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A PHENOMENOLOGICAL ANALYSIS OF ψ PARTICLE DECAYS

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A B S T R A C T

We use standard ideas on meson decays (internal and spin symmetries of the quark model, and vector meson dominance) to analyze data on a large class of decays of the ψ and related particles in the charm model. We find no evidence against exact SU(4) for hadronic vertices or against vector dominance. SU(4) must then be broken in photon couplings and a systematic rule emerges. The level of consistency with experiment is suggestive that the ψ behaves similarly to other hadrons.

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1. INTRODUCTION

In this paper we apply standard ideas on meson decays to a large class containing almost all two-body decays of the ψ and some related particles. Our purpose is to discover whether the properties of ψ -particles bear any resemblance to the established phenomenology of ordinary hadrons, apart from the question of their small widths.

We make use of general algebraic aspects of the quark model (internal symmetries and spin symmetries) that are relatively free of the most uncertain features of quark models, i.e. of explicit wave functions and quark dynamics. Of necessity, considerable use is also made of vector meson dominance. We fully recognize that the large extrapolations involved could generate large deviations from these models: this is one of the principal questions to which we address the data.

The decays considered (there are two minor exceptions) are those which for kinematical reasons can proceed only through a single partial wave: their Lorentz structures are unique. These cases have the least sensitivity to explicit quark motions, but this restriction prevents us from discussing in the present context a number of interesting decays, such as $\psi' \rightarrow \chi\gamma$, $\psi \rightarrow \pi^+\pi^-\omega(\phi)$, etc.

We assume the charm¹⁾ ($c\bar{c}$) interpretation of the ψ (3.1), with ψ' (3.7) a radial excitation; higher ψ -bumps are not discussed. For their small widths, we assume the validity of the Okubo-Zweig-Iizuka rule²⁾, and the near purity of these states, with small mixing of ordinary quark pairs as the origin of decays into ordinary hadrons³⁾. Considerable attention is focussed on this mixing in the ψ , and on mixing amongst η , η' and χ (2.8) [identified as the η_c].

As is widely recognized, unitarity forces an intrinsic breaking of the Zweig rule (as it does for SU(6) symmetries), even if it is exact in lowest order. Physical mesons then deviate from ideal quark states, and we presume it to be reasonable to take the Zweig rule as exact in the vertices connecting these mixed states. In this view, breaking of the Zweig rule has a universal property for a given meson.

It is clear that our choice of a phenomenological approach has limited predictive power^{*)}. Nevertheless there are situations where clear contradictions with experiment could show up but in fact do not; and one case where the model is ambiguous and is indeed contradicted by the data. On balance we feel that the level of consistency with experiment is quite surprising and lends credence to the view that the ψ -particles are quark systems like ordinary hadrons.

*) The reader can count that input : output $\approx$ 1:1.

A number of other authors⁴⁻¹⁴⁾ have made similar studies from various points of view. Our approach is different in that we argue directly from the data, searching for regularities and a simple over-all view, irrespective of the detailed nature of quark or hadron¹⁴⁾ dynamics. Some overlap is inevitable, but for completeness we give most of our results.

Kinematical conventions are given in Appendix A; our quark model symmetry assumptions in Appendix B; and some details of an evaluation of electromagnetic effects in $\psi \rightarrow$ hadrons decays in Appendix C. As is discussed in Appendix B, we find it convenient to make use of the successful relations for ordinary mesons

$$\begin{aligned} g_{\omega \rho^0 \pi^0} &= 2 g_{\rho^0 \pi^+ \pi^-} \\ &= 2 g_\rho \end{aligned} \quad (1.1)$$

$$g_{\omega \pi^0 \gamma} / e = 2 \quad (1.2)$$

These normalize the quark model couplings to vector dominances and follow from $SU(6)_W$ and VDM.

2. MIXING IN THE η AND η'

Let ε_η and $\varepsilon_{\eta'}$ be the amplitudes of $(c\bar{c})$ in the physical η and η' mesons. They are parameters which we take from $\psi \rightarrow \eta \gamma$ and $\eta' \gamma$: as argued by Harari⁷⁾ and below, the $(c\bar{c})$ content of the $\eta(\eta')$ provides the dominant mechanism. We parametrize the transition moments from the quark model [Appendix B, Eq. (B.2)]:

$$\begin{aligned} \langle \gamma^\omega | \mu | \psi \rangle &\equiv \frac{g_{\psi \gamma^\omega \gamma}}{2 m_\psi} \\ &= \frac{4}{3} \frac{\sqrt{4\pi\alpha}}{2 m_c^*} \varepsilon_{\gamma^\omega} \end{aligned} \quad (2.1)$$

The effective c-quark mass m_c^* is a parameter; we take the charge $Q_c = (2/3)e$ but clearly only Q_c/m_c^* enters (2.1). We find

$$\mathcal{E}_{\gamma^{(1)}} = (0.800 \text{ GeV}^{-1}) m_c^* g_{\psi\eta\gamma}$$

$$|\mathcal{E}_\eta| = 0.827 (m_c^* \text{ in GeV}) \sqrt{\Gamma(\psi \rightarrow \gamma\gamma) \text{ in keV}} \times 10^{-2} \quad (2.2)$$

$$|\mathcal{E}_{\gamma'}| = 0.919 (m_c^* \text{ in GeV}) \sqrt{\Gamma(\psi \rightarrow \gamma'\gamma) \text{ in keV}} \times 10^{-2}.$$

For example, if $\Gamma(\psi \rightarrow \eta\gamma) = 0.1 \text{ keV}$ and $m_c^* = 2 \text{ GeV}$,

$$|\mathcal{E}_\eta| \approx \frac{1}{2} \% \quad (2.3)$$

Using vector meson dominance (VDM), it is possible to go much further, determining ϵ_η , $\epsilon_{\eta'}$ and Q_c/m_c^* from the experimental rates of

$$\psi' \rightarrow \psi \gamma = 9.7 \pm 1.8 \text{ keV}^{15)} \quad (2.4)$$

$$\psi \rightarrow \eta \gamma = 90 \pm 24 \text{ eV}^{16, 17)} \quad (2.5)$$

$$\psi \rightarrow \eta' \gamma = 225 \pm ? \text{ keV}^{16, 17, 18)} \quad (2.6)$$

$$\left. \begin{aligned} \psi \rightarrow e^+e^- &= 4.8 \pm 0.6 \text{ keV}^{19)} \\ \psi' \rightarrow e^+e^- &= 2.1 \pm 0.3 \text{ keV}^{19)} \end{aligned} \right\} \quad (2.7)$$

That VDM is not a bad approximation is already provided by the agreement between $\sigma_{\text{tot}}(\psi N)$ from ψ -photoproduction and nuclear shadowing²⁰⁾.

2.1 VDM analysis

First we note that both the ψ - and ψ' -poles must be present, and they must very nearly cancel. For the ψ -pole alone, and guessing $g_{\psi\psi\eta} \approx g_{\psi'\psi\eta}$, gives, with (2.4), $\Gamma(\psi \rightarrow \eta\gamma) = 3.9 \pm 0.4 \text{ keV}$; or, by taking the rate (2.5) as input, $|g_{\psi\psi\eta}| \approx 1/6 |g_{\psi'\psi\eta}|$. With the ψ' -pole alone, $\Gamma(\psi \rightarrow \eta\gamma) \approx 1.4 \pm 0.4 \text{ keV}$. These numerical disparities require the ψ -pole to cancel against higher states. Of necessity, we take only ψ and ψ' , neglecting higher contributions [the $\psi(4.4)$ is negligible; in effect the ψ' -term is the ψ' plus higher poles and continuum].

Secondly, the $(c\bar{c})$ mixing in η and η' must be the dominant mechanism⁷⁾. For ω and ϕ pole dominance of $\psi \rightarrow \eta(\eta')\gamma$ (breaking the Zweig rule) yields rates $\psi \rightarrow \eta\omega \oplus \eta\phi$ of $\approx 250 \text{ keV}$, and $\psi \rightarrow \eta'\omega \oplus \eta'\phi \approx 900 \text{ keV}$, far larger than the ψ total width. Similarly from experiment we find that $|g_{\psi'\psi\eta}|$ is larger than similar

hadronic couplings by a factor ~ 100 , and is itself ~ 100 times smaller than the SU(4) value of $|g_{\psi\psi\chi}|$ as is suggested by $|\varepsilon_\eta| \sim 10^{-2}$ in (2.3). This is evidence that the η meson belongs to the ψ ($c\bar{c}$) system, as well as to the ordinary hadrons⁷⁾.

The hadronic couplings are denoted $\frac{g_{\psi\psi\eta}}{m_\psi}$, etc., see Eq. (A.1) of Appendix A. VDM reads

$$\frac{g_{\psi\gamma\gamma}}{m_\psi} \approx \sqrt{4\pi\alpha} \left(\frac{1}{g_\psi} \frac{g_{\psi\psi\gamma}}{m_\psi} + \frac{1}{g_{\psi'}} \frac{g_{\psi'\psi\gamma}}{m_{\psi'}} \right) \quad (2.8)$$

and similarly SU(4) gives

$$\frac{g_{\psi\psi\eta}}{m_\psi} = \sqrt{2} \varepsilon_\eta \frac{g_{\omega\rho\pi\pi}}{m_\omega} \quad (2.9)$$

so by Eq. (2.1)

$$\sqrt{4\pi\alpha} g_{\psi\psi\gamma} = \frac{3}{2\sqrt{2}} \frac{m_c^*}{m_\gamma} g_{\omega\rho\pi\pi} g_{\psi\gamma\gamma}$$

and Eq. (2.8) is then

$$\begin{aligned} g_{\psi\gamma\gamma} &\approx \frac{-g_{\psi'\psi\gamma} \frac{m_\psi}{m_{\psi'}} \sqrt{4\pi\alpha}}{\left\{ \frac{3}{2\sqrt{2}} \frac{g_{\omega\rho\pi\pi}}{g_\psi} \right\} \frac{m_c^*}{m_\omega} - 1} \\ &\approx \frac{1.0 \times 10^{-2}}{\frac{m_c^*}{m_\omega} - 1} \end{aligned} \quad (2.10)$$

A convenient form is

$$\begin{aligned} \Gamma(\psi \rightarrow \gamma\gamma) &= \frac{\Gamma(\psi' \rightarrow e^+e^-) \Gamma(\psi' \rightarrow \psi\gamma) \times (5.00 \times 10^{-2} \text{ keV}^{-1})}{\left(\left\{ 1.01 \pm 0.15 \right\} \frac{m_c^*}{m_\omega} - 1 \right)^2} \\ &\approx \frac{1.02 \pm 0.33 \text{ keV}}{\left(\frac{m_c^*}{m_\omega} - 1 \right)^2} \end{aligned} \quad (2.11)$$

The minus sign in Eq. (2.10) is the cancellation discussed above, predicted here if $m_c^* > m_w$; in our quark model view of magnetic transitions (see the Appendix B), m_c^*/m_w has the significance of the quark mass ratio $m_c^*/2m_{u,d}^*$, and has a numerical coefficient in Eq. (2.11) consistent with 1 ^{*)}.

From the rates (2.4 to 2.7), we fit

$$m_c^* = 3.4 \pm 0.8 \text{ GeV} \quad (2.12)$$

$$\frac{m_c^*}{m_{u,d}^*} = 8.7 \pm 2.0$$

$$\begin{aligned} g_{\psi\psi\eta} &= m_c^* \times (0.17 \pm 0.04 \text{ GeV}^{-1}) \\ &= 0.58 \pm 0.14 \end{aligned} \quad (2.13)$$

compared to

$$|g_{\psi'\psi\eta}| = 0.81 \pm 0.07 \quad (2.14)$$

and

$$\begin{aligned} \epsilon_\eta &= (0.85 \pm 0.23) \times 10^{-2} \\ &\approx 1\% \end{aligned} \quad (2.15)$$

with $g_{\psi\psi\eta}$ and ϵ_η having the same signs, opposite to $g_{\psi'\psi\eta}/g_{\psi'}$; both $g_{\psi\psi\eta}$ and ϵ_η are very insensitive to m_c^* in the range of Eq. (2.12).

For $\psi \rightarrow \eta'\gamma$, Eq. (2.2) gives

$$\left| \frac{\epsilon_{\eta'}}{\epsilon_\eta} \right| \approx 1.75 \pm ?$$

$$|\epsilon_{\eta'}| \approx (1.5 \pm ?) \times 10^{-2} \quad (2.16)$$

The values (2.14) and (2.16) are not dissimilar to those obtainable from analysis of SU(4) mass formulae^{4, 5)}, but smaller than suggested by Harari⁷⁾.

) It is only in this number that SU(4) symmetry for hadronic couplings enters, and $Q_c = \frac{2}{3}$ in Q_c/m_c^ .

2.2 Interpretation

The value $m_c^* \approx m_\psi \approx 3$ GeV in Eq. (2.14) is larger than expected. However, we recall from Appendix B, Section (i), that m_c^* is actually only a dimensional constant that fixes the scale of the magnetic coupling through m_c^*/Q_c . The value $m_c^* \approx m_\psi$ that we find can be directly understood in terms of the well-known fact of large SU(4) breaking in the ψ -photon coupling g_ψ^{-1} ; the experimental value from Eq. (2.7)

$$g_\psi^{\text{expt.}} = 12.0 \pm 0.8$$

is twice the SU(4) value for $Q_c = 2/3$:

$$(g_\psi Q_c)^{SU(4)} = g_\rho \left(\frac{2}{3} - \left(-\frac{1}{3}\right) \right) \frac{1}{\sqrt{2}}$$

$$\text{i.e. } g_\psi^{SU(4)} = \frac{3}{2\sqrt{2}} g_\rho = 6.02 \pm 0.24$$

So either SU(4) is broken for g_ψ , or $|Q_c| = 1/3$ ¹¹⁾.

Now the coefficient of m_c^*/m_ω in Eqs. (2.10) and (2.11) used exact SU(4) for the full hadronic coupling between mesons, and is further proportional to the electromagnetic coupling $(g_\psi Q_c)^{-1}$ whose value we see to be precisely $1/2$ of $(g_\psi Q_c)^{-1} SU(4)$.

We therefore observe that the same SU(4) breaking that weakens the ψ -coupling into the electric current also seems to weaken its magnetism, even if SU(4) is exact for hadronic vertices. This conclusion ^{*}) of course also relies on the VDM model which could be very wrong.

Note that the difference between having $m_c^*/m_\omega = 4.4$ (as above) and $= 2.2$ is a factor 8 in the rate $\psi \rightarrow \eta\gamma$. Thus although VDM or SU(4) could be at fault, our interpretation above appears to be reasonable. Exact SU(4) with $|Q_c| = 1/3$ ¹²⁾ allows us $m_c^* = 1.65$ GeV, but the theoretical motivation ¹⁾ for such a mass is then absent.

^{*}) The same observation has been made by Chan et al. ¹³⁾ on the same phenomenological grounds in a different (dual Pomeron) model.

3. MIXING IN THE ψ

3.1 Mixing parameters

For the ψ to be an SU(3) singlet (e.g. $c\bar{c}$), the decay $\psi \rightarrow K\bar{K}$ is forbidden by SU(3) symmetry. An easy way to see this is to make SU(3) rotations on $\psi \rightarrow \pi^0\pi^0$, forbidden by Bose symmetry and $C = -1$. Indeed, the very small upper limits on $\psi \rightarrow K_L K_S$ ²¹⁾ have suggested that the ψ must be very close to a singlet of SU(3), as expected for a $(c\bar{c})$ system.

