in α in the IR sense.
- 18) This is because the variation in t of $\rho(t)$ is proportional to the Reggeon slope which is in general one order of magnitude smaller than η .

- 19) L. Matsson - Phys.Rev. D9, 2894 (1974) ; ibid D10, 2027 (1974).
- 20) Part of this generalization in case of real emission was formulated in :
K.E. Eriksson, L. Matsson and G. Peressutti - ITP Preprint 71-2, Göteborg (1971), unpublished.
- 21) F.E. Low - Phys.Rev. 110, 947 (1958).
- 22) C. Bourrely, J. Soffer and D. Wray - CNRS Preprint, Marseille (1974), p. 597.
- 23) H. Feshbach and D.R. Yennie - Nuclear Phys. 37, 150 (1962).
- 24) L.I. Schiff - Phys.Rev. 103, 443 (1956).
- 25) M.M. Islam - Nuovo Cimento 5A, 315 (1971).
- 26) J.L. Cardy - Nuclear Phys. B33, 139 (1971).
- 27) I.G. Halliday, J. Huskins and C.T. Sachrajda - Nuclear Phys. B83, 189 (1974).
- 28) N.N. Khuri and T. Kinoshita - Phys.Rev. 137B, 720 (1965) ; ibid 140B, 706 (1965).

FIGURE CAPTIONS

- Figure 1 Virtual radiative corrections to a strong kernel.
- Figure 2 One-photon exchange amplitude with virtual radiative corrections.
- Figure 3 One-photon exchange.
- Figure 4 Dressed photon propagator.
- Figure 5 Strong amplitude with virtual corrections of arbitrary order.
- Figure 6 Pure electromagnetic amplitude of arbitrary order.
- Figure 7 Generalized ladder graph.
- Figure 8 Vertex and self-energy corrections.
- Figure 9 Virtual "emission" from external prong.
- Figure 10 "Emission" from the strong kernel.
- Figure 11 Diffractive excitation.

FIG.1

FIG.2

FIG.3

$$\text{wavy line} \times \square \times \text{wavy line} = \text{wavy line} \times \text{wavy line} \times \text{wavy line} + \text{wavy line} \times \bigcirc \times \text{wavy line} + \dots$$

FIG.4

FIG.5

FIG.6

FIG. 7

FIG. 8

FIG. 9

FIG. 10

(a)

(b)

FIG. 11