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ASYMPTOTIC BEHAVIOUR AND QUANTUM CORRECTIONS IN THE REGGEON
CALCULUS ABOVE THE CRITICAL INTERCEPT

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A B S T R A C T

A way to account for the leading quantum corrections to the Reggeon Green's functions above the critical intercept is proposed, with the results of Bronzan as a first non-trivial order. The results are in agreement with those obtained recently in a quantum formalism with appropriate boundary conditions. The relation to Gribov diagrams is briefly discussed.

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In this note we use available information (supplemented by some assumptions) on the classical solutions of the Reggeon field theory above the critical intercept to derive the leading asymptotic behaviour of the scattering amplitude, including quantum corrections. Our results agree with those obtained recently in a more sophisticated approach ¹⁾, and we think that they help to make the s channel interpretation of the latter more transparent, because of close connection with the perturbative expansion around the original ($|0\rangle$) vacuum.

Our point of view has been explained in detail in the previous paper ²⁾. According to it, the scattering amplitude is given by the Reggeon Green's functions corresponding to the $|0\rangle$ vacuum state (with vanishing field expectation values) which is unstable above the critical intercept. The Green's functions corresponding to the stable vacua: $|+\rangle$ and $|-\rangle$ (with the field expectation values $\langle\varphi\rangle=0$, $\langle\varphi^+\rangle=-iv$ and $\langle\varphi\rangle=-iv$, $\langle\varphi^+\rangle=0$, respectively) have no direct physical interpretation because they are evaluated with unphysical boundary conditions which contradict the usual s channel interpretation of the Reggeon calculus. We note that in the formalism of Ref. 1), with the boundary conditions appropriate to the physical scattering, the $|0\rangle$ vacuum is stable and the $|0\rangle$ vacuum Green's functions are well defined. This removes one of the possible objections to our conjecture that the scattering amplitude is given by the $|0\rangle$ vacuum Green's functions. Still this does not prove that this conjecture is necessarily correct. This is because our argument is based on the correspondence between the topological expansion in the underlying theory and the (formal) Reggeon perturbative expansion ²⁾ which are both divergent above the critical intercept.

The method we use is equivalent to calculating the functional integral over the Reggeon fields φ^+ , φ only at certain moments of time: $y_1, y_2, \dots, y_n$ and assuming classical propagation in between. This may be easily understood in terms of the Reggeon diagrams in the (y, b) representation: if we cut all internal lines passing through $y_1, y_2, \dots, y_n$ the diagram must separate into the trees. The quantum effects due to the "largest" loops (those with large rapidity span $\sim y_k - y_{k-1}$) are included. As we expect that the ultra-violet behaviour of the Reggeon field does not affect the infra-red region we are interested in, the contribution of the "small" loops can hopefully be neglected and the above approximation should be valid provided that the time intervals $y_k - y_{k-1}$ are not too large (on the other hand, we want them large enough to use asymptotic formulae for the classical solutions).

Consider the generating functional for the $|0\rangle$ vacuum Green's functions which contribute to the two-particle scattering amplitude at $s = e^Y$:

$$\Gamma_{0,Y}[J^+, J] \equiv \langle 0 | T \exp \left\{ -i \int d^2b [J^+(b) \varphi^+(0,b) + J(b) \varphi(Y,b)] \right\} | 0 \rangle \quad (1)$$

Our method is based on the following representation of Γ :

$$\Gamma_{0,Y}[J^+, J] = D[G, G^+] \Gamma_{0,Y_1}[J^+, G] \Gamma_{Y_1,Y}[G^+, J] \quad (2)$$

and its immediate generalization

$$\Gamma_{0,Y}[J^+, J] = \prod_{i=1}^n D[G_i, G_i^+] \Gamma_{0,Y_1}[J^+, G_1] \Gamma_{Y_1,Y_2}[G_1^+, G_2] \dots \Gamma_{Y_n,Y}[G_n^+, J] \quad (3)$$

Here

$$D[G, G^+] \equiv \exp \left\{ - \int d^2b \frac{\delta}{\delta G(b)} \frac{\delta}{\delta G^+(b)} \right\} \Big|_{G, G^+ = 0} \quad (4)$$

These are exact (if formal) expressions. What we do is assume the classical approximation for all Γ factors on the right side. The $\Gamma[J^+, J]$ as given by (3) corresponds then to the n^{th} approximation. We shall also write $\Gamma^{(0)}$, $\Phi^{(0)}$ for the classical ($n=0$) case.