In that case, $\psi \rightarrow \rho\pi$ and $\psi \rightarrow K^*K$ (all charge states) would be in the ratio $4/3 \times 0.86 \approx 1.15$; experiment gives $\sim 1/3$ (SPEAR ²¹⁾, and $\sim 2/3$ (DASP ²²⁾. Both differ from 1.15. The paradox is easily resolved by the Zweig rule: the dominant SU(3) singlet ($c\bar{c}$) part of the ψ is inoperative in decays of ψ to ordinary hadrons; in the above ratio the issue is the SU(3) nature of the ordinary quark pairs ($q\bar{q}$) in the ψ . Writing

$$|\psi\rangle \approx (c\bar{c}) + \frac{\epsilon_{NS}}{\sqrt{2}}(u\bar{u} + d\bar{d}) + \epsilon_S(s\bar{s}) \quad (3.1)$$

the Zweig rule and SU(3) symmetry give

$$\epsilon_S / \epsilon_{NS} = \frac{1}{\sqrt{2}} (2N - 1) \quad (3.2)$$

where

$$N = \pm \sqrt{\frac{B.R.(\psi \rightarrow K^*K, \text{all})}{B.R.(\psi \rightarrow \rho\pi, \text{all})} \times 0.870}$$

The corresponding mixing angle between SU(3) 1 and 8 components is

$$\theta_{1,8} = \tan^{-1} \left(\sqrt{2} \left(\frac{1-N}{N+\frac{1}{2}} \right) \right)$$

Recent data on N from SPEAR²¹⁾ and DASP²²⁾ are not quite consistent, so we have to give results for both:

$$N(\text{SPEAR}) = \pm \sqrt{\frac{0.44 \pm 0.10}{1.3 \pm 0.3} \times 0.870} = \pm (0.54 \pm 0.14)$$

$$N(\text{DASP}) = \pm \sqrt{\frac{0.82 \pm 0.24}{1.2 \pm 0.3} \times 0.870} = \pm (0.77 \pm 0.25)$$

by which

$$\begin{aligned} \frac{\varepsilon_s}{\varepsilon_{NS}} &= \left. \begin{array}{l} 0.06 \pm 0.20 \quad (\text{SPEAR}) \\ 0.38 \pm 0.35 \quad (\text{DASP}) \end{array} \right\} \text{if } + \text{ sign : (i)} \\ \text{or} &= \left. \begin{array}{l} -1.50 \pm 0.20 \quad (\text{SPEAR}) \\ -1.80 \pm 0.35 \quad (\text{DASP}) \end{array} \right\} \text{if } - \text{ sign : (ii)} \end{aligned} \quad (3.3)$$

corresponding to

$$\begin{aligned} \theta_{1,8} &= \left. \begin{array}{l} 32 \pm 10^\circ \quad (\text{SPEAR}) \\ 14 \pm 14^\circ \quad (\text{DASP}) \end{array} \right\} \text{if } + \text{ sign : (i)} \\ \text{or} &= \left. \begin{array}{l} -(90 \pm 2)^\circ \quad (\text{SPEAR}) \\ -(84 \pm \frac{3}{7})^\circ \quad (\text{DASP}) \end{array} \right\} \text{if } - \text{ sign : (ii)} \end{aligned}$$

For both data, solution (ii) ($N < 0$) means pure octet mixing. Solutions (i) ($N > 0$) differ. With SPEAR it is consistent with $\theta_{1,8} = \tan^{-1} 1/\sqrt{2}$: no strange quarks in the ψ . With DASP $\theta_{1,8}$ is smaller and consistent with zero: pure singlet mixing. These two distinct solutions (i) and (ii) are sharply distinguished by calculating $\psi \rightarrow K\bar{K}$, which proves to exclude case (ii) *).

3.2 $\psi \rightarrow K\bar{K}$ and tests of SU(4)

Applying SU(4) to the full couplings (Appendix A)

$$g_{\psi K^+ K^-} \equiv g_{\psi K_L K_S} = \varepsilon_{NS} \left(1 - \sqrt{2} \frac{\varepsilon_s}{\varepsilon_{NS}} \right) \frac{1}{2} g_{\rho^0 \pi^+ \pi^-} \quad (3.4)$$

$$\text{and} \quad g_{\psi \rho^0 \pi^0} = \varepsilon_{NS} \frac{m_\psi}{m_\omega} g_{\omega \rho^0 \pi^0}$$

*) This case is also unlikely intuitively, and in simple models of the mixing based on essentially SU(3) singlet forces (e.g. Pomeron^{13,14} and gluon²³ models). It was disregarded by Rosner⁶).

From Eqs. (3.2) and (1.1),

$$\frac{1}{2} \left(1 - \sqrt{2} \frac{\mathcal{E}_S}{\mathcal{E}_{NS}} \right) = 1 - N$$

$$\frac{g_{\rho^0 \pi^+ \pi^-}}{g_{\omega^0 \rho^0 \pi^0}} = \frac{1}{2}$$

giving

$$g_{\psi K_L K_S} = (1 - N) \frac{1}{2} \frac{m_\omega}{m_\psi} g_{\psi \rho^0 \pi^0} \quad (3.5)$$

so

$$B.R. (\psi \rightarrow K_L K_S, \text{ or } K^+ K^-) = B.R. (\psi \rightarrow \rho \pi, \text{ all}) \times (1 - N)^2 \times 1.1 \times 10^{-2}$$

$$= \left. \begin{array}{l} (0.30 \pm 0.23) \times 10^{-4} \text{ (SPEAR)} \\ (0.07 \pm 0.15) \times 10^{-4} \text{ (DASP)} \end{array} \right\} \text{ if (i)}$$

(3.6)

$$\text{or} \quad = \left. \begin{array}{l} (3.5 \pm 1.0) \times 10^{-4} \text{ (SPEAR)} \\ (4.2 \pm 1.6) \times 10^{-4} \text{ (DASP)} \end{array} \right\} \text{ if (ii)}$$

The data

$$B.R. (\psi \rightarrow K_L K_S) < 0.8 \times 10^{-4} \quad 21)$$

$$B.R. (\psi \rightarrow K^+ K^-) = (1.4 \pm 1.4) \times 10^{-4} \quad 22)$$

excludes case (ii) [pure octet mixing; $N < 0$] while (i) is fully consistent with both data on N .

Our use of $SU(4)$ eliminated direct use of the scale of $|\mathcal{E}_{NS}|$: it is given by Eq. (3.4):

$$\begin{aligned} |\mathcal{E}_{NS}| &= 10^{-4} \times (1.3 \pm 0.2) \\ \text{so } |\mathcal{E}_S| &= 10^{-4} \times (0.08 \pm 0.27) \quad \text{(SPEAR)} \\ \text{or} &= 10^{-4} \times (0.48 \pm 0.54) \quad \text{(DASP)} \end{aligned}$$

(3.7)

Another, more surprising, successful test of SU(4) is to compare $\psi \rightarrow K^{**}K$ to $A_2 \rightarrow \rho\pi$ (or to $K^{**} \rightarrow K^*\pi$ or ρK). These D-wave decays have a phase space $\propto |p^{c.m.}|^5$, enhancing $\psi \rightarrow K^{**}K$ by ~ 300 . For the ordinary tensor mesons, SU(3) works well²⁴); for the ψ ,

$$g_{\psi K^{**} \bar{K}} = (\varepsilon_{NS} - \sqrt{2} \varepsilon_S) \frac{1}{2} g_{A_2^0 \rho^+ \pi^-} \quad (3.8)$$

so for either the charged or neutral modes,

$$\Gamma(\psi \rightarrow K^{**} \bar{K} + \bar{K}^{**} K) = 2 (1-N)^2 |\varepsilon_{NS}|^2 \frac{5}{3} \left(\frac{p_{\psi}}{p_{A_2}} \right)^5 \Gamma(A_2^0 \rightarrow \rho^+ \pi^-)$$

$$= (1-N)^2 |\varepsilon_{NS}|^2 \times \frac{10}{3} \times 174 \times (36 \pm 4 \text{ MeV})$$

$$= \left. \begin{array}{ll} (7.4 \pm 3.9) \times 10^{-2} \text{ keV} & (\text{SPEAR}) \\ (1.8 \pm 1.0) \times 10^{-2} \text{ keV} & (\text{DASP}) \end{array} \right\} \text{if (i)}$$

$$\text{or} = \left. \begin{array}{ll} 0.84 \pm 0.13 \text{ keV} & (\text{SPEAR}) \\ 1.10 \pm 0.25 \text{ keV} & (\text{DASP}) \end{array} \right\} \text{if (ii)}$$

so

$$\text{B.R.}(\psi \rightarrow K^{**} \bar{K} + \bar{K}^{**} K) = \left. \begin{array}{ll} (1.08 \pm 0.55) \times 10^{-3} & (\text{SPEAR}) \\ (0.27 \pm 0.27) \times 10^{-3} & (\text{DASP}) \end{array} \right\} \text{if (i)}$$

(3.9)

$$\text{or} = \left. \begin{array}{ll} (12.5 \pm 2.0) \times 10^{-3} & (\text{SPEAR}) \\ (16 \pm \frac{5}{3}) \times 10^{-3} & (\text{DASP}) \end{array} \right\} \text{if (ii)}$$

The data is

$$\begin{aligned} &< 1.4 \times 10^{-3} \quad ^{21)} \quad (\text{SPEAR}) \\ &< 3.3 \times 10^{-3} \quad ^{22)} \quad (\text{DASP}) \end{aligned}$$

so again (ii) is excluded and (i) works well for both data.

We conclude that the mixing model for Zweig rule breaking, and SU(4) for hadronic couplings, work well.

3.3 Tests of $SU(6)_W$

W-spin conservation is a natural symmetry of the quark model for collinear processes such as 2-body decays^{25, 26}. Transverse motion (e.g. of the quarks) breaks the symmetry, (ℓ -breaking)^{27, 28}, but not in decays with a single partial wave. Breaking of $SU(3)$ or $SU(4)$ in the hadron masses also breaks W-spin²⁶: mass factors must be present but their choice depends on postulated (e.g. free quark²⁶) equations of motion. We consider two examples.

a) $\psi \rightarrow K\bar{K}$ (P-wave only)

It is simple to see that our previous $SU(4)$ analysis of this decay contradicts W-spin conservation. For the analogue of the successful $SU(6)_W$ relation Eq. (1.1)

$$g_{\rho^0 \pi^+ \pi^-} = \frac{1}{2} g_{\omega \rho^0 \pi^0}$$

is⁶)

$$g_{\psi K_L K_S} = \frac{1}{2} \left(1 - \sqrt{2} \frac{\epsilon_S}{\epsilon_{N_S}}\right) \frac{1}{2} g_{\psi \rho^0 \pi^0} \quad (3.10)$$

which differs from the $SU(4)$ relation (3.5) by the factor $(m_\omega/m_\psi) = 1/4$. Rates 16 times any of those in Eq. (3.6) are excluded by the data. Our procedure requires an extra factor $1/4$ in Eq. (3.10). The model of Sakita and Wali²⁶ give $1^{1/8}/3 = 3^{1/8}$ for this factor, making the rates $(3/2)^2$ times those Eq. (3.6). This also agrees with experiment for solution (i): their mass-breaking model is not excluded by the data, but it is inconsistent with $SU(4)$ for couplings.

b) $\psi \rightarrow K^* \bar{K}^*$ (P- and F-waves)

These are 7 helicity amplitudes, but only one [for helicity $0 \rightarrow 0 + 0$; see Eq. (A.6)] has the F-wave. The P-wave term is kinematically similar to $\psi \rightarrow K\bar{K}$; the $SU(3)$ factor is identical.

With exact W-spin symmetry, for each charge

$$B.R. (\psi \rightarrow K^* \bar{K}^*) = B.R. (\psi \rightarrow K \bar{K}) \times 7 \times 0.645$$

$$\begin{aligned}
 &= \left. \begin{aligned} &(0.14 \pm 0.11) \times 10^{-3} && (\text{SPEAR}) \\ &(0.03 \pm 0.07) \times 10^{-3} && (\text{DASP}) \end{aligned} \right\} \text{if (i)} \\
 \\
 \text{or} &= \left. \begin{aligned} &(1.6 \pm 0.4) \times 10^{-3} && (\text{SPEAR}) \\ &(1.9 \pm 0.7) \times 10^{-3} && (\text{DASP}) \end{aligned} \right\} \text{if (ii)}
 \end{aligned}
 \tag{3.11}$$

Experiment²¹⁾ is $< 0.6 \times 10^{-3}$.

Again, (ii) is excluded but not (i): $SU(6)_W$ works without contradiction. It may be that the breaking does have less scope in this case, but we do not accept (3.11) as a serious estimate.

3.4 $\psi \rightarrow \omega\eta$, etc.

These test our view, that transitions breaking the Zweig rule occur only through small deviations from ideal mixing. The above decays can proceed by both the $(q\bar{q})$ mixing in the ψ as described above, and by $(c\bar{c})$ in the η (or η') and ω . In either case, we can not get large⁷⁾ rates.

In the case of $\psi \rightarrow \omega\eta(\eta')$, the $(q\bar{q})$ mechanism dominates over the $(c\bar{c})$: the couplings are of order 5×10^{-3} and 5×10^{-5} respectively. The rates are therefore quite small, being essentially $SU(3)$ rotations on $\psi \rightarrow \rho\pi$:

$$B.R.(\psi \rightarrow \eta\omega) = B.R.(\psi \rightarrow \rho\pi, \text{all}) \times \left\{ \frac{1/9}{(1/6)} \right\} \times 0.89 = \begin{cases} 0.13 \pm 0.03 \% \\ (0.19 \pm 0.05 \%) \end{cases}
 \tag{3.12}$$

$$B.R.(\psi \rightarrow \eta'\omega) = B.R.(\psi \rightarrow \rho\pi, \text{all}) \times \left\{ \frac{2/9}{(1/6)} \right\} \times 0.69 = \begin{cases} 0.20 \pm 0.05 \% \\ (0.15 \pm 0.03 \%) \end{cases}$$

where the upper (lower) numbers correspond to the η and η' mesons being unmixed [or mixed by -10° ²⁹⁾ as suggested by the meson $(\text{mass})^2$ formula, by the rate $\eta \rightarrow 2\gamma$ ^{29,24)}, and by high energy η -production³⁰⁾].

For $\psi \rightarrow \phi \eta$ (or η'), there is a large correction from non-ideal ω - ϕ mixing, even if the same ($q\bar{q}$) mechanism were dominant:

$$\text{B.R.}(\psi \rightarrow \eta \phi) = \text{B.R.}(\psi \rightarrow \rho \pi, \text{all}) \times \left\{ \begin{aligned} &\frac{4}{9} \left[\frac{\epsilon_S}{\epsilon_{NS}} - \frac{\epsilon_\phi}{2} \right]^2 \\ &\left(\frac{1}{3} \left[\frac{\epsilon_S}{\epsilon_{NS}} - \frac{\epsilon_\phi}{\sqrt{2}} \right]^2 \right) \end{aligned} \right\} \times 0.755 \quad (3.13)$$

$$\text{B.R.}(\psi \rightarrow \eta' \phi) = \text{B.R.}(\psi \rightarrow \rho \pi, \text{all}) \times \left\{ \begin{aligned} &\frac{2}{9} \left[\frac{\epsilon_S}{\epsilon_{NS}} + \epsilon_\phi \right]^2 \\ &\frac{1}{3} \left[\frac{\epsilon_S}{\epsilon_{NS}} + \frac{\epsilon_\phi}{\sqrt{2}} \right]^2 \end{aligned} \right\} \times 0.555$$

where

$$|\phi\rangle \approx (\bar{s}s) + \frac{\epsilon_\phi}{\sqrt{2}} (u\bar{u} + d\bar{d}) .$$

With small quantities ϵ_S/ϵ_{NS} and ϵ_ϕ of the same size ($\sim 10^{-1}$), cancellations can render the calculation meaningless. If factors like $[\epsilon_S/\epsilon_{NS} - \epsilon_\phi/2]$ etc. in Eq. (3.13) are of order 10^{-2} , ($c\bar{c}$) mixing in $\eta(\eta')$ and ϕ provides a competing mechanism, (and the branching ratios would be very small, $\leq 0.001\%$).

As an example, we take ϵ_S/ϵ_{NS} from DASP in Eq. (3.3) (i), and

$$\epsilon_\phi = + (0.80 \pm 0.10) \times 10^{-1}$$

obtained from $\phi \rightarrow \rho\pi$ and $g_{\omega\rho\pi}$ given in Eq. (B.9). (This gives an ω - ϕ mixing angle = $35.25^\circ + (4.6 \pm 0.6)^\circ \approx 40^\circ$; close to 39° from the Gell-Mann-Okubo (mass)² formula, and to $41 \pm 3^\circ$ from $\omega(\phi) \rightarrow e^+e^-$. This ϵ_ϕ makes $\Gamma(\phi \rightarrow \pi^0\gamma) = 13 \pm 3$ keV; experiment is 5.9 ± 2.1 keV).

Then

$$B.R. (\psi \rightarrow \gamma \phi) = \begin{cases} (0.05 \pm 0.13) \% \\ (0.033 \pm 0.030) \% \end{cases}$$

and $= 0.044 \pm ? \% \text{ (experiment }^{31})$.

$$B.R. (\psi \rightarrow \eta' \phi) = \begin{cases} (0.034 \pm 0.050) \% \\ (0.047 \pm 0.040) \% \end{cases}$$

and $\leq 0.030 \pm ? \% \text{ (experiment }^{31})$.

In view of the uncertainties, the DASP value $\epsilon_S/\epsilon_{NS} \approx 0.4$ is in good agreement with the type of magnitude observed. The SPEAR value $\epsilon_S/\epsilon_{NS} \approx 0.06 \pm 0.20$ give rates orders of magnitude too small (but with huge errors), and is afflicted by a comparable ($c\bar{c}$) mechanism.

We conclude (tentatively) that the ($q\bar{q}$)-model agrees with experiment, for the DASP mixing parameter. A firmer test would be $\psi \rightarrow \eta(\eta')\omega$.

3.5 Electromagnetic interference

With rates as small as those encountered in "strong" decays $\psi \rightarrow AB$ (amplitude T_{strong}), electromagnetism is potentially competitive through $\psi \rightarrow \gamma^* \rightarrow AB$ [amplitude $T_Y(A,B)$] interfering with T_{strong} .³²⁾

Calculations of T_Y are presented in Appendix C. The issue is complicated because of sensitivity to all of the following: SU(3) properties of the ψ , SU(3) breaking in T_{strong} , in the normalization ($q^2 = 0$) and q^2 -dependence of the (transition) form factors $F_{AB}(q^2)$, and the phase of T_Y relative to T_{strong} . Here we summarize the main points to emerge from Appendix C. $\psi \rightarrow \pi^+\pi^-$ is a special case: only T_Y occurs and $F_\pi(q^2)$ is well measured, agreeing with the ρ^0 -pole model out to $q^2 = 3.0 \text{ (GeV)}^2$.

We find

$$|g_{\psi\pi^+\pi^-}| = 5.04 \times 10^{-4}$$

and $B.R.(\psi \rightarrow \pi^+\pi^-) = 0.75 \times 10^{-4}$

in good agreement with experiment²²⁾ $[(1 \pm 7) \times 10^{-4}]$.

For $\psi \rightarrow K\bar{K}$, with ρ , ω , ϕ -pole model of the $F_{K\bar{K}}(q^2)$ form factors, T_γ alone gives

$$\begin{aligned} B.R.(\psi \rightarrow K^+K^-) &= 1.0 \times 10^{-4} \\ (\text{Experiment}^{22}) &= (1.4 \pm 1.4) \times 10^{-4} \end{aligned}$$

and $B.R.(\psi \rightarrow K_L K_S) = 0.05 \times 10^{-4}$

$$(\text{Experiment}^{21}) < 0.8 \times 10^{-4} .$$

Our hadronic calculation (3.4) should therefore be viewed with caution, at least for K^+K^- . We note, however, that this form factor model seems to be wrong (Appendix C); and the conclusion excluding the pure octet solution for mixing in ψ is in any case unchanged.

Now take $\psi \rightarrow VP$. The algebraic structure of the amplitudes are given in Appendix C, with the one photon terms denoted by $\epsilon_{VP} \times [\text{SU}(3) \text{ factor}]$. Our discussion proceeds in stages.

With SU(3) exact, all ϵ_{VP} are equal (but not necessarily real). Then

$$T_\gamma(K^*K)^\pm = -\frac{1}{2} T_\gamma(K^*K)^0 \quad (3.14)$$

so T_γ here could not dominate, as experiment is²¹⁾

$$\sqrt{\frac{(B.R. \psi \rightarrow (K^*K)^\pm)}{(B.R. \psi \rightarrow (K^*K)^0)}} = \begin{matrix} 1.00 & +.25 \\ & -.15 \end{matrix} \quad (3.15)$$

With SU(3) broken only in the $q^2 = 0$ normalizations [$\lambda^* = 1/2$ in Eq. (B.11)]

$$T_{\gamma} (K^* K)^{\pm} \equiv - T_{\gamma} (K^* K)^0 \quad (3.16)$$

$$\equiv \frac{3}{2} T_{\gamma} (\rho^0 \pi^0) \quad (3.17)$$

$$\equiv \frac{1}{2} T_{\gamma} (\omega^0 \pi^0) \quad (3.18)$$

(phase space factor relating rate of Eq. (3.16) to Eqs. (3.17) and (3.18) is 0.86).

Equation (3.16) agrees with the data, but not Eq. (3.17). Therefore, T_{strong} must be present as well, in agreement with observations comparing on- and off-resonance production²²⁾. But SU(3) forces T_{γ} to have opposite phases in the two (K^*K) charge states, Eq. (C.1), so Eq. (3.15) cannot be satisfied if T_{γ} competes with T_{strong} .

Now consider the specific pole model for transition form factors $F_{VP}(q^2)$. As $m_{\phi} > m_{\omega, \rho}$, SU(3) is broken as a function of q^2 , but at $q^2 = m_{\psi}^2$ it turns out (Appendix C) that all ε_{VP} are actually equal within 20% ($\approx -2.3 \times 10^{-3}$), except for $\varepsilon(K^*K)^{\pm}$ which is smaller by a factor 8.

These magnitudes are such that $|T_{\gamma}|^2$ alone accounts for $1/18$ of the rate $\psi \rightarrow \rho^0 \pi^0$; $1/300$ of $(K^*K)^{\pm}$ but the whole of $(K^*K)^0$. Any fit to one (K^*K) rate with both T_{strong} and T_{γ} consequently forces the other rate to be zero or four times it; so contradicting experiment, Eq. (3.15).

Whether or not both T_{strong} and T_{γ} occur, the pole model for T_{γ} predicts [see Eq. (C.1)]

$$\frac{\Gamma(\psi \rightarrow \gamma' \phi^0)}{\Gamma(\psi \rightarrow \gamma \phi^0)} = 0.735 \times \left\{ \frac{1/2}{(1)} \right\} \quad (3.19)$$

$$\frac{\Gamma(\psi \rightarrow \gamma' \omega^0)}{\Gamma(\psi \rightarrow \gamma \omega^0)} = 0.77 \times \left\{ \frac{2}{(1)} \right\}$$

(there is a correction for non-ideal ω - ϕ mixing). This is more general than the pole model (e.g. from nonet symmetry).

Experiment is in accord with Eq. (3.19); but not with the pole model for it gives absolute rates ~ 2 to 3 times too high taking T_Y alone. Fitting to $T_Y + T_{\text{strong}}$ gives $|\epsilon_S/\epsilon_{NS}| \gtrsim 1$ or $\lesssim 0.2$, but the kaon rates cannot still fit experiment.

We conclude that the systematics of the data, which are well reproduced by the $(q\bar{q})$ -mixing model of T_{strong} ($\psi \rightarrow$ hadron pairs), are destroyed by significant interference with T_Y , considered either at the SU(3) level or more specifically through the simple pole model of transition form factors. We believe these transition form factors $F_{VP}(q^2)$ fall faster than the diagonal pion form factor $F_\pi(q^2)$, just as $G_{N\Delta}^-(q^2)_M$ falls faster than $G_N(q^2)_M$ for the baryons.

A direct measure of the form factors at $q^2 = m_\psi^2$ would be to search for $\omega\pi^0$, $\rho^0\eta$, $\rho^0\eta'$ and $A_2^\pm\pi^\mp$ final states. The pole model gives

$$\begin{aligned} B.R. (\psi \rightarrow \omega \pi^0) &= 2 \times 10^{-3} \\ B.R. (\psi \rightarrow \rho^0 \eta) &\approx B.R. (\psi \rightarrow \rho^0 \eta') \\ &\sim \left(\frac{1}{2} \text{ to } 1\right) \times 10^{-3} \\ B.R. (\psi \rightarrow A_2^+ \pi^- + A_2^- \pi^+) &= 3.8 \times 10^{-3} \\ (\text{Experiment}^{22}) &< 4.3 \times 10^{-3} \end{aligned}$$

and from T_Y alone we similarly find

$$\begin{aligned} B.R. (\psi \rightarrow K^{*+} K^- + K^{*-} K^+) &= 4.0 \times 10^{-3} \\ B.R. (\psi \rightarrow K^{*0} \bar{K}^0 + \bar{K}^{*0} K^0) &= 0.2 \times 10^{-3} \\ (\text{Experiment for either}^{21}) &< 1.4 \times 10^{-3} \end{aligned}$$

3.6 $\psi \rightarrow p\bar{p}$ and other baryon pairs^{22, 33, 34)}

This provides an interesting test of the model used so far. There are two independent couplings, e.g.

$$T(\psi \rightarrow p \bar{p}) = \epsilon_\mu(p_\psi) \bar{u}(\bar{p}) \left[f_1 \gamma^\mu + \frac{f_2}{4M_N} (\gamma^\mu \not{p}_\psi - \not{p}_\psi \gamma^\mu) \right] u(p)$$

and we define the standard Sachs couplings for definite helicities:

$$g_M = f_1 + f_2$$

$$g_E = f_1 + \frac{p_\psi^2}{4 M_N^2} f_2$$

We normalize g_E and g_M to the nucleon electric charges and magnetic moments by Sakurai's universality argument³⁵⁾ (VDM), which is certainly well confirmed for the ρ^0 -Regge pole and meson pole couplings³⁶⁾ [the ω -pole is less clearly identified and the couplings are confused]. We find

$$g_E = \frac{g_M}{0.880} = \frac{3}{2} g_\rho \epsilon_{NS}$$

$$\text{so } f_1 = 0.91 g_E \quad (3.20)$$

$$f_2 = 0.07 g_E \approx 0$$

The rate

$$\Gamma(\psi \rightarrow p \bar{p}) = \left(g_M^2 + \frac{2 M_N^2}{m_\psi^2} g_E^2 \right) \frac{|\vec{p}^{c.m.}|}{6\pi} \quad (3.21)$$

is clearly proportional to $|\epsilon_{NS} g_\rho|^2$; as also is $\Gamma(\psi \rightarrow \rho\pi)$ since $g_{\psi\rho\pi} \propto \epsilon_{NS} g_{\omega\rho\pi} \propto \epsilon_{NS} g_\rho$. We therefore normalize $\psi \rightarrow p\bar{p}$ by $\psi \rightarrow \rho\pi$:

$$B.R.(\psi \rightarrow p \bar{p}) = \frac{B.R.(\psi \rightarrow \rho\pi, \text{all})}{1.58} \left| \frac{\vec{p}^{c.m.}}{6\pi} \right|^2 \left(\frac{3}{2} \right)^2 \left((0.880)^2 + \frac{2 M_N^2}{m_\psi^2} \right)$$

$$= (0.12 \pm 0.03) \%$$

$$\text{Experiment} = (0.21 \pm 0.04) \% \quad (\text{SPEAR}^{33})$$

$$= (0.23 \pm 0.03) \% \quad (\text{DASP}^{22})$$

Our result is a little low, but of the correct order of magnitude. We note the corresponding angular distribution

$$\propto (1 + \cos^2 \theta) + 0.48 (1 - \cos^2 \theta)$$

is quite consistent with experiment^{17, 22)}.

The above argument may be extended to all of the baryon octet, taking the quark model [SU(6)] magnetic moments. This is equivalent to making SU(3) rotations on the proton, with $D/F = 0$ for g_E , and $D/F = 3/2$ for g_M as follows from $\mu_p/\mu_n = -3/2$. The Zweig rule $g_{E,M}(\phi^0 \bar{N}N) \equiv 0$ relates singlet to octet couplings.

In these cases, an ambiguity arises over the choice of couplings $f_{1,2}$ or $g_{E,M}$ etc. to which universality is applied³⁷⁾; and there may be SU(3) breaking in the moments [see Eq. (B.11)]. These questions do not affect $\psi \rightarrow p\bar{p}$, as $f_2(N\bar{N}) \approx 0$.

In fact, the ambiguity over couplings is hidden by errors on ϵ_S/ϵ_{NS} , and SU(3) breaking turns out to have a negligible effect ($< 10\%$ in $g_{E,M}$)²⁴⁾. We therefore give results for the choice g_E and g_M , exact SU(3) but for both SPEAR and DASP values of ϵ_S/ϵ_{NS} in Table 1 below. Rates for $\psi \rightarrow \gamma^* \rightarrow B\bar{B}$ using correctly normalized dipole form factors $G_{E,M}(m_\psi^2)$ are also given: they are negligible, in agreement with direct off-resonance measurements^{17, 33)}.

Table 1

Branching ratios for $\psi \rightarrow$ baryon pairs

	Branching ratio from universality taking parameters from		Experiment	One photon Branching ratio
	SPEAR	DASP		
$p\bar{p}$	$0.12 \pm 0.03 \%$	$0.12 \pm 0.03 \%$	$0.21 \pm 0.04\% \text{ }^{33)}$ $0.23 \pm 0.03\% \text{ }^{22)}$	$p : 0.17 \times 10^{-2}\%$ $n : 0.78 \times 10^{-3}\%$
$\Lambda\bar{\Lambda}$	$0.002 \begin{smallmatrix} + 0.003 \\ - 0.001 \end{smallmatrix} \%$	$0.07 \pm 0.13 \begin{smallmatrix} 0.04 \\ 0.04 \end{smallmatrix} \%$	$0.16 \pm 0.63\% \text{ }^{33)}$	$0.12 \times 10^{-3}\%$
$\Sigma\bar{\Sigma}$ (one charge)	$0.018 \pm 0.002 \%$	$0.014 \pm 0.003 \%$	$< 0.063\% (?) \text{ }^{33)}$	$\Sigma^+ : 0.14 \times 10^{-2}\%$ $\Sigma^0 : 0.15 \times 10^{-3}\%$ $\Sigma^- : 0.20 \times 10^{-3}\%$
$\Xi\bar{\Xi}$ (one charge)	$0.0007 \pm 0.0004\%$	$0.0016 \begin{smallmatrix} + 0.0096\% \\ - 0.0013\% \end{smallmatrix}$		$\Xi^0 : 0.49 \times 10^{-3}\%$ $\Xi^- : 0.17 \times 10^{-3}\%$
$\Lambda\bar{\Sigma}^0 + \bar{\Lambda}\Sigma^0$			$< 0.63 \times 10^{-1}\% \text{ }^{33)}$	$0.93 \times 10^{-3}\%$

$\psi \rightarrow \Lambda \bar{\Lambda}$ is low for both values of ϵ_S/ϵ_{NS} ; but for the DASP value it agrees with experiment within errors and also with $\psi \rightarrow p \bar{p}$, as expected from a near singlet ψ state. For the SPEAR value $\epsilon_S/\epsilon_{NS} \sim 1/10$, it is low as SU(3) [exact or broken form²⁴)] forces the ideal ω to decouple from the coupling g_M which normally controls the rate (3.21). $\psi \rightarrow \Xi \bar{\Xi}$ is very small with large errors, as strange quarks dominate the decay of high strangeness particles.