The Reggeon Green's functions are given by the functional derivatives of $\Gamma[J^+, J]$ with respect to J^+ and J . We can use (2) to write the two-point function as follows

$$\langle 0 | T \varphi^+(0,0) \varphi(Y,0) | 0 \rangle = D[G, G^+] \Phi_{0,0;Y_1}^+[G] \Phi_{Y_1,Y,0}[G^+] \quad (5)$$

Here $\Phi^+[G]$, $\Phi[G^+]$ are the expectation values of the Reggeon fields in presence of the time-localized external sources. For instance,

$$\Phi_{0,0;Y_1}^+[G] = \langle 0 | T \varphi^+(0,0) \exp \left\{ -i \int d^2b G(b) \varphi(Y_1,b) \right\} | 0 \rangle \quad (6)$$

is the expectation value of $\varphi^+(0,0)$ with the absorbing external sources corresponding to the term $-iG(b)\delta(y-y_1)\varphi(y,b)$ in the Lagrangian density functions. In the classical approximation the expectation values Φ^+, Φ become the classical fields $\Phi^{+(0)}, \Phi^{(0)}$. We shall make one more assumption, suggested by the results of Ref. 3) :

$$\begin{aligned}\Gamma_{o,y}^{(o)}[J^+,G] &\approx \exp\{-i\int d^2b J^+(b)\Phi_{o,b;y}^{+(o)}[G]\} \\ \Gamma_{y,Y}^{(o)}[G^+,J] &\approx \exp\{-i\int d^2b J(b)\Phi_{y;Y,b}^{(o)}[G^+]\}\end{aligned}\quad (7)$$

This is supposed to be valid provided that J, J^+ are small enough. We then obtain the $n=1$ approximation for any Green's function in terms of the classical fields alone

$$\langle o|T\prod_{i=1}^K\varphi^+(o,b_i)\prod_{j=1}^L\varphi(y,b_j)|o\rangle \approx D[G,G^+]\prod_{i=1}^K\Phi_{o,b_i;y_i}^{+(o)}[G]\prod_{j=1}^L\Phi_{y_i;Y,b_j}^{(o)}[G^+]\quad (8)$$

This approximation is essentially equivalent to that of Bronzan ⁴⁾. In our notation, his results are

$$\langle o|T\varphi^+(o,g)\varphi(y,b)|o\rangle \approx -\langle\varphi^+\rangle\langle\varphi\rangle\Theta(2\sqrt{\alpha'\Delta}Y-|B|)\quad (9)$$

$$\langle o|T[\varphi^+(o,g)]^K[\varphi(y,b)]^L|o\rangle \approx -\langle\varphi^+\rangle^K\langle\varphi\rangle^L\Theta(2\sqrt{\alpha'\Delta}Y-|B|)\quad (10)$$

neglecting non-leading terms. Here $\Delta=\alpha_0-1$, α_0 is the bare Pomeron intercept and α' the bare Pomeron slope. $\langle\varphi^+\rangle$ and $\langle\varphi\rangle$ are the vacuum expectation values in the tree approximation. In the case of pure three-Pomeron interaction : $\langle\varphi^+\rangle=\langle\varphi\rangle=2\Delta/ig$ where g is the triple Pomeron coupling (note that $\langle\varphi^+\rangle$ and $\langle\varphi\rangle$ refer to two different vacuum states ²⁾).

To see how these results appear in our formalism, we first rewrite the expectation values Φ^+, Φ in terms of the "shifted" fields :

$$\Phi^+[G] = \langle\varphi^+\rangle + \chi^+[G] \quad , \quad \Phi[G] = \langle\varphi\rangle + \chi[G]\quad (11)$$

As $\Phi[0] = \Phi^+[0] \equiv 0$ we have $\chi^+[0] = \langle \varphi^+ \rangle$, $\chi[0] = -\langle \varphi \rangle$. Substituting (11) into (5), we get

$$\langle 0 | T \varphi^+(0, y) \varphi(y, y) | 0 \rangle = -\langle \varphi^+ \rangle \langle \varphi \rangle + D[G, G^+] \chi_{0, y, y_1}^+[G] \chi_{y_1, y, y}^-[G^+] \quad (12)$$