For such a simple and not entirely well defined model, agreement for rates within a factor 2 is, we feel, not at all bad. The model clearly prefers the DASP value $\epsilon_S/\epsilon_{NS} \sim 0.4 \pm 0.3$, consistent with the fact that their spectrometer affords better kaon identification than the SPEAR detector. The $(q\bar{q})$ -mixing in the ψ appears to be close to an SU(3) singlet.

4. $\psi \rightarrow x\gamma$ and $x \rightarrow \gamma\gamma$

4.1 $\psi \rightarrow x\gamma$

The parameter m_c^* in Eq. (2.12) predicts the rate. For convenience, we use $m_c^*/m_{u,d}^*$, thereby taking $\omega \rightarrow \pi^0\gamma$ as normalization. The Clebsch-Gordan factor between these is $2Q_c = 4/3$ [Eq. (B.2)],

$$\begin{aligned} \langle x | \mu_- | \psi \rangle &= \langle \pi^0 | \mu_- | \omega \rangle \frac{4}{3} \left(\frac{m_{u,d}^*}{m_c^*} \right) \\ &= (0.52 \pm 0.03) \left(\frac{m_{u,d}^*}{m_c^*} \right) \text{ GeV}^{-1} \end{aligned} \quad (4.1)$$

$$\Gamma(\psi \rightarrow x\gamma) \text{ in keV} = (28.3 \pm 1.5) \left(\frac{m_{u,d}^*}{m_c^*} \right)^2 \bar{p}_\gamma^3 \quad (4.2)$$

where the photon momentum p_γ in 100 MeV/c units is

$$\begin{aligned} \bar{p}_\gamma &= (\bar{m}_\psi - \bar{m}_x) \left(1 - \frac{m_\psi - m_x}{2 m_\psi} \right) \approx \bar{m}_\psi - \bar{m}_x \\ &\sim 2.5 \end{aligned}$$

The rate (4.2) is sensitive to $(m_c^*/m_{u,d}^*)$, and extremely so to m_x . Figure 1 shows the dependences. Using $m_c^*/m_u^* = 8.4$ from Eq. (2.12),

$$\begin{aligned}
 \Gamma(\psi \rightarrow x \gamma) &= 8.9 \pm 0.5 \text{ keV} \quad \text{for } m_x = 2.80 \text{ GeV} \\
 &= 6.7 \pm 0.3 \quad " \quad " \quad " = 2.83 \quad " \quad (4.3) \\
 &= 5.25 \pm 0.3 \quad " \quad " \quad " = 2.85 \quad "
 \end{aligned}$$

i.e. $\Gamma(\psi \rightarrow x \gamma) \sim 7 \pm 2 \text{ keV}$.

This is just consistent with the experimental limit $< 7 \text{ keV}$ (99% C.L.)³⁸⁾. We note that $m_x = 2.75 \text{ GeV}$ makes $\Gamma = 15 \pm 1 \text{ keV}$, which is too big.

[Equation (4.1) is equivalent to the use of VDM, with couplings fixed by the $\psi \rightarrow \eta \gamma$ analysis of Section 2:

$$g_{\psi \psi x} = \frac{g_{\psi \psi \eta}}{\varepsilon_\eta} = \sqrt{2} g_{\omega \rho \pi^0} \frac{m_\psi}{m_\omega} = 63.7 \pm 4.5$$

$$g_{\psi \psi' x} = \frac{g_{\psi \psi' \eta}}{\varepsilon_\eta} = - (95 \pm 25) \text{ (times the sign of } g_{\rho \pi^0})$$

$$\begin{aligned}
 \text{so } \frac{g_{\psi \psi x}^2}{4\pi} &= (3.24 \pm 0.45) \times 10^2 \\
 \frac{g_{\psi \psi' x}^2}{4\pi} &= (7.2 \pm 3.2) \times 10^2 \quad (4.4)
 \end{aligned}$$

These couplings are large, contribute with opposite signs, to give a rate agreeing with Eq. (4.3)²⁴⁾. Their values are required later.]

Returning to the quark model and VDM procedure of Section 2, we note a parameter free relation:

$$\frac{\Gamma(\psi' \rightarrow \psi \gamma)}{\Gamma(\psi \rightarrow \gamma \gamma)} = (9.5 \pm 1.5) \left(\frac{(2.68 \pm 0.40) \bar{p}_\gamma^{3/2}}{(\Gamma(\psi \rightarrow x \gamma) \text{ in keV})^{1/2}} - 1 \right)^2$$

4.2 $x \rightarrow \gamma \gamma$

The VDM formula is

$$\begin{aligned}
 \frac{g_{x \gamma \gamma}}{m_x} &= \sqrt{4\pi\alpha} \sum_{\psi, \psi', \dots} \frac{g_{\psi x \gamma}}{m_\psi g_\psi} \\
 &= 4\pi\alpha \frac{4}{3} \frac{1}{g_\psi} \frac{1}{m_c^*} + \dots \quad (4.5)
 \end{aligned}$$

using Eq. (4.1). Keeping only the ψ -pole, the rate would be

$$\begin{aligned} \Gamma(x \rightarrow \gamma\gamma) &= \frac{g_{x\gamma\gamma}^2}{4\pi} \frac{m_x}{16} \\ &= (1.1 \pm 0.1) \text{ keV}. \end{aligned} \quad (4.6)$$

Now we discuss the ψ' -pole.

As remarked in Section 5.2 below, we do expect

$$\left| \frac{g_{\psi'x\gamma}}{g_{\psi'}} \right| / \left| \frac{g_{\psi x\gamma}}{g_{\psi}} \right| \ll 1$$

but the experimental limits on $\psi' \rightarrow x\gamma$ ¹⁶⁾ give only

$$\left| \frac{g_{\psi'x\gamma}}{g_{\psi'}} \right| / \left| \frac{g_{\psi x\gamma}}{g_{\psi}} \right| < \frac{0.4}{\sqrt{b}} \quad (4.7)$$

where we write

$$b \equiv 10^3 \times \text{B.R.}(x \rightarrow \gamma\gamma).$$

From the experimental numbers^{16,17)}

$$\text{B.R.}(\psi \rightarrow x\gamma) \text{ B.R.}(x \rightarrow \gamma\gamma) \approx 1.5 \times 10^{-4} \quad (4.8)$$

$$\text{B.R.}(\psi \rightarrow x\gamma) \text{ B.R.}(x \rightarrow \gamma\gamma) \sim \text{B.R.}(\psi \rightarrow \gamma\gamma) \text{ B.R.}(\gamma \rightarrow \gamma\gamma) \quad (4.9)$$

Eqs. (4.3) and (2.5) give

$$\begin{aligned} b &\approx 1.5 && \text{from (4.8)} \\ b &\sim 3 && \text{from (4.9)} \end{aligned} \quad (4.10)$$

so we assume it reasonable to suppose $b \approx 1$ or larger in Eq. (4.7) *).

*) As most models (at least) have $\Gamma(\psi \rightarrow x\gamma)$ large, they would also like b larger.

The experimental numbers

$$\Gamma(\psi' \rightarrow x\gamma) \text{ B.R. } (x \rightarrow \gamma\gamma) \leftarrow 0.084 \text{ keV}^{(16)} \\ \approx 0.02 \text{ to } 0.04 \text{ keV}^{(18)} \quad (4.11)$$

enable us to estimate the effect of the ψ' -pole on $x \rightarrow \gamma\gamma$, as a function of the rate $\psi' \rightarrow x\gamma$. This is shown in Fig. 2: two values of $\Gamma(x \rightarrow 2\gamma)$ for each $\Gamma(\psi' \rightarrow x\gamma)$, as the signs of couplings are not known.

It is interesting that both Eq. (4.11) and the limit $\lesssim 50 \text{ keV}$ on $\psi' \rightarrow x\gamma$ from unidentified ψ' -decays eventually give a useful constraint: e.g.

$$\begin{aligned} \Gamma(x \rightarrow \gamma\gamma) &= 1.0 \text{ or } 1.1 \text{ keV} \quad \text{when } \Gamma(\psi' \rightarrow x\gamma) = 4 \text{ keV and } b \leq 20 \\ &1.55 \text{ " } 0.7 \text{ " " " " } = 21 \text{ " " } b \leq 4 \\ &1.8 \text{ " } 0.6 \text{ " " " " } = 42 \text{ " " } b \leq 2. \end{aligned}$$

We therefore believe

$$\Gamma(x \rightarrow \gamma\gamma) \approx 1.0 \pm 0.5 \text{ keV} \quad (4.12)$$

and so with Eq. (4.10)

$$\begin{aligned} \Gamma(x \rightarrow \text{all}) &\approx 0.7 \pm 0.3 \text{ MeV} \\ &\sim 1 \text{ MeV.} \end{aligned} \quad (4.13)$$

We have checked that the contribution to $x \rightarrow \gamma\gamma$ from ordinary quark pairs ($q\bar{q}$) in the x contributes only $\approx 1/50$ of the value $g_{x\gamma\gamma} \approx 9 \times 10^{-3}$ obtaining from Eq. (4.5) [we use the mixing strength of Section 6.2 below].

We may remark that since $m_c^* \approx m_x$, Eq. (4.12) must be similar to the result of applying SU(4) to the dimensionless $g_{x\gamma\gamma}$:

$$\begin{aligned} g_{x\gamma\gamma} &= \frac{4\sqrt{2}}{3} g_{\pi^0\gamma\gamma} \\ \text{gives } \Gamma(x \rightarrow \gamma\gamma) &= \frac{32}{9} \frac{m_x}{m_{\pi^0}} \Gamma(\pi^0 \rightarrow \gamma\gamma) \\ &= 0.56 \pm 0.07 \text{ keV} , \end{aligned} \quad (4.14)$$

[which is $\approx 1/2$ of Eq. (4.12) largely because g_ψ is twice the SU(4) value obtained from g_ρ , which gives nearly the correct $\pi^0 \rightarrow \gamma\gamma$ ²⁴⁾].

It is clear from Eq. (4.13), that SU(4) must fail when applied to the full coupling ($g_{x\gamma\gamma}/m_x$), as then $\Gamma(x \rightarrow \gamma\gamma) \approx 250$ keV, despite the fact that this works well at the SU(3) level. This paradox is a point of some importance, as discussed in Section 7.

5. ψ' DECAYS

5.1 $\psi' \rightarrow P^0\gamma$ ($P^0 \equiv \eta, \eta'$ or $x(2.8)$)

Here our phenomenological approach, and the data, are rather too limited to draw many conclusions, but the data are adequate to provide a consistency test of the VDM model of Section 3, and to specify ψ' -couplings.

The situation is summarized in Table 2. The upper limits from SPEAR³⁹⁾ are an order of magnitude above those of DASP^{16,17)}. We first observe that both ψ - and ψ' -VDM poles must again be present, and must nearly cancel, in all three cases. The rates from one pole (ψ or ψ') are also entered in Table 2: they far exceed the DASP limits but might be acceptable with SPEAR. Again the number $b \equiv (\text{B.R. } x \rightarrow 2\gamma) \times 10^3$ and we have $b \gtrsim 1$ as from Eq. (4.10) ^{*}).

Table 2

Decays $\psi' \rightarrow P^0\gamma$: rates and coupling ($\psi'\psi'P^0$) from VDM

$\psi' \rightarrow P^0\gamma$	Rates (a) DASP; (b) SPEAR	Rates from one VDM pole	$ g_{\psi'\psi'P^0} $ Range allowed by VDM and rates (value for exact cancellation)
$\psi' \rightarrow \eta\gamma$	< 0.30 keV (a) < 5.0 keV (b)	5.1 ± 1.6 keV	1.03 ± 0.33 to 1.67 ± 0.33 (1.34 ± 0.12)
$\psi' \rightarrow \eta'\gamma$	< 3.2 keV (a) (< 100 keV (b)?)	20 ± 10 keV	1.4 ± 0.8 to 3.3 ± 0.8 (2.4 ± 0.8)
$\psi' \rightarrow x\gamma$	< $84/b$ keV (a) $\approx 20/b$ keV (a) ¹⁸⁾ < $600/b$ keV (b)	5 ± 7 MeV $- 3$	136 to 176 if $b = 1$ 147 to 165 if $b = 5$ 116 to 196 if $b = 1/4$ (160 ± 60)

*) In fact, provided $b \gtrsim 1$, it is $\psi' \rightarrow x\gamma$ that provides the tightest limits:
 $|g_{\psi'x\gamma}| < \sqrt{0.10}$. See also Ref. 9.

The DASP limits are sufficiently restrictive to put significant limits on the $\psi'\psi'P^0$ couplings from the near cancellation of ψ - and ψ' -VDM poles.

Exact cancellation

$$\frac{g_{\psi'\psi'P^0}}{g_{\psi'\psi'P^0}} = - \frac{g_{\psi}}{g_{\psi'}} \approx - \frac{1}{\sqrt{3}} \quad (5.1)$$

gives the numbers in brackets in Table 1.

Thus the entries in Table 2 are consistent with the expected pattern

$$\frac{g_{\psi'\psi'\eta'}}{g_{\psi'\psi'\eta}} = \frac{\epsilon_{\eta'}}{\epsilon_{\eta}} \sim 2$$

$$\frac{g_{\psi'\psi'X}}{g_{\psi'\psi'\eta}} = \frac{1}{\epsilon_{\eta}} \approx 10^2 \times (1.2 \pm 0.3)$$

these numbers being determined from ψ -decays.

Finally, one expects (after allowing for phase space)

$$\frac{B.R.(\psi' \rightarrow \eta' \gamma)}{B.R.(\psi' \rightarrow \eta \gamma)} = 0.99 \frac{B.R.(\psi \rightarrow \eta' \gamma)}{B.R.(\psi \rightarrow \eta \gamma)} \approx 4 \quad (5.2)$$

and

$$\frac{B.R.(\psi' \rightarrow X \gamma)}{B.R.(\psi' \rightarrow \eta \gamma)} = \frac{6.85 \times 10^{-2}}{|\epsilon_{\eta}|^2} \approx 1.0 \times 10^3$$

The data cannot yet test (5.2), but the limits in Table 2 have orders of magnitude in line with Eq. (5.2) and $b > 1/4$, consistent with our requirements $b \gtrsim 1$.

5.2 Non-relativistic quark model

Here one expects the $\psi'P^0\gamma$ couplings to be much smaller than the corresponding $\psi P^0\gamma$ cases, owing to the orthogonality of the spatial wave functions. This of course neglects recoil, an important point since the photon momentum $p_{\gamma} \approx 1.7$ (1.5) GeV for $\psi'(\psi) \rightarrow \gamma\eta$ or η' .

Table 3 gives the values of $g^2/4\pi\alpha$ for $\psi' \rightarrow P^0\gamma$, corresponding to the experimental rates in Table 2. For comparison we also give the values for $\psi \rightarrow P^0\gamma$; note that for $\omega \rightarrow \pi^0\gamma$, $g^2/4\pi\alpha = 4$.

Table 3

$(\psi' P^0 \gamma)$ couplings from limits on rates

Meson P^0	$g^2/4\pi\alpha$ for $\psi' \rightarrow P^0 \gamma$, from (a) DASP; (b) SPEAR data	$g^2/4\pi\alpha$ for $\psi \rightarrow P^0 \gamma$
η	$< 2.6 \times 10^{-4}$ (a) $< 44 \times 10^{-4}$ (b)	1.1×10^{-4} (a)
η'	$< 34 \times 10^{-4}$ (a) $(< 1,000 \times 10^{-4} \text{ (b)?})$	5×10^{-4} (a)
x	$< 1.1/b$ (a) $\approx 0.6/b$ (a) ¹⁸⁾ $< 8.8/b$ (b)	1.6 (calculated)

The data are clearly inadequate to challenge the above expectation. Very small ψ' rates also ensure in some dual models^{13, 40)}; they vanish also if ψ' is made of different heavy quarks, by the Zweig rule.