If we assume the functional integral representation for $\chi^+[G]$ and $\chi[G]$:

$$\chi_{0, y, y_1}^+[G] = \int [dz] \tilde{\chi}_{0, y, y_1}^+[z] e^{-\int d^2 k z(k) G(k)} \quad (13)$$

the second term in (12) becomes

$$\begin{aligned} D[G, G^+] \chi_{0, y, y_1}^+[G] \chi_{y_1, y, y}^-[G^+] &= \\ &= \int [dz] \int [dz^+] \tilde{\chi}_{0, y, y_1}^+[z] \tilde{\chi}_{y_1, y, y}^-[z^+] e^{-\int d^2 k z(k) z^+(k)} = \\ &= \int [dz] \tilde{\chi}_{0, y, y_1}^+[z] \chi_{y_1, y, y}^-[z] \end{aligned} \quad (14)$$

Here $\int [dz]$ denotes the functional integration over real functions $z(\underline{p})$. Formally, (13) may be obtained from the usual path integral representation of the field expectation value with $z(\underline{p})$ corresponding to $i\varphi(y_1, \underline{p})$, provided that the integration contour can be rotated to encircle real positive $z(\underline{p})$ axis. This choice of the integration contour may be also obtained from the requirement that the functional integrals are well defined around corresponding classical solutions.

The leading order results (9), (10) follow from rather intuitive picture of the long-range behaviour of the classical Reggeon field. Consider the case of the field $\varphi(y, \underline{p})$ with the external (emitting) sources acting at the time $y=0$. Then $\Phi_{0, y, \underline{p}}[G]$ is the solution of the classical equations of motion with the initial condition $\varphi(0, \underline{p}) = -iG(\underline{p})$ (and the constraint $\varphi^+ \equiv 0$ because there are no absorbing external sources). We are interested in the case $G(\underline{p}) \geq 0$ so that the stable fixed point can possibly be reached.

What we expect is that after the time $\sim 1/\Delta$ (which is enough to reach the fixed point $\varphi \approx \langle \varphi \rangle$ from any initial value except for $\varphi \approx 0$ ³⁾) the large scale structure of the field becomes controlled by the fixed points, that is, either $\varphi \approx \langle \varphi \rangle$ or $\varphi \approx 0$ at $(y, \underline{p})$. The second fixed point is unstable; consequently $\varphi(y, \underline{p}) \approx 0$ is possible only if the

initial conditions have $G(\underline{b}) \approx 0$ everywhere within the correlation distance $\sim 2\sqrt{\alpha'}/\Delta$ from $\underline{b}$. When y increases beyond $1/\Delta$ the $\varphi \approx \langle \varphi \rangle$ region will expand at the rate of one correlation length per one unit of time ($\sim 1/\Delta$), that is like $\sim 2\sqrt{\alpha'}\Delta y$. All this can be made exact in a simple (classical) lattice model with the lattice spacing of order of the correlation length, if the intersite interaction is between the field variables which are averaged over the time $\sim 1/\Delta$. A rigorous justification would require appropriate block spin formalism

We conclude that

$$\Phi_{0;Y,\underline{B}}[G] \approx \Phi_{0;Y,\underline{B}}[\Theta G] \approx \begin{cases} \langle \varphi \rangle & \text{if } \Theta G > G_{\min} \text{ at some point} \\ 0 & \text{if } \Theta G < G_{\min} \text{ everywhere} \end{cases} \quad (15)$$

Here $\Theta(\underline{b})G(\underline{b}) = \Theta(2\sqrt{\alpha'}\Delta Y - |\underline{B} - \underline{b}|)G(\underline{b})$ and $G_{\min} \sim \langle \varphi \rangle e^{-\Delta Y} \approx 0$. As explained before, Y should be greater than $\sim 1/\Delta$. Together with analogous conjecture for $\Phi^+[G]$, (15) is enough to derive (9) and (10) provided that the functional integration in (14) is not dominated by $z < G_{\min}$ (the leading behaviour of the tree diagrams suggests that it is rather $z \sim e^{\Delta Y}$ which dominate the integral).