5.3 $\psi' \rightarrow \omega x$

The mechanism from $(c\bar{c})$ mixing in the ω is found to dominate the $(q\bar{q})$ -mechanism of Section 3.4.

Let ϵ_ω be the $(c\bar{c})$ -amplitude in the ω . We take

$$g_{\psi' \omega x} \approx g_{\psi' \psi \gamma} \frac{\epsilon_\omega}{\epsilon_\gamma}$$

so by Eqs. (2.14) and (2.15)

$$\frac{g_{\psi' \omega x}^2}{4\pi} \approx \left(7.2 + \frac{8.8}{3.5} \right) \times 10^2 \times |\epsilon_\omega|^2.$$

The phase space is also similar to $\psi' \rightarrow \psi \eta$, the rate

$$\begin{aligned} \Gamma(\psi' \rightarrow \omega x) &= \frac{g_{\psi' \omega x}^2}{4\pi} \frac{|\vec{P}|^3}{3 m_{\psi'}^2} \\ &= |\epsilon_\omega|^2 |\vec{P}|^3 \times (17.5 \pm \frac{21.2}{8.5} \text{ GeV}^{-2}) \end{aligned}$$

being very sensitive to m_x :

$$\begin{aligned}
 |\vec{p}| &= 1.91 \times \sqrt{1 - \frac{m_x}{2.901 \text{ GeV}}} \quad \text{GeV} \\
 &= 0.30 \pm 0.05 \quad \text{GeV} \\
 \text{taking } m_x &= 2.83 \pm 0.03 \quad \text{GeV} , \\
 \text{c.o.f. } |\vec{p}| &= 0.196 \text{ GeV in } \psi' \rightarrow \psi \eta
 \end{aligned}$$

so

$$\begin{aligned}
 \Gamma(\psi' \rightarrow \omega x) &= |\epsilon_\omega \times 10^4|^2 \times (0.5_{-0.3}^{+1.2}) \quad \text{keV} \\
 &\sim \frac{1}{2} \text{ to } 1 \quad \text{keV}
 \end{aligned} \tag{5.3}$$

taking

$$\epsilon_\omega \approx 10^{-4} \approx \epsilon_{NS}$$

as may be expected from Eq. (3.7), [and the meson masses in broken $SU(4)$ ^{4,5)}.]

In any case, ϵ_ω cannot exceed 10^{-3} , in view of a reported limit⁴¹⁾

$\Gamma(\psi' \rightarrow \omega x) < 20 \text{ keV}$, and the limit $\lesssim 50 \text{ keV}$ set by unidentified ψ' -decays.

For $\psi' \rightarrow \eta\omega$, $\eta'\omega$, $\eta\phi$ and $\eta'\phi$, branching ratios are expected to be about a tenth of those in Eqs. (3.19) and (3.12), i.e. very small, of order $10^{-2}\%$ or less, compared to experimental upper limits of 2.2% and 4.3% for $\eta\omega$ and $\eta'\omega$ respectively⁴¹⁾.

This is because identified ψ' decays have widths about $1/3$ of corresponding ψ decays⁴¹⁾ [just as $\Gamma(\psi' \rightarrow e^+e^-) \approx 1/2 \Gamma(\psi \rightarrow e^+e^-)$ ¹⁹⁾], so branching ratios are $\approx 1/10$ so large [e.g. $\text{B.R.}(\psi' \rightarrow p\bar{p}) = 0.024 \pm 0.008\%$ ⁴¹⁾]. It is curious to note that according to Eq. (5.1), ψ' couplings to other $(c\bar{c})$ systems are larger than those of ψ , by just such a factor $\approx \sqrt{2}$ to $\sqrt{3}$: the values are

$$\begin{aligned}
 \left| \frac{g_{\psi'\psi P^0}}{g_{\psi\psi P^0}} \right| &= \begin{array}{ll} 1.40 \pm 0.25 & \text{for } P^0 = \eta \\ 1.50 \pm 0.25 & \text{" } P^0 = X \end{array} \\
 \\
 \left| \frac{g_{\psi'\psi' P^0}}{g_{\psi'\psi P^0}} \right| &= \begin{array}{ll} 1.65 \pm 0.25 & \text{for } P^0 = \eta \\ 1.70 \pm 0.20 & \text{" } P^0 = X \end{array} .
 \end{aligned}$$

5.4 Radiative transitions of $x'(3.45)$

We identify the pseudo-scalar partner of ψ' to be $x'(3.45)$, observed^{4,2)} as an enhancement in inclusive γ -spectra of ψ' -decays and as a clustering of events in the $(\psi\gamma)$ -mass in $\psi' \rightarrow \psi\gamma\gamma$ with one of the photons converting.

Then the quark model as used in Section 4.1 gives

$$\begin{aligned}\Gamma(\psi' \rightarrow x'\gamma) &= 2.3 \pm 0.9 \text{ keV} \\ \text{for } m_c^* &= 3.4 \pm 0.8 \text{ GeV.}\end{aligned}$$

For $x' \rightarrow 2\gamma$, we take VDM and as discussed in Section 4.2 believe it reasonable to take only the ψ' -pole here. As a rough guide we then get

$$\begin{aligned}\frac{g_{x'\gamma\gamma}}{m_{x'}} &= (1.8 \pm 0.7) \times 10^{-3} \text{ GeV}^{-1} \\ \text{for } m_c^* &= 3.4 \pm 0.8 \text{ GeV.}\end{aligned}$$

$$\text{or } \Gamma(x' \rightarrow \gamma\gamma) = (0.7 \pm 0.2) \text{ keV.}$$

For $x' \rightarrow \psi\gamma$, we expect a very small coupling as in $\psi' \rightarrow x\gamma$, and may guess

$$\langle \psi | \mu | x' \rangle \approx \langle x | \mu | \psi' \rangle.$$

The rate $x' \rightarrow \psi\gamma$ is bounded by the $\psi' \rightarrow x\gamma$ limit, and we find

$$\Gamma(x' \rightarrow \psi\gamma) = 0.25 \Gamma(\psi' \rightarrow x\gamma).$$

Thus

$$\Gamma(x' \rightarrow \gamma\gamma) < 2.1 \text{ keV};$$

and

$$\begin{aligned}\text{for } \mathcal{L}_S &\approx 4 \text{ keV}, \\ \Gamma(\psi' \rightarrow x\gamma) &\approx 20 \text{ keV, etc.}\end{aligned}$$

The above rough estimates also take ψ' and $x'(3.45)$ as pure S-states; mixing with the D-states is potentially important, as well as modifications to the ψ' state from the nearby continuum⁴³). But we believe that the above rates are not unreasonable.

6. DECAYS OF $x(2.8)$ TO HADRONS

We write

$$|x\rangle \approx (c\bar{c}) + \frac{\delta_{NS}}{\sqrt{2}} (u\bar{u} + d\bar{d}) + \delta_S (s\bar{s}) \quad (6.1)$$

As is discussed below and explicitly occurs in dynamical models of the mixing^{13,14,22,44}), δ_{NS} and δ_S are energy dependent and so cannot be obtained from the order 10^{-2} ($c\bar{c}$)-amplitudes in η and η' . From the data $|\delta_{NS}| \lesssim 10^{-3}$ seems reasonable. This feature prevents us from discussing the meson masses, and limits the otherwise detailed predictions for two-body x -decays.

6.1 x -decays

For $x \rightarrow (\rho^0, \omega^0, \phi^0)\gamma$, the quark model transition moments from Appendix B are

$$\begin{aligned} \langle \rho^0 | \mu | x \rangle &= 3 \langle \omega^0 | \mu | x \rangle = 2 \left(\frac{e}{2m_\omega} \right) \delta_{NS} \\ \langle \phi^0 | \mu | x \rangle &= -\frac{2}{3} 2 \left(\frac{e}{2m_\omega} \right) \left(\frac{m_{u,d}^*}{m_s^*} \right) \delta_S \end{aligned} \quad (6.2)$$

Using VDM and SU(3), the $x \rightarrow VV'$ couplings are found:

$$g_{x\rho^0\rho^0} = -g_{x\rho^+\rho^-} = g_{x\omega^0\omega^0} = g_{\omega^0\rho^0\pi^0} \delta_{NS}$$

$$g_{x\phi^0\phi^0} = \sqrt{2} g_{\omega^0\rho^0\pi^0} \delta_S$$

$$g_{xK^{*+}K^{*-}} = -g_{xK^{*0}\bar{K}^{*0}} = (\delta_{NS} + \sqrt{2}\delta_S) \frac{1}{2} g_{\omega^0\rho^0\pi^0} \frac{m_{K^*}}{m_\omega}$$

Also

$$g_{xK^*K} = \frac{1}{2} (\delta_{NS} - \sqrt{2}\delta_S) g_{\rho^0\pi^+\pi^-}$$

Denoting $\bar{\delta}_{NS} \equiv 10^3 \times \delta_{NS}$, $\bar{\delta}_S \equiv 10^3 \times \delta_S$, the rates are

$$\Gamma(X \rightarrow \rho^0 \rho^0) = |\bar{\delta}_{NS}|^2 \times 14.0 \text{ keV}$$

$$\Gamma(X \rightarrow \rho^+ \rho^-) = |\bar{\delta}_{NS}|^2 \times 28.0 \text{ keV}$$

$$\Gamma(X \rightarrow \omega^0 \omega^0) = |\bar{\delta}_{NS}|^2 \times 13.7 \text{ keV}$$

$$\Gamma(X \rightarrow \phi^0 \phi^0) = |\bar{\delta}_S|^2 \times 15.7 \text{ keV}$$

$$\begin{aligned} \Gamma(X \rightarrow K^{*+} K^{*-}) &= \Gamma(X \rightarrow K^{*0} \bar{K}^{*0}) \\ &= |\bar{\delta}_{NS} + \sqrt{2} \bar{\delta}_S|^2 \times 5.5 \text{ keV} \end{aligned}$$

$$\Gamma(X \rightarrow \rho^0 \gamma) = |\bar{\delta}_{NS}|^2 \times 107 \text{ eV}$$

$$\Gamma(X \rightarrow \omega^0 \gamma) = |\bar{\delta}_{NS}|^2 \times 11.9 \text{ eV}$$

$$\Gamma(X \rightarrow \phi^0 \gamma) = |\bar{\delta}_S|^2 \left(\frac{m_{u,d}^*}{m_s^*} \right)^2 \times 39.6 \text{ eV}$$

$$\begin{aligned} \Gamma(X \rightarrow K^{*0} \bar{K}^0 + \bar{K}^{*0} K^0) &= \Gamma(X \rightarrow K^{*+} K^- + K^{*-} K^+) \\ &= |\bar{\delta}_{NS} - \sqrt{2} \bar{\delta}_S|^2 \times 0.92 \text{ keV}. \quad (6.3) \end{aligned}$$

6.2 Mixing parameters

$|\delta_{NS}/\epsilon_{NS}|$ may be estimated by comparing the rates of x and ψ decaying to ordinary hadrons. By analogy with $V \rightarrow e^+e^-$ in the quark model⁴⁵⁾, we assume

$$\begin{aligned}\Gamma(x \rightarrow \text{ordinary hadrons}) &\approx C m_x (|\delta_{NS}|^2 + \frac{1}{2} |\delta_S|^2 + \frac{1}{2} \delta_{NS} \delta_S) \\ &\sim 1 \text{ MeV} \quad (\text{cf. (4.13)}) .\end{aligned}$$

$$\begin{aligned}\Gamma(\psi \rightarrow \text{ordinary hadrons}) &\approx \frac{C}{3} m_\psi (|\epsilon_{NS}|^2 + \frac{1}{2} |\epsilon_S|^2 + 0.7 \epsilon_{NS} \epsilon_S) \\ &\approx 40 \text{ keV} \quad (\text{say}) .\end{aligned}$$

(6.4)

The numerical factors in Eq. (6.4) come from calculating the relative phase spaces of a large number of channels with different SU(3) quantum numbers. Then roughly

$$\left| \frac{\delta_{NS}}{\epsilon_{NS}} \right| \sim 3 \quad (\text{for } |\delta_S| \ll |\delta_{NS}|) \quad (6.5)$$

$$|\delta_{NS}| \sim 4 \times 10^{-4} \quad (6.6)$$

being $\sim 10^{-2}$ of the value obtaining from $(c\bar{c})$ mixing in the η and η' , Eqs. (2.15) and (2.16): for their amplitudes would have given $\Gamma(x \rightarrow \text{all}) \sim 1 \text{ GeV}$.

In support of the above rough argument, we may try SU(6)_W to get an order of magnitude estimate of the one exclusive channel, $x \rightarrow p\bar{p}$, on which any evidence exists^{16, 17, 46)}:

$$\Gamma(x \rightarrow p\bar{p}) \leq \Gamma(x \rightarrow \gamma\gamma) \approx 1 \text{ keV}$$

$$\text{and} \quad \equiv \frac{g_{xpp}^2}{4\pi} |\vec{p}|$$

for a Yukawa coupling

$$T(X \rightarrow p \bar{p}) = i g_{X p \bar{p}} \bar{u}(p) \gamma_5 u(p)$$

which is related by $SU(6)_W^{28)}$ to the g_M coupling of $\psi \rightarrow p \bar{p}$, Eq. (3.20): both are S-waves:

$$g_{X p \bar{p}} = \frac{1}{\sqrt{2}} \delta_{NS} \frac{3}{2} g_p \times 0.880$$

We find

$$|\delta_{NS}| \lesssim 6.5 \times 10^{-4}$$

in good agreement with Eq. (6.6); and from which Eq. (6.3) gives

$$B.R.(X \rightarrow \rho^0 \rho^0) \approx 2 \times 10^{-3}$$

This is consistent with a reported experimental limit⁴⁵⁾ from SPEAR:

$$B.R.(X \rightarrow 2\pi^+ 2\pi^-) < \frac{5 \times 10^{-3}}{B.R.(\psi \rightarrow x \gamma)} \\ < 5 \times 10^{-2} \quad \text{from our Eq. (4.3).}$$

The problem⁴⁷⁾ of excessive $x \rightarrow 4\pi^\pm$ decays is avoided.

It is clear, just on phenomenological grounds, that the $(q\bar{q})$ mixing amplitudes between pseudoscalar mesons must decrease by a factor of order 100 between $1/2$ -1 GeV and 3 GeV.

In dual models of the mixing^{13,14)} this is explicit, following from the energy dependence s^α of Regge propagators. These models have another channel [the Pomeron (disconnected quark-line) sector]. In the asymptotic freedom model^{23,44)} there is explicit energy dependence in the effective coupling, and there is the gluon sector as the extra channel. In our more limited approach, these features prevent a discussion of meson masses, or from evaluating in detail what would otherwise be firm predictions for the rates (6.3).

If we accept Eqs. (6.3) and (6.4) together, the branching ratios for $x \rightarrow$ meson pairs depend only on δ_S/δ_{NS} which can therefore in principle be measured. Figure 3 shows this dependence. With $|\delta_{NS}| \approx 0.4 \times 10^{-3}$, the rate

$x \rightarrow \rho^+ \rho^-$ is 4.5 keV. The others are less if $0 \leq \delta_S/\delta_{NS} \leq 1/\sqrt{2}$ which corresponds to one's anticipation of mainly SU(3) singlet mixing. In this range the rates $x \rightarrow \rho^0 \rho^0$ and $K^* K^*$ vary by a factor of two, and $\phi\phi$ by more, so the model is open to tests as these rates are correlated.

7. CONCLUSIONS AND SUMMARY

For radiative decays we have used together both VDM and the quark model, with SU(4) symmetry broken through effective quark masses m_q^* . We find this breaking for photons to be essential even when SU(4) is exact for couplings amongst hadrons.