We can also obtain the many-point functions (8) for any $\underline{b}_1, \underline{B}_j$. The most compact expression is

$$\langle 0 | T \prod_{i=1}^k [\langle \varphi^+ \rangle - \varphi^+(\underline{a}, \underline{b}_i)] \prod_{j=1}^l [\langle \varphi \rangle - \varphi(\underline{Y}, \underline{B}_j)] | 0 \rangle \approx \langle \varphi^+ \rangle^k \langle \varphi \rangle^l \prod_{i=1}^k \prod_{j=1}^l [1 - \Theta(2\sqrt{\alpha'}\Delta Y - |\underline{B}_j - \underline{b}_i|)] \quad (16)$$

In particular, the Bronzan's result (10) applies when $|\underline{b}_i - \underline{B}_j| < 2\sqrt{\alpha'}\Delta Y$ for any i, j . In this case

$$\Gamma_{0,Y}[J^+, J] = 1 - (1 - e^{-i\langle \varphi^+ \rangle \int_A d^2 \underline{b} J^+(\underline{b})}) (1 - e^{-i\langle \varphi \rangle \int_B d^2 \underline{b} J(\underline{b})}) \quad (17)$$

may be used as a generating functional. The integration regions in (17) must be completely within the distance $\lesssim 2\sqrt{\alpha'}\Delta Y$ from each other.

If the external particles correspond to the pointlike sources of the Reggeon field, the scattering amplitude is factorizable. In the case of exponential sources,

$$T_{AB}(Y, \underline{B}) = -(1 - e^{-i\beta_A \langle \varphi^+ \rangle})(1 - e^{-i\beta_B \langle \varphi \rangle}) \Theta(2\sqrt{\Delta} Y - |\underline{B}|) \quad (18)$$

where β_A, β_B are the coupling constants to the external particles. Though (18) is obtained in the $n=1$ approximation, it essentially agrees with the result of more complete quantum calculation¹⁾. The exception is the case of zero transverse dimensions. In this case, the quantum formalism of Ref. 5) yields

$$\Gamma_{0,Y}[J^+, J] = 1 - e^{-\delta Y} (1 - e^{-iJ^+ \langle \varphi^+ \rangle}) (1 - e^{-iJ \langle \varphi \rangle}) \quad (19)$$

where $\delta \sim (\Delta^2/g^2) e^{-\frac{1}{2}(\Delta/g)^2}$ ¹⁾. In the $n=1$ approximation the factor $e^{-\delta Y}$ does not appear (we note that the classical equations of motion can be solved in zero dimensions and the $n=1$ calculations can be done explicitly, with no assumptions). The conclusion is that further quantum corrections must be included.

To account for some of them we calculate the two-point function in the $n=1$ approximation. We shall use asymptotic classical results for $\Gamma_{Y_1, Y_2}[G_1^+, G_2]$ ³⁾:

$$\Gamma_{Y_1, Y_2}[G_1^+, G_2] = \begin{cases} \exp[-G_1^+ (\frac{2\Delta}{g} - \frac{G_1^+}{2})] & G_1^+ < G_2, G_1^+ < \frac{2\Delta}{g} \\ \exp[-G_2 (\frac{2\Delta}{g} - \frac{G_2}{2})] & G_2 < G_1^+, G_2 < \frac{2\Delta}{g} \\ \exp[-\frac{1}{2}(\frac{2\Delta}{g})^2] & G_1^+, G_2 > \frac{2\Delta}{g} \end{cases} \quad (20)$$

The two-point function for $n=2$ is given by

$$\begin{aligned} \langle 0 | T \varphi^+(0, \underline{g}) \varphi(Y, \underline{B}) | 0 \rangle = \\ = -\langle \varphi^+ \rangle \langle \varphi \rangle + \int [dz] \int [dz^+] \tilde{\chi}_{0, \underline{g}; Y_1}^+[z] \tilde{\chi}_{Y_2, \underline{B}}[z^+] \Gamma_{Y_1, Y_2}[z, z^+] \end{aligned} \quad (21)$$

In zero transverse dimensions the functional integrals $\int [dz]$, $\int [dz^+]$ are replaced by usual integrations over z, z^+ . The integral transforms $\tilde{\chi}, \tilde{\chi}^+$ are

$$\tilde{\chi}_{0,y}[z] = \tilde{\chi}_{0,y}^+[z] = \frac{2\Delta}{ig} \left[\frac{e^{-\Delta y}}{1 - e^{-\Delta y}} \delta(z) - A_y e^{-z A_y} \Theta(z) \right] \quad (22)$$

where $A_y = (2\Delta/g)(e^{-\Delta y}/1 - e^{-\Delta y})$. When y_1 and $Y - y_2$ in (21) are large enough the integral is dominated by large z, z^+ so that $\Gamma[z, z^+]$ is a constant. The result is

$$\langle 0 | T \varphi^+(0) \varphi(Y) | 0 \rangle \approx - \langle \varphi^+ \rangle \langle \varphi \rangle (1 - e^{-\frac{1}{2}(\frac{2\Delta}{g})^2}) \quad (23)$$