Magnetic transitions proportional to m_q^{*-1} lead to $m_c^* = 3.3$ GeV for the charmed quark (charge $2/3$), by fitting to $\psi \rightarrow \eta\gamma$ and $\psi' \rightarrow \psi\eta$ rates. Numerically, m_c^* is twice the value expected because the ψ -photon coupling $g_{\psi\gamma}^{-1}$ has half its SU(4) value. The large m_c^* reduces $\psi \rightarrow x\gamma$ to $\lesssim 10$ keV; a much smaller experimental value would invalidate the model.

The result $m_c^* \approx m_\psi$ is equivalent to applying exact SU(4) to $g_{\psi x\gamma}$, $g_{\psi\eta\gamma}$, $g_{x\gamma\gamma}$, etc. in electromagnetism; but to our full couplings $g_{\psi\psi\eta}/m_\psi$ etc., for hadrons. Such a rule would be untenable as it is contradicted by the ordinary mesons²⁴⁾, where it is remarkable that SU(3) works very well, but only for $g_{\rho^0\gamma\gamma}/m_{\rho^0}$ in π^0 and $\eta \rightarrow 2\gamma$:

$$\begin{aligned} \frac{g_{\eta\gamma\gamma}/m_\eta}{g_{\pi^0\gamma\gamma}/m_{\pi^0}} &= \frac{1}{\sqrt{3}} = 0.577 \quad (\text{unmixed } \eta, \eta') \\ &= \frac{5-\sqrt{2}}{3\sqrt{2}} = 0.845 \quad (-10^\circ \text{ mixed } \eta, \eta') \end{aligned} \quad (7.1)$$

$$\text{and} \quad = 0.78 \pm 0.08 \quad (\text{experiment});$$

even though $m_\eta/m_{\pi^0} = 4.08$ which is comparable to $m_\psi/m_\omega = 3.94$ in our uses of SU(4). Similarly, $m_{K^+}/m_{\pi^+} = 3.54$ in the dimensional axial current couplings:

$$\begin{aligned} \frac{f_{K^+}}{f_{\pi^+}} &= 1 \quad (\text{SU(3)}) \\ &= 1.12 \pm 0.02 \quad (\text{experiment}). \end{aligned}$$

Thus SU(3) can work well despite large mass differences. We believe that this is also true for hadronic couplings in SU(4). Magnetic photon couplings do have SU(4) broken, but (in a physically meaningful) way through effective quark masses.

The quark mass parametrization is a way of making a rule that systematizes the application of SU(3) or SU(4) to both hadronic and photon vertices. Our point is that it is possible to find a set of simple conventions for which SU(4) works.

Vector dominance using ψ - and ψ' -poles also seems to work without contradiction. This agrees with its success in giving consistent values of the ψ -nucleon total cross-section, through both VDM in ψ -photoproduction, and nuclear shadowing²⁰⁾.

The $(q\bar{q})$ -mixing model of two-body ψ -decays works well when tested through SU(4) relations. Tests would be in $\psi \rightarrow \eta\omega$ and $\eta'\omega$. It also works for $\psi \rightarrow \eta\phi$ and $\eta'\phi$ if we take the mixing parameter ϵ_S/ϵ_{NS} from DASP; but not from SPEAR. The universal coupling model for $\psi \rightarrow p\bar{p}$ and $\Lambda\bar{\Lambda}$ works tolerably with the DASP value ϵ_S/ϵ_{NS} (rates are both a factor of 2 too small), but not at all with SPEAR.

Viewed over-all through this model, the data favour the DASP value $\epsilon_S/\epsilon_{NS} \approx 0.4^{+0.3}_{-0.2}$ (consistent with an SU(3) singlet ψ). This reduces $\psi \rightarrow K\bar{K}$ and $\psi \rightarrow K^{*\bar{K}}$, and increases $\psi \rightarrow \eta\phi$ and $\eta'\phi$, compared to the SPEAR value $\epsilon_S/\epsilon_{NS} \approx 0.1 \pm 0.2$ (consistent with no strange quarks in the ψ). This situation is in line with the fact that the DASP spectrometer affords better kaon identification than the SPEAR detector.

$SU(6)_W$ [exact or ℓ -broken²⁷⁾] for this $(q\bar{q})$ -component of ψ is inconsistent with our use of SU(4), and also with experiment on $\psi \rightarrow K\bar{K}$. The mass-broken scheme of Sakita and Wali²⁶⁾ does not contradict experiment but still contradicts SU(4).

These conclusions neglect interference from $\psi \rightarrow \gamma^* \rightarrow$ hadrons. This is a potentially serious point. As far as we could, the situation was assessed in Section 3.5 based on calculations given in outline in Appendix C, to which the reader may refer. It is possible that this interference is present and the results of Section 3 thereby wrong. We believe however that it is negligible, as the pattern of rates expected from one photon interference is absent from the data.

We found the mixing of $\eta_c \equiv x$ into η and η' to be of order 1%, but of $\eta(\eta')$ in η_c smaller by a factor ~ 100 . The mixing amplitudes are energy dependent^{13,14,44)}, and the states η , η' , η_c do not form a complete basis: there is also the gluon²³⁾ or Pomeron^{13,14)} channel in models.

Evaluation of $\psi' \rightarrow \omega x$ by comparing to $\psi' \rightarrow \psi\eta$ suggests a smaller energy dependence of mixing amongst ψ , ω and ϕ : the $(c\bar{c})$ in the ω cannot be more than 10 times that of ω in ψ . The precise limit depends critically on the value of m_x .

In our VDM assessment of $\psi' \rightarrow P^0 \gamma$ ($P^0 \equiv \eta, \eta'$ or χ) there is just one interesting conclusion from the data. As for the ψ -decays, so also here, the ψ - and ψ' -VDM poles must cancel almost completely in all cases. The data are tight enough to extract $(\psi' \psi P^0)$ -couplings. It is curious that (in all cases)

$$\left| g_{\psi' \psi' P^0} \right| > \left| g_{\psi' \psi P^0} \right| > \left| g_{\psi \psi P^0} \right| .$$

ψ' seems to couple more strongly to other $(c\bar{c})$ systems than does ψ : and less strongly to $(q\bar{q})$ and to (e^+e^-) by the same factor ≈ 2 . This may be an artifact of VDM (the left inequality comes from $|g_{\psi'}/g_{\psi}| \approx 1.65$), but if real it suggests that $\sigma(\psi N \rightarrow \psi' N) > \sigma(\psi N \rightarrow \psi N)$ [for $\sqrt{s}$ above threshold and the $t_{\min}$ limitation]. This could well be related to the apparent sharp threshold in ψ -production at $\sqrt{s} = 4.8 \text{ GeV}$ ^{48, 49}).

In summary, the data on ψ -decays do seem to be in line with standard quark model symmetry relations, and with vector dominance using just ψ and ψ' poles. It seems quite consistent with the data on ψ to have the Zweig rule and SU(4) exact at hadronic vertices, with decays proceeding through mixing in the meson wave functions. For electromagnetic vertices, SU(4) is broken by a factor 2, for a charmed quark charge $|Q_c| = 2/3$ (but not if $|Q_c| = 1/3$). Our analysis in no way determines Q_c . A crucial test of our analysis using only ψ - and ψ' -VDM poles is the consequence of a rate $\approx 7 \text{ keV}$ for $\psi \rightarrow \chi \gamma$: if it were 2 keV ^{18, 42}), it would prove our treatment to be wrong.

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APPENDIX A

KINEMATICS

Amplitudes for $V' \rightarrow VP$ and $V \rightarrow PP'$ are kinematically unique (P-waves)
($V \equiv 1^{+-}$ and $P \equiv 0^{-+}$ mesons):

$$T(V' \rightarrow VP) = i \frac{g_{V'VP}}{m_V} \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu(V')^* \epsilon_\nu(V) p_\rho(V') p_\sigma(V) \quad (A.1)$$

$$\text{and} \quad = i g_{V'VP} (\vec{\epsilon}(V')^* \wedge \vec{\epsilon}(V)) \cdot \vec{p}(V) \quad (A.1')$$

in the rest frame (i.e. transverse vector mesons).

$$T(V \rightarrow PP') = g_{VPP'} \epsilon(V)_\mu (p(P) - p(P'))^\mu \quad (A.2)$$

$$\text{and} \quad = 2 g_{VPP'} \vec{\epsilon}(V) \cdot \vec{p}(P) \quad (A.2')$$

in the rest frame (i.e. longitudinal vector meson).

SU(3) relations are

$$\begin{aligned} g_{VPP'} &= g_f \text{Trace } \lambda(V) [\lambda(P), \lambda(P')] \\ g_{V'VP} &= g_d \text{Trace } \lambda(P) \{ \lambda(V'), \lambda(V) \} \end{aligned} \quad (A.3)$$

where $\lambda(V)$ etc., are the standard Gell-Mann matrices for meson nonets and the traces are most rapidly calculated from quark-line graphs⁵⁰⁾ (Zweig's rule).

SU(6)_W symmetry relates²⁶⁾

$$g_d = 2 g_f \quad (A.4)$$

$$\text{e.g. : } g_{\omega\rho^+\pi^-} = 2 g_{\rho^+\pi^+\pi^-} \quad (A.5)$$

More generally [compare to Eqs. (A.1') and (A.2')], $SU(6)_W$ asserts equal magnitudes for all kinematically allowed helicity amplitudes, in any collinear frame, associated with three $(\underline{35} \otimes \underline{1}, L=0)$ $SU(6) \otimes O(3)$ meson states²⁸⁾ [up to the $SU(3)$ factors of Eq. (A.3)].

For example, take $V(1) \rightarrow V(2) + V(3)$: seven helicity amplitudes, each equal in a scale set by Eq. (A.4). This decay has two partial waves: P- and F-waves:

$$\begin{aligned} T(V(1) \rightarrow V(2) + V(3)) &= \int_{V(1)V(2)V(3)} \left(\epsilon_{(1)} \cdot \epsilon_{(2)}^* \epsilon_{(3)}^* (\mathcal{P}_{(1)} - \mathcal{P}_{(2)}) + \right. \\ &\quad \left. + G_{V(1)V(2)V(3)} \left(\epsilon_{(1)} \cdot (\mathcal{P}_{(2)} - \mathcal{P}_{(3)}) \epsilon_{(2)} \cdot (\mathcal{P}_{(3)} - \mathcal{P}_{(1)}) \epsilon_{(3)} \cdot (\mathcal{P}_{(1)} - \mathcal{P}_{(2)}) \right) \right) \end{aligned} \quad (A.6)$$

Unlike $V' \rightarrow VP$ and $V \rightarrow PP'$, these equalities are affected by the intrinsic breaking of W-spin conservation associated with transverse quark motion in the mesons [quark pair creation²⁷⁾ with $\Delta L_Z = \pm 1$ occurs as well as with $\Delta L_Z = 0$ ²⁸⁾].

This upsets the $SU(6)_W$ relation between P- and F-waves in Eq. (A.6). However, only one of seven amplitudes (for the pure helicity zero transition)

$$V(1, \lambda=0) \rightarrow V(2, \lambda=0) + V(3, \lambda=0)$$

has the F-wave [the G-term in Eq. (A.6)]. Therefore W-spin non-conservation has little scope to show up. Note that the dominant g-term in Eq. (A.6) is a Yang-Mills coupling which is similar to $V \rightarrow PP'$ (Eq. A.2).

For $T \rightarrow PV$ or $V \rightarrow TP$ ($T \equiv 2^{++}$ meson), we have a D-wave only with f-coupling:

$$T(V \rightarrow TP) = \kappa g_{TPV} \epsilon^{\mu\nu\rho\sigma} p_\mu(V) p_\nu(T) \epsilon_\rho(V) \epsilon_{\sigma\kappa}^*(T) p^\kappa(V) \quad (A.7)$$

Formulae for rates are

$$\Gamma(V' \rightarrow VP) = \frac{1}{3} \frac{g_{V'VP}^2}{4\pi} \left| \frac{\vec{P}}{M_{V'}} \right|^3 M_{V'} \quad (A.8)$$

$$\Gamma(V \rightarrow PP') = \frac{2}{3} \frac{g_{VPP'}^2}{4\pi} \left| \frac{\vec{p}}{M_V} \right|^3 M_V \quad (\text{A.9})$$

$$\Gamma(V \rightarrow \tau P) = \frac{1}{3} \frac{g_{V\tau P}^2}{4\pi} \frac{|\vec{p}|^5}{2} \quad (\text{A.10})$$

and also

$$\Gamma(V \rightarrow e^+e^-) = \frac{\alpha^2}{3} \frac{4\pi}{g_V^2} M_V \quad (\text{A.11})$$

where

$$T(V \rightarrow e^+e^-) \equiv \frac{M_V^2}{g_V} \frac{\sqrt{4\pi\alpha}}{p_V^2 + i\epsilon} \bar{v}(e^+) \not{\epsilon}(V) u(e^-).$$

[Formulae (A.8) and (A.9) also apply to $P \rightarrow V'V$ and $P \rightarrow VP'$ with the factor $(1/3) \rightarrow 1$; for identical particles $(1/3) \rightarrow (1/2)$.]

APPENDIX B

QUARK MODEL

1) Radiative decays $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$

These are magnetic transitions with amplitudes (A.1) and transition moments

$$\langle P | \mu_- | V_+ \rangle = \frac{g_{VP\gamma}}{2 m_V} \quad (B.1)$$

so $g_{VP\gamma}/e$ is like the g-factor (moment in units of the vector meson magneton $e/2m_V$).

In the quark model ($q\bar{q}$ -meson states are $|V_{a\bar{b}\pm}\rangle = (a \bar{b})_{S_Z=\pm 1}^{S=0}$, $|P_{a\bar{b}}\rangle = (a\bar{b})^{S=0}$, with $L=0$) the transition occurs by the real photon's magnetic field $\vec{B} = i \vec{p}(\gamma) \wedge \vec{\varepsilon}(\gamma)$ turning over the Pauli spin of the a - or $\bar{b}$ -quark:

$$\langle P_{a'\bar{b}'} | \mu_- | V_{a\bar{b},+} \rangle = \frac{e}{2} \left(\frac{Q_a}{m_a^*} + \frac{Q_{\bar{b}}}{m_{\bar{b}}^*} \right) \delta_{a,a'} \delta_{b,b'} \quad (B.2)$$

to compare with the magnetic moment of the vector meson

$$\begin{aligned} \mu_V &= \langle V_{a\bar{b},+} | \mu_- | V_{a\bar{b},+} \rangle \\ &= \frac{e}{2} \left(\frac{Q_a}{m_a^*} - \frac{Q_{\bar{b}}}{m_{\bar{b}}^*} \right) \end{aligned} \quad (B.3)$$

In the limit of exact $SU(3)$ $m_u^* \equiv m_d^* = m_s^*$, (B.2) and (B.3) are $SU(6)$ results; they also obtain in exact^{25,26)} and ℓ -broken²⁷⁾ $SU(6)_W$, where $|P\rangle \equiv -(a\bar{b})_{W_Z=0}^{W=1}$ is in the W -spin triplet with the $S_Z = \pm 1$ vector mesons. In this limit the vector meson magnetic moment measured in units of the quark magneton

$$\mu_V = (Q_a - Q_{\bar{b}}) \left(\frac{e}{2m_u^*} \right)$$

equals the vector meson charge Q_V .

In this limit m_u^* is a reduced matrix element, constant for each class of decays between mesons in the same L -multiplet (and so of similar P_γ). We take this standard algebraic view^{51-54,45)} of Eq. (B.2), but allow the additional possibility⁵³⁾ of $SU(3)$ breaking by $m_s^* > m_u^* \equiv m_d^*$ through the physical interpretation of the m_a^* being related to (transverse) masses of quarks bound in mesons.⁴⁴⁾

Note that Eq. (B.2) is not trivial, even if intuitive. Derivation^{51, 54)} from the SU(6) algebraic content of relativistic quark currents $\bar{q}\gamma_\mu Qq$ between hadrons requires recognition of the spin rotation on quarks with $p_\perp \neq 0$, on transforming from rest to $p_z = \infty$, as pointed out long ago by Gell-Mann⁵⁴⁾. Derivation from the non-relativistic quark model⁵³⁾ requires Eq. (B.2) to be multiplied by the overlap integral

$$\int d^3\vec{r} \psi_P^*(\vec{r}) e^{i\vec{P}_\gamma \cdot \vec{r}} \psi_V(\vec{r}) \quad (B.4)$$

which is a form factor allowing for absorption of momentum p_γ by the recoiling meson: this can be a serious factor as $p_\gamma \sim \langle |\vec{r}| \rangle^{-1}$. It would destroy the nice agreement with experiment that $g_{\omega\pi^0\gamma}/e = 2$, Eq. (B.9) below, and does not resolve the disagreements with experiment in other $V \rightarrow P\gamma$ decays (Section 3.b below). Therefore we retain the algebraic view of magnetic transitions (B.2). With exact SU(3), Eq. (B.2) is also an SU(3)-nonet symmetry result for ideally mixed ω and ϕ ⁵²⁾; and also a VDM result as this subsumes SU(3)⁵²⁾.

2) Normalization from $\omega \rightarrow \pi^0\gamma$

For $\omega \rightarrow \pi^0\gamma$ with ideally mixed ω^0 , Eq. (B.2) is

$$\langle \pi^0 | \mu | \omega \rangle = \frac{g_{\omega\pi^0\gamma}}{2 m_\omega} = \frac{e}{2 m_{u,d}^*} \quad (B.5)$$

Vector dominance (VDM) reads ($e \equiv \sqrt{4\pi\alpha} = 1/3.30$)

$$e^{-1} g_{\omega\pi^0\gamma} = \frac{g_{\omega^0\rho^0\pi^0}}{g_\rho} \quad (B.6)$$

and

$$g_{\rho^0\pi^+\pi^-} = g_\rho \quad (B.7)$$

(experiment: 6.03 ± 0.14 and 5.68 ± 0.23).

Now SU(6)_W [Eq. (A.4)], exact or ℓ -broken, asserts

$$g_{\omega^0\rho^0\pi^0} = 2 g_{\rho^0\pi^+\pi^-} \quad (B.8)$$

Thus by applying VDM to both sides of Eq. (B.8),

$$e^{-1} g_{\omega\pi^0\gamma} = 2 \quad (B.9)$$

and experimentally $= 2.00 \pm 0.06$ from $\Gamma(\omega \rightarrow \pi^0\gamma) = 880 \pm 50$ keV.

As Eq. (B.9) works so well, we accept Eq. (B.8), to provide the normalizations of VVP and VP γ couplings used in this paper:

$$g_{\omega^0 \rho^0 \pi^0} = 2 g_\rho = 11.4 \pm 0.5 \quad (\text{B.10})$$

$$m_{u,d}^* = \frac{1}{2} m_\omega = 391 \text{ MeV.} \quad (\text{B.11})$$

We note that the VDM model of Gell-Mann et al.⁵⁵⁾ for $\omega \rightarrow 3\pi$ agrees well, giving $|g_{\omega^0 \rho^0 \pi^0}| = (1.06 \pm 0.14)$ times Eq. (B.10); and that the quark model for nucleon magnetic moments is a little off (B.11), giving a magnetic coupling 1.16 times (B.11) from $(m_{u,d}^*)_{\text{baryons}} = M_N/2.79 \approx 336 \text{ MeV.}$

3) SU(3) symmetry breaking

The quark formula (B.2) suggests a pattern of breaking that we use in this paper: heavier quarks have less magnetism. For SU(3), such breaking in the current operator gives to the magnetic dipole coupling of the photon SU(3)-components additional to the usual U-spin zero member of an octet:

$$\gamma \sim \sqrt{\frac{3}{2}} \left[\frac{8}{\sqrt{2}} \right]_{U=0} + \frac{(1-\lambda^*)}{3\sqrt{3}} \left(\left[\frac{1}{\sqrt{2}} \right] - \sqrt{2} \left[\frac{8}{\sqrt{2}} \right]_{I=0} \right)$$

There is some evidence that the breaking parameter

$$\lambda^* \equiv \frac{m_{u,d}^*}{m_s^*} \quad (\text{B.12})$$

is less than unity.

3.a) For baryons in the quark model,

$$\mu_\Lambda = -\frac{1}{3} \frac{e}{2m_s^*} = -(0.67 \pm 0.06) \frac{e}{2M_p} \quad (\text{experiment})$$

$$\mu_p = -\frac{3}{2} \mu_n = \frac{e}{2m_{u,d}^*} = 2.79 \frac{e}{2M_p} \quad (\text{experiment})$$

giving for baryons

$$\lambda_{\text{baryons}}^* = 0.72 \pm 0.06 \approx \frac{1}{\sqrt{2}}.$$

The values of μ_{Σ^+} , μ_{Ξ^-} and $\mu_{\Sigma^0\Lambda}$ are consistent²⁴⁾ with this^{*}).

3.b) $V \rightarrow P\gamma$ decays have rates⁵⁶⁾ that contradict exact SU(3)^{57,6,58)}. An inspection²⁴⁾ of the new data (on $K^{*0} \rightarrow K^0\gamma$ and $\phi \rightarrow \eta\gamma$) and amplitudes given by Eq. (B.2) shows that an adequate fit can be achieved (see Table 4 below) if

$$\lambda_{\text{mesons}}^* \approx \frac{1}{2}$$

except of course for $\rho^- \rightarrow \pi^-\gamma$. This decay is clearly independent of λ^{**} , and of kinematical differences from $\omega \rightarrow \pi^0\gamma$; it could only be fitted by large deviations from ideal mixing in the ω ⁵⁹⁾, or breaking of isospin^{60)**)}. According to Eq. (B.12) the standard predictions⁵²⁾ are modified and are listed in Table 4 for reference [in brackets for η and η' with -10° mixing²⁹⁾].

Note that the ratio

$$\frac{\Gamma(K^{*0} \rightarrow K^0\gamma)}{\Gamma(K^+ \rightarrow K^+\gamma)} = \left(\frac{1 + \lambda^*}{2 - \lambda^*} \right)^2$$

depends on λ^* , unlike many approaches to SU(3) breaking: as λ^* is reduced from 1, the K^{*+} rate rises and the K^{*0} rate falls. This distinctive feature could be checked in $e^+e^- \rightarrow (K^*\bar{K} + \bar{K}^*K)$ or by Primakoff production $K \rightarrow K^*$ on nuclear targets at very high energy where the hadron exchange background has fallen away.

Similarly, for charmed mesons, using Eq. (B.2),

$$D^0 \equiv (\bar{u}c), \quad D^+ \equiv (\bar{d}c), \quad F^+ \equiv (\bar{s}c), \text{ etc.,}$$

$$\frac{\sigma(e^+e^- \rightarrow D^{*0}\bar{D}^0 + \bar{D}^{*0}D^0)}{\sigma(e^+e^- \rightarrow D^{*+}D^- + D^{-*}D^+)} = 4 \frac{(1 + \lambda_c^*)^2}{(1 - 2\lambda_c^*)^2}$$

) The m_q^ differ for mesons and baryons, but the ratios may not. The spacing in the baryon decuplet gives $m_s^{**} - m_{u,d}^{**} = 147$ MeV. For mesons, $\lambda^* = 0.765$ from (m_u/m_ϕ) .

**) The predicted ratio

$$\frac{T(\rho^{\pm 0} \rightarrow \pi^{\pm 0}\gamma)}{T(\omega^0 \rightarrow \pi^0\gamma)} = \frac{\frac{2}{3} - \frac{1}{3}}{\frac{2}{3} + \frac{1}{3}} = \frac{1}{3}$$

reflects only the charges and equal masses of the u- and d-quarks, and ideal $\omega(\rho^0) = 1/\sqrt{2}(u\bar{u} + d\bar{d})$. This clear result shows why⁵⁷⁾ it is not possible⁵⁸⁾ to fit the rates in the standard framework.

where

$$\lambda_c^* = \frac{m_{u,d}^*}{m_c^*}$$

The ratio is $8.5_{-3.2}^{+2.4}$ for our $\lambda_c^* = 0.115_{-0.180}^{+0.035}$ of Eq. (2.12), not the SU(4) symmetry value of 16. This is relevant to $D^{*\pm}$ searches in e^+e^- annihilation.

Similarly,

$$\sigma'(e^+e^- \rightarrow F^{+*} F^- + F^{-*} F^+) = \sigma'(e^+e^- \rightarrow D^{+*} D^- + D^{-*} D^+)$$

$$= \frac{(1 - 2\lambda_c^*)^2}{8\lambda_c^{*2}} \sigma'(e^+e^- \rightarrow \psi x)$$

where

$$\sigma' \equiv \sigma / |p_{meson}^{c.m.}|^3$$

Table 4

Decay	Predicted rates (keV)	Remarks	Experiment (keV) (old values in brackets)
$\omega \rightarrow \pi^0 \gamma$	870.0	By $SU(6)_W$ and VDM From $\phi \rightarrow \rho \pi$ Data could allow ⁵⁶⁾ $\Gamma = 30$ to 80 keV	880 ± 50 (old 870 ± 50)
$\omega \rightarrow \eta \gamma$	4.4 (6.9)		8.9 ± 40
$\phi \rightarrow \eta \gamma$	$\left\{ \begin{array}{l} 224 \lambda^{*2} \\ (167 \lambda^{*2}) \end{array} \right\}$		65 ± 15 (old 126 ± 46 and 306 ± 80)
$\phi \rightarrow \eta' \gamma$	$0.51 \lambda^{*2}$ ($0.77 \lambda^{*2}$)		
$\phi \rightarrow \pi^0 \gamma$	12 ± 3		5.7 ± 2.1 (old 10 ± 5)
$\rho^\pm \rightarrow \pi^\pm \gamma$	91		35 ± 10
$\rho^0 \rightarrow \pi^0 \gamma$	93		
$\rho^0 \rightarrow \eta \gamma$	36 (55)		< 160
$K^{*0} \rightarrow K^0 \gamma$	$214 \left(\frac{1 + \lambda^*}{2} \right)^2$		75 ± 35
$K^{*\pm} \rightarrow K^\pm \gamma$	$52 (2 - \lambda^*)^2$		< 80
$\eta' \rightarrow \rho^0 \gamma$	153 (115)		$(274 \pm 22) \times (\Gamma_{\text{tot. in MeV}})$
$\eta' \rightarrow \omega \gamma$	13.8 (10.4)		$< 80 \times (\Gamma_{\text{tot. in MeV}})$
$\pi^0 \rightarrow 2\gamma$	7.60×10^{-3} (8.8 ± 1.7) $\times 10^{-3}$	PCAC anomaly Vector dominance	$(7.92 \pm 0.9) \times 10^{-3}$
$\eta \rightarrow 2\gamma$	$\left\{ \begin{array}{l} 0.174 \pm 0.020 \\ (0.372 \pm 0.042) \end{array} \right\}$	By $SU(3)$ from $\pi^0 \rightarrow 2\gamma$	0.324 ± 0.046 (1.00 ± 0.22)
$\eta' \rightarrow 2\gamma$	$\left\{ \begin{array}{l} 13.5 \pm 1.5 \\ (11.6 \pm 1.3) \end{array} \right\}$	By $SU(3)$ from $\pi^0 \rightarrow 2\gamma$	$(20.0 \pm 3.0) \times (\Gamma_{\text{tot. in MeV}})$
$\frac{\eta' \rightarrow \rho^0 \gamma}{\eta' \rightarrow 2\gamma}$	$\left\{ \begin{array}{l} 21 \pm 2 \\ (18 \pm 2) \end{array} \right\}$	By $SU(6)$ and VDM	15 ± 4

ONE PHOTON INTERFERENCE

The decay $\psi \rightarrow \gamma^* \rightarrow AB$ has amplitude

$$T_{\gamma}(AB) = \frac{4\pi\alpha}{g_{\psi}} F_{AB}(m_{\psi}^2)$$

$F_{AB}(q^2)$ is the (transition) form factor; for $\psi \rightarrow VP$, $T_{\gamma}(VP) \equiv g_{\psi VP}/m_{\psi}$ and F_{VP} has normalization

$$F_{VP}(0) = \frac{g_{VP\gamma}}{e m_V}$$

$$\left(\text{where } \frac{g_{\omega\pi^0\gamma}}{e} = 2 \right)$$

in terms of the magnetic transition couplings of Appendices A and B. Writing

$$\begin{aligned} (g_{\psi\rho^0\pi^0})_{\text{strong}} &\equiv g_0 \\ \frac{F_{VP}(m_{\psi}^2)}{F_{VP}(0)} &\equiv \mathcal{E}_{VP} \end{aligned}$$

we find the following algebraic structure for the couplings:

$$\begin{aligned} g(\rho^{\pm}\pi^{\pm}) &\equiv g(\rho^0\pi^0) = g_0 + \frac{2}{3} \mathcal{E}_{\rho\pi} \\ g(K^*K)^0 &= \frac{1}{2} \left(1 + \sqrt{2} \frac{\mathcal{E}_0}{\mathcal{E}_{NS}} \right) g_0 - \frac{2}{3} (1 + \lambda^*) \mathcal{E}_{K^*K^0} \\ g(K^*K)^{\pm} &= \frac{1}{2} \left(1 + \sqrt{2} \frac{\mathcal{E}_{\pm}}{\mathcal{E}_{NS}} \right) g_0 + \frac{2}{3} (2 - \lambda^*) \mathcal{E}_{K^*K^{\pm}} \end{aligned}$$

For η and η' , we give unmixed (and -10° mixed) cases:

$$\begin{aligned} g(\phi^0\eta) &= \begin{Bmatrix} -\frac{2}{\sqrt{3}} \\ (-1) \end{Bmatrix} \left(\frac{\mathcal{E}_S}{\mathcal{E}_{NS}} g_0 - \frac{2\sqrt{2}}{3} \lambda^* \mathcal{E}_{\phi\eta} \right) \\ g(\phi^0\eta') &= \begin{Bmatrix} \sqrt{\frac{2}{3}} \\ (1) \end{Bmatrix} \left(\frac{\mathcal{E}_S}{\mathcal{E}_{NS}} g_0 - \frac{2\sqrt{2}}{3} \lambda^* \mathcal{E}_{\phi\eta'} \right) \end{aligned}$$

and also

$$\begin{aligned}
 g(\omega^0 \eta) &= \left\{ \frac{1}{\sqrt{3}} \right\} \left(g_0 + \frac{2}{3} \varepsilon_{\omega^0 \eta} \right) \\
 g(\omega^0 \eta') &= \left\{ \frac{\sqrt{3}}{(\frac{1}{\sqrt{2}})} \right\} \left(g_0 + \frac{2}{3} \varepsilon_{\omega^0 \eta'} \right) \\
 g(\rho^0 \eta) &= \left\{ \frac{2}{(\sqrt{2})} \right\} \varepsilon_{\rho \eta} \\
 g(\rho^0 \eta') &= \left\{ \frac{2\sqrt{3}}{(\sqrt{2})} \right\} \varepsilon_{\rho \eta'} \\
 g(\omega^0 \pi^0) &= 2 \varepsilon_{\omega^0 \pi^0} \quad (C.1)
 \end{aligned}$$

We have taken exact SU(3) for strong vertices and parametrized for an octet component $[(\varepsilon_S/\varepsilon_{NS}) \neq (1/\sqrt{2})]$ in the ψ . (These two effects cannot be separated.) Allowance for SU(3) breaking in the radiative meson couplings is included by $\lambda^* \equiv m_S^*/m_{u,d}^* \neq 1$, Eq. (B.12). Our results agree with earlier work³²).