It coincides with the quantum result : $-\langle \varphi^+ \rangle \langle \varphi \rangle e^{-\delta Y} \approx -\langle \varphi^+ \rangle \langle \varphi \rangle (1 - \delta Y)$ provided that $Y \sim g/\Delta^2$. At higher Y the $n=2$ approximation is not sufficient.

To get some results for higher n it is convenient to use the $n=1$ results in (2), (3) instead of the classical ($n=0$) ones. For example,

$$\Gamma_{0,Y}[J^+, J] = D[G, G^+] \Gamma_{0,Y_1}^{(1)}[J^+, G] \Gamma_{Y,Y}^{(1)}[G^+, J] \quad (24)$$

reproduces essentially the $n=2$ result (23). The only difference is that instead of $e^{-\frac{1}{2}(2\Delta/g)^2}$ we have $e^{-(2\Delta/g)^2}$. This is because of the approximation (7) and can be remedied.

For arbitrary n we obtain

$$\Gamma_{0,Y}[J^+, J] = 1 - (1 - e^{-v^2})^{n-2} (1 - e^{-iJ^+ \langle \varphi^+ \rangle}) (1 - e^{-iJ \langle \varphi \rangle}) \quad (25)$$

($v = 2\Delta/g$). The extra factor $(1 - e^{-v^2})^{n-2}$ we identify with $(1 - Y_0 \delta)^{Y/Y_0} \approx e^{-\delta Y/Y_0}$ where $Y_0 \sim g/\Delta^2$. This agrees with (19) provided that we always use the approximation corresponding to $n \sim Y/Y_0$ (higher n approximation should not be used because we must keep Y_0 large enough to justify the asymptotic form of all classical or $n=1$ factors).

We can do similar calculation in two transverse dimensions. The $n=1$ result for $\Phi[J^+]$ can be obtained from (17) :

$$\Phi_{0,Y,B}[J^+] = \langle \varphi \rangle \left(1 - e^{-i\langle \varphi^+ \rangle \int d^2b J^+(b) \Theta(2\sqrt{\alpha'} Y - |B-b|)} \right) \quad (26)$$

[again in the approximation (7)]. The $n=2$ two-point function is then

$$\langle 0 | T \varphi^+(a, \underline{b}) \varphi(Y, \underline{B}) | 0 \rangle \approx - \langle \varphi^+ \rangle \langle \varphi \rangle \left(1 - e^{-i\langle \varphi^+ \rangle S} \right) \Theta(2\sqrt{\alpha'} Y - |B|)$$

where S is the area of the region $|\underline{b}| < 2\sqrt{\alpha'} \Delta y_1$, $|B - \underline{b}| < 2\sqrt{\alpha'} \Delta (Y - y_1)$. We see that, contrary to (23), the extra term vanishes as $Y \rightarrow \infty$. Unfortunately the higher n calculation is much more difficult in this case [we cannot use Γ_{y_1, y_2} as given by (17) because of the restriction on the integration regions].

We summarize the conclusions as follows. First, the method we use is formally equivalent to summing the usual Gribov diagrams (with no unphysical tadpoles²⁾) and it is not changed by our approximations. The results are in agreement with those of Ref. 1). This we regard as a confirmation that the quantum formalism of Ref. 1) preserves the formal relation to the $|0\rangle$ vacuum perturbative expansion, as proposed in Ref. 2).

Another point is that our procedure for quantum corrections (with Bronzan's results as a first non-trivial order) is able to produce the $e^{-\delta Y}$ factor in zero dimensions⁵⁾ while the usual loop expansion is not⁶⁾. Finally, from the expressions like (5) which involve the expectation values with different boundary conditions (Φ is evaluated with only emitting external sources and Φ^+ with only absorbing ones) we can see how the asymmetric fixed points (the $|+\rangle$ and $|-\rangle$ vacuum states) affect the $|0\rangle$ vacuum Reggeon Green's functions. The results of Bronzan⁴⁾ emerge in a natural way.

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