Now we specialize to the ρ^0, ω, ϕ -pole model for the $F_{VP}(m_\psi^2)$, using SU(3) for hadronic couplings g_{VVP} . The ε_{VP} are then real at $q^2 = m_\psi^2$:

$$\begin{aligned}
 \varepsilon_{\phi^0 \eta} &\equiv \varepsilon_{\phi^0 \eta'} = -2.8 \times 10^{-3} \\
 \varepsilon_{\omega^0 \eta} &\equiv \varepsilon_{\omega^0 \eta'} = -2.1 \times 10^{-3} \\
 \varepsilon_{\rho^0 \eta} &\equiv \varepsilon_{\rho^0 \eta'} = -2.0 \times 10^{-3} \\
 &\equiv \varepsilon_{\omega^0 \pi^0} \\
 \text{and} \\
 \varepsilon_{K^* K^0} &= -2.5 \times 10^{-3} \\
 \varepsilon_{\rho \pi} &= -2.1 \times 10^{-3} \\
 \varepsilon_{K^* K^\pm} &= -0.30 \times 10^{-3} \\
 &\approx \frac{1}{7} \varepsilon_{\rho \pi} \quad (C.2)
 \end{aligned}$$

For comparison, the experimental rates give

$$\begin{aligned}
 |g(\rho^0 \pi^0)| &= (5.8 \pm 0.8) \times 10^{-3} \\
 |g(K^* K)^0| &= (4.6 \pm 0.5) \times 10^{-3} \quad (\text{SPEAR}) \\
 |g(K^* K)^\pm| &= (4.6 \pm 0.5) \times 10^{-3} \quad (\text{SPEAR}) \\
 &\quad \text{or} \quad = (6.2 \pm 0.9) \times 10^{-3} \quad (\text{DASP}) \\
 |g(\gamma \phi)| &= (2.2 \pm ?) \times 10^{-3} \\
 |g(\gamma' \phi)| &\leq (2.1 \pm ?) \times 10^{-3}
 \end{aligned}$$

$\varepsilon_{(K^* K)^\pm}$ is so small because SU(3) forces a cancellation of poles to reproduce the smaller $q^2 = 0$ normalization. The identities in (C.2) are between form factors containing the same single pole.

For $\psi \rightarrow K\bar{K}$ and $\pi^+\pi^-$,

$$\begin{aligned}
 g_{\psi(K\bar{K})} &= \frac{1}{16} \left(1 - \sqrt{2} \frac{\varepsilon_S}{\varepsilon_{NS}} \right) + \varepsilon_{(K\bar{K})} \\
 g_{\psi \pi^+\pi^-} &= \varepsilon_{\pi^+\pi^-}
 \end{aligned}$$

In the pole model,

$$\begin{aligned}
 \varepsilon_{K^+K^-} &= -0.65 \times 10^{-3} \\
 \varepsilon_{K_L K_S} &= -0.15 \times 10^{-3} \\
 \varepsilon_{\pi^+\pi^-} &= -0.504 \times 10^{-3}
 \end{aligned}$$

Again $\varepsilon_{K_L K_S}$ is smaller because $F_{K_L K_S}(0) \equiv 0$.

The ρ^0 -pole model of $F_\pi(q^2)$ agrees with experiments from $q^2 = -2$ to $+3.0$ (GeV)² ⁶¹), e.g.:

$$\begin{aligned}
 |F_\pi(q^2)| &= 0.069 \quad (\text{in model}) \\
 &= 0.13 \pm_{13}^{06} \quad (\text{expt.})
 \end{aligned}$$

For $K\bar{K}$, the pole model is in less good shape, giving the wrong sign for the $K_L K_S$ charge (radius)^{2 62)}. This is a consequence of SU(3) being broken in the model at $q^2 \neq 0$ by $m_\phi > m_{\rho,\omega}$. The pole model seems to be wrong even for the kaon form factor.

REFERENCES

- 1) S.L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D 2, 1285 (1970).
M.K. Gaillard, B.W. Lee and J.L. Rosner, Rev. Mod. Phys. 47, 277 (1975).
- 2) S. Okubo, Phys. Letters 5, 165 (1963).
G. Zweig, CERN preprints TH.401 and 412 (1964) [unpublished].
J. Iizuka, Prog. Theor. Phys. Suppl. 37-38, 21 (1966).
- 3) L. Clavelli and T.C. Yang, Phys. Letters 57B, 83 (1975).
H.J. Lipkin, Phys. Letters 60B, 371 (1976).
- 4) A. Kazi, G. Kramer and D.H. Schiller, Nuovo Cimento Letters 15, 120 (1976),
and DESY reports 75/10 and 75/11 (1975) and 76/18 (1976).
- 5) V.S. Mathur, S. Okubo and S. Borchardt, Phys. Rev. D 11, 2572 (1975).
E. Takasugi and S. Oneda, Phys. Rev. D 13, 70 (1976).
- 6) J.L. Rosner, Minnesota preprint (1975) of talk to APS Seattle Conference,
August 1975.
- 7) H. Harari, Phys. Letters 60B, 172 (1976).
- 8) R.N. Cahn and M. Chanowitz, Phys. Letters 59B, 277 (1975).
- 9) L. Clavelli, Bordeaux preprint (1975).
- 10) T.F. Walsh, Nuovo Cimento Letters 14, 290 (1975) and DESY report 76/13
(April 1976).
- 11) M. Singer, Wisconsin preprint C00-488 (1975).
- 12) F. Ravndal, Nuovo Cimento Letters 14, 432 (1975).
- 13) H.M. Chan et al., Phys. Letters 60B, 367 (1976); Phys. Letters 60B, 469 (1976).
- 14) T. Inami et al., Phys. Letters 61B, 60 (1976).
D.P. Roy, Phys. Letters 62B, 315 (1976).
J.F. Bolzan et al., Ohio preprints (1976).
J. Pasupathy, Phys. Rev. Letters 58B, 71 (1975).
C. Rosenzweig, Phys. Rev. D 13, 3080 (1976).
- 15) W. Tannenbaum et al., Phys. Rev. Letters 36, 402 (1976).
- 16) B. Wiik and J. Heintze, in Proc. Internat. Symposium on Lepton-Photon Inter-
actions at High Energies, Stanford University, 1975 (Ed.: W.T. Kirk)
Stanford University, 1975).
- 17) G. Wolf, DESY preprint (unpublished), invited talk at the International
Meeting on Storage Ring Physics, Flaine, France, February, 1976.
- 18) Reports to Aachen Neutrino Conference, June 1976.
- 19) A.M. Boyarski et al., Phys. Rev. Letters 34, 1357 (1975).
V. Lüth et al., Phys. Rev. Letters 35, 1124 (1975).
- 20) D.M. Ritson, SLAC-PUB-1728 (March 1976).

- 21) B. Jean-Marie et al., Phys. Rev. Letters 36, 291 (1976).
F. Vannucci et al., SLAC-PUB-1724 and LBL-4699 (March 1976).
- 22) W. Braunschweig et al., DESY report 76/28 and Phys. Letters 57B, 279 (1975).
- 23) T. Appelquist and H.D. Politzer, Phys. Rev. Letters 34, 43 (1975).
- 24) R.L. Heimann (unpublished).
- 25) H.J. Lipkin and S. Meshkov, Phys. Rev. Letters 14, 670 (1965).
K.J. Barnes, P. Carruthers and F. von Hippel, Phys. Rev. Letters 14, 82 (1965).
- 26) B. Sakita and K.C. Wali, Phys. Rev. Letters 14, 404 (1965), and Phys. Rev. 139, B1355 (1965).
- 27) J.L. Rosner, Phys. Reports 11C, 187 (1974).
E.W. Colglazier and J.L. Rosner, Nuclear Phys. B27, 349 (1971).
L. Micu, Nuclear Phys. B10, 521 (1969).
A. Hey, J.L. Rosner and J. Weyers, Nuclear Phys. B61, 205 (1973).
J.L. Rosner, Phys. Rev. D 6, 1781 (1972).
- 28) R. Carlitz and M. Kislinger, Phys. Rev. D 2, 336 (1970).
- 29) N. Isgur, Phys. Rev. D 13, 122 (1976).
A. Bramon and M. Greco, Phys. Letters 48B, 137 (1974).
- 30) A.D. Martin and C. Michael, Phys. Letters 37B, 513 (1971).
G.C. Fox and C. Quigg, Annu. Rev. Nuclear Sci. 21, 219 (1973).
- 31) F. Vanucci et al., SLAC-LBL preprint (May 1976), [submitted to Phys. Rev. Letters].
- 32) S. Rudaz, Phys. Rev. D 14, 298 (1976).
H. Kowalski and T.F. Walsh, Phys. Rev. D 14, 852 (1976).
- 33) G. Goldhaber et al., LBL-4224, SLAC-PUB-1622 (1975).
A.M. Boyarski et al., SLAC-PUB-1599, LBL-3897 (1975).
- 34) G.J. Feldman and M.L. Perl, Phys. Reports 5C, 235 (1975).
- 35) J.J. Sakurai, Currents and Mesons (University of Chicago Press, 1969).
J.J. Sakurai, Ann. Phys. (NY), 11, 1 (1960).
M. Gell-Mann, Phys. Rev. 125, 1067 (1962).
R.P. Feynman, Photon-hadron interactions, (Benjamin, Reading, Mass., 1972).
- 36) C.B. Chiu, Annu. Rev. Nuclear Sci. p. 255 (1972).
C.B. Chiu, Nuclear Phys. B30, 477 (1971).
G. Höhler and E. Pietarinen, Nuclear Phys. B95, 210 (1975).
- 37) W-K Tung, Phys. Rev. Letters 26, 1531 (1969).
- 38) J.W. Simpson et al., Phys. Rev. Letters 35, 699 (1975).
- 39) E.B. Hughes et al., Phys. Rev. Letters 36, 76 (1976).
- 40) E. Etim and M. Greco, CERN preprint TH. 2174.
- 41) W. Tannenbaum, SLAC-PUB-1722, LBL-4695 (February, 1976).

- 42) D. Aschman, CERN seminar (May, 1976);
V. Lüth, talk given at Neutrino Conference, Aachen (June 1976).
B. Wiik, Rapporteur's talk, Internat. Conf. on High Energy Physics, Tbilisi (July 1976).
- 43) E. Eichten et al., Phys. Rev. Letters 36, 500 (1976).
- 44) A. de Rujula, H. Georgi and S.L. Glashow, Phys. Rev. D 12, 147 (1975).
A. de Rujula and S.L. Glashow, Phys. Rev. Letters 34, 46 (1975).
- 45) R. van Royen and V.F. Weisskopf, Nuovo Cimento 50A, 617 (1967).
- 46) G. Wolf, talk at Flaine meeting (March 1975) [unpublished].
- 47) M. Kugler, reported by H. Harari, ref. 7.
- 48) D.M. Ritson, SLAC-PUB-1928 (March 1976).
R. Prepost and W. Lee (1975) in ref. 16, p. 213 and 241.
B. Gittelman et al., Phys. Rev. Letters 35, 1616 (1975).
U. Camerini et al., Phys. Rev. Letters 35, 483 (1975).
B. Knapp et al., Phys. Rev. Letters 34, 1040 (1975).
T. Nash et al., Phys. Rev. Letters 36, 1233 (1976).
- 49) D. Horn, Phys. Letters 58B, 323 (1975).
V. Barger and R.J.N. Phillips, Phys. Letters 58B, 433 (1975).
- 50) G. Zweig, in ref. 2.
H. Harari, Phys. Rev. Letters 22, 562 (1969).
J.L. Rosner, Phys. Rev. Letters 22, 689 (1969).
D.E. Neville, Phys. Rev. Letters 22, 494 (1969).
- 51) F.J. Gilman and I. Karliner, Phys. Rev. D 10, 2194 (1974).
A. Love and D.V. Nanopoulos, Phys. Letters 45B, 507 (1973).
H.J. Melosh IV, Phys. Rev. D 9, 1095 (1974), and California Institute of Technology Thesis (1973).
- 52) A.A. Anisovitch et al., Phys. Letters 16, 194 (1965).
W.E. Thirring, Phys. Letters 16, 335 (1965); Acta Physica Austriaca Suppl. II (1966).
L.D. Soloviev, Phys. Letters 16, 345 (1965).
S. Okubo, Phys. Letters 4, 14 (1963).
G. Morpurgo, in Proc. Internat. School of Physics "Ettore Majorana", Erice, 1968 (Academic Press, NY and London, 1969), and Proc. Internat. School of Physics, Erice, 1971 (Editrice compositori, Bologna, 1973).
A. Dar and V.F. Weisskopf, Phys. Letters 26B, 670 (1968).
R.H. Dalitz and D.G. Sutherland, Nuovo Cimento 37, 1777 (1965).
- 53) C. Becchi and G. Morpurgo, Phys. Rev. 140 B687 (1965).
- 54) M. Gell-Mann, Phys. Rev. Letters 14, 77 (1965).
S. Badier and C. Bouchiat, Phys. Letters 15, 96 (1965).
H.J. Melosh IV, ref. 51.
- 55) M. Gell-Mann, D. Sharp and W.G. Wagner, Phys. Rev. Letters 8, 261 (1962).
- 56) R. Gobbi et al., Phys. Rev. Letters 33, 1450 (1975).
W.C. Carithers et al., Phys. Rev. Letters 35, 349 (1975).
C. Bemporad, Proc. Internat. Symposium on Lepton-Photon Interactions at High Energies, Stanford University, 1975 (Ed. W.T. Kirk) [Stanford University 1975].

- 57) R.G. Moorhouse, CERN preprint TH. 2103 (Dec. 1975), to be published in Proc. 1975 Triangle Meeting, Bratislava.
- 58) P.J. O'Donnell, Phys. Rev. Letters 36, 177 (1976).
B.J. Edwards and A.N. Kamal, Phys. Rev. Letters 36, 241 (1976) and Alberta preprints (1976).
D.H. Boal and R. Torgerson, Nuovo Cimento Letters 15, 417 (1976).
S. Ono, Nuovo Cimento Letters 14, 569 (1975).
A.N. Mitra, Phys. Rev. D 14, 855 (1976).
D.H. Boal, R.H. Graham and J.W. Moffatt, Phys. Rev. D 13, 3107 (1976).
G.J. Gounaris, Phys. Letters 63B, 307 (1976).
T. Barnes, Phys. Letters 63B, 65 (1976).
- 59) D.H. Boal, R.H. Graham and J.W. Moffatt, Phys. Rev. Letters 36, 714 (1976).
- 60) G. Grunberg and F.M. Renard, Montpellier preprint (1976).
- 61) D. Bollini et al., Nuovo Cimento Letters 14, 418 (1975).
M. Basile et al., CERN preprint (April 1976), "The Present Knowledge of $|F_\pi|$ and the value of a low-energy e^+e^- storage ring".
- 62) F. Dydak et al., Nuclear Phys. B102, 253 (1976).

Figure captions

- Fig. 1 : Rate $\psi \rightarrow x\gamma$ as a function of the quark mass ratio $m_c^*/m_{u,d}^*$, for various values of m_x (GeV). The value $m_c^*/m_{u,d}^* = 8.7 \pm 2.0$ obtained from our model of $\psi \rightarrow \eta\gamma$ is shown.
- Fig. 2 : Rate $x \rightarrow \gamma\gamma$ (two values) against the rate $\psi' \rightarrow x\gamma$. The upper scale is the corresponding upper limit on $b \equiv 10^3 \times \text{B.R.} (x \rightarrow \gamma\gamma)$ from the data of Ref. 16.
- Fig. 3 : Branching ratios of decays $x \rightarrow VV$ as a function of δ_S/δ_{NS} . ($\rho\rho$) is for $(\rho^0\rho^0)$, or $\frac{1}{2} \times (\rho^+\rho^-)$, or $(\omega^0\omega^0)$; also for $131 \times (\rho^0\gamma)$ and for $1200 \times (\omega^0\gamma)$. (K^*K^*) is for either neutral or charged modes. ($\phi\phi$) is also for $400 \times (\phi^0\gamma)$.

Fig. 1

Fig. 